Collaboration and Competition in Interest-Driven Cyber Ecosystems

by

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Abstract

The next-generation cyber ecosystems are characterized by novel distributed paradigms composed of various network entities, such as individuals, devices, and software. The efficiency, scalability, and security of the systems highly rely on the interconnection of these autonomous and intelligent entities. These coexisting entities can be cooperative, selfish, or malicious, and make local-information-based decisions on their own behalf. Conventional optimization methods or security approaches dealing with pure engineering issues are either inefficient or impractical when applied to decentralized architectures in practice. Therefore, integrated approaches combining multiple scientific and engineering disciplines, including engineering, economics, and operations research, are required to transform the vision of modern cyber system paradigms into reality.

This research addresses the practical concerns of the cyber ecosystem entities, analyzes their complex interactions and develops multi-agent models using mathematical tools including optimization and game theory. The research goal is to support interdisciplinary decision-making in real-world cyber systems. First, I propose a flexible network infrastructure sharing framework to incentivize cooperation between operators in a volatile and competitive market. Second, I explore the attackers' profit-driven behaviors and study the economic value of traded target information in security games. Finally, I design an incentive mechanism for crowdsourcing to facilitate the cooperation between the service providers and the service users.

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Chapter 1

Introduction

The modern cyberspace, like the other ecosystems, is a distributed organism that is composed of a variety of diverse participants-devices, individuals, firms and governments. Prominent examples of these ecosystems are IoT-based health monitoring systems, autonomous vehicles, and smart grids. The efficiency, scalability and security of the system highly rely on both the behaviors of and the relationships among the entities: First, these coexisting entities are self-interested, which means that their behaviors are driven by their own interests and their decisions are made based on local information. Second, these autonomous and intelligent entities, who generate and also consume data in the system, are closely interconnected [1]. They can be collaborative and/or competitive. Such characteristics require a comprehensive and interdisciplinary understanding of the cyber ecosystem when dealing with the emerging engineering and economic issues. Conventional pure engineering approaches are very limited due to the neglect of the heterogeneous interest-seeking behaviors and the complex interactions in the ecosystem. They tend to be either inefficient or impractical when applied in practice. For example, ad-hoc vehicle network designs which overlook the non-cooperative behaviors of selfish or malicious nodes are vulnerable to threats that can lead to life-threatening circumstances. Similarly, in IoT systems, a crowdsourcing mechanism that disregards the heterogeneous privacy concerns of the participants would result in free-riding behaviors which may cause the failure of the group decision support system.

Therefore, integrated approaches combining multiple disciplines, including engineering, economics and operations research, are required to transform the vision of cyber ecosystem paradigms into reality. Such approaches can, in principle, explain underlying economics, provide realistic mechanism designs and guarantee the network sustainability. The goal of this research is to support interdisciplinary decision-making in real-world networks by exploring the incentive-driven behaviors and dynamic interactions and developing multi-agent models that generate practical and reliable solutions for the cyber ecosystems.

The focus of this research is to study the collaboration and competition issues in the cyber ecosystems using mathematical tools such as optimization and game theory. In particular, we are interested in the collaborations in three domains of the cyber ecosystem: network infrastructure sharing, information trading in cyber security, and data sharing through crowdsourcing in Internet of Things systems. The key contributions of this research are as follows:

- Developed a utility-based framework representing the participants' attribute profiles and preferences by integrating economic theory with engineering practice. The framework identifies the endogenous incentives and interests of selfish individuals in the network, constructs their utility functions and models the structured relationship between them. It allows us to understand the roles of different entities and evaluate the practicability and stability of the network.

- Developed a game-theoretical model to represent the interactive behaviors and optimize the strategic and operational decisions of the network entities. The key idea is to seek potential system-wide solutions by finding the equilibrium behaviors of the individuals, such as the operators' marketing strategies, and the hacker's attack strategies in collaborative attacks. It gives a new dimension to the characterization of the network behavior and sustainability.
- Devised a mechanism that significantly improves the practical efficiency and performance of the cyber ecosystems. The mechanism provides incentives to promote global optimal behaviors, flexible cooperation strategies, resource and profit sharing schemes.

All of the work is devoted to find state-of-the-art solutions to issues surrounding the next generation of cyber systems. It addresses the agents' practical concerns such as their economic payoffs, especially when they have common or conflicting interests, enabling us to improve the performance, reliability, and robustness of the innovative distributed systems.

The rest of this dissertation is organized as follows. In Chapter 2, the related literature is reviewed. In Chapter 3, we study the network infrastructure sharing framework to incentivize cooperation between operators in a volatile and competitive market. In Chapter 4, we study the attackers' profit-driven behaviors with a particular emphasis on the role of traded target information. Chapter 5 studies the collaboration between service providers and service users in the form of crowdsourcing and the incentive mechanism design.

Chapter 2

Literature Review

2.1 Network Infrastructure Sharing

In recent years, NIS has emerged as an important research area with interest from both academic and industrial. Existing literature on sharing models have identified a wide range of factors that determine how network infrastructure are allocated and affect the effectiveness of sharing strategy. Current studies typically focus on examining them in one of the two categories: service market profile and infrastructure market profile. The studies in the former include but are not limited to the service demand pattern, market shares, regulations on the market concentration, interference and traffic cost [2, 3, 4]. For instance, [5] argues that the market shares of the operators should be taken into consideration to guarantee coalition stability, but most models assume fixed market shares or user numbers [5, 6, 7]. [8] shows that the existence of tight competition regulations on the market concentration could reduce the benefits of jointly updating and managing the operators' networks, while how the market competition among service providers affects their network resource sharing decisions has not been investigated. The studies in the later refer to the relationships among the operators which cannot be avoided when multiple operators are considered in resource sharing or trading market. The existing literature normally considers the price competition/cooperation among multiple resource suppliers [9, 10, 11, 7]. Our study tries to investigate both the competition in services and the cooperation in resource sharing. Moreover, how the factors in both markets, including the uncertain demand risks, the long-term backup contract design, and the potential redundant resource capacity, affect the sharing decision configurations and economic performances is explored in our research.

In determining how the resources or benefits should be allocated among the operators in NIS, one stream of research, mostly in the area of spectrum sharing for unlicensed bands, looks for the sharing rules that lead to fairness and efficiency [12, 13, 14, 15]. Another stream of research conducts a cooperative game approach [5, 16]. However, many challenges still remain in real world NIS, which include preserving a liable, mutually beneficial relationship for the operators, reserving an appropriate backup capacity and ensuring supply in cases of uncertain resource demand. To combat these challenges, we generalize and develop the well-structured partnership between partners to ensure supply availability in NIS mechanism under demand uncertainty. Different from many papers in the literature, our interest is in the interactions between the operators participating in both the resource sharing cooperation and the service competition. Through backup contract design and parameter optimization, the operators' effort and willingness to collaborate should increase.

The long-term contract design between operators in NIS is still relatively under-explored. There has been some work that relies on different types of contracts to incentivize spectrum sharing, like revenue sharing [17], and insurance contracts [3]. However, the contract parameters are assumed to be exogenous, which determine how the economic benefits are allocated between the partners. Our work tries to optimize the contract parameters for a realistic profit allocation mechanism. [7] obtains the optimal service prices to maximize the cross-carrier VNO's profit, but how the resource allocation can be implemented through pricing for maximum ISPs' profit are not fully studied. Of particular relevance is the work of [18] on the spectrum reservation contract. Their contract is assigned between two different types of players: one third-party broker and one unlicensed white space device; while our model focuses on the co-opetition relationships between two operators providing similar services to the users. An altogether different approach to the spectrum sharing/trading problem is the one taken by the mechanism design literature. There, the primary users offer a direct mechanism that allocates its resources as a function of secondary users' reports of their private information, such as transmission efficiency [19], and preference for a given spectrum quality [20]. The contract parameters are optimized to minimize the impact of the private information for the resource providers. Our work contributes to this literature by applying the theory of backup contract design to the problem of network infrastructure sharing in an uncertain and competitive environment. Our contract-based approach provides a richer form of characterizing relationships among operators than the previous auction-based approach, and enables us to evaluate the feasibility and effectiveness of cooperation among competing operators.

2.2 Information Trading in Black Markets

Much of the research on security game has assumed that the attacker makes a perfect observation of the defense policy over potential targets and therefore been able to explore the value of commitment for the defender in a Stackelberg game framework [21]. Realizing that this assumption rarely holds in real-world domains, existing studies are turning their interests into the scenario of incomplete, inaccurate, or uncertain information. Some work has proposed the version of the security game with bounded memory [22] or imperfect observations [23]. Others have assumed that the target information gained by the attackers can be learned more accurately by conducting a period of surveillance [24, 25]. A more recent study which discusses the defender's strategic revelation of its commitment, further shows the importance of target information [26]. However, none of the above studies consider the possibility of purchasing information from a data broker in black markets. The value and the impacts of such an information service have hardly been addressed. Although there is already a study evaluating the value of customer information for the retailers in the consumer market [27]. Their results cannot be applied to the security problem because the target in the security problem may not be exclusive to the attackers as the set supply of merchandise is to the consumers.

This work focuses on the information market in the context of the hacker community. The hacker community is both devastating and prevalent because it facilities cooperation and allows for specialization among attackers, leading to more advanced and more economically efficient attacks. We can discern a growing interest among researchers in the enigmatic hacker community. Some studies have focused on the organization of the community, such as identifying the key actors [28], discovering the types of collaborative attack patterns [29] and evaluating its sustainability [30]. Others provide a window into the society by microscopically analyzing the behaviors of the attackers, mostly addressing their cooperation in the form of a coalition. Current studies assume that the attackers are heterogeneous with respect to non-task-specific efficiency, resource allocation, or skill sets, and a coalition is formed for more attacks or to gain higher total utility [31, 32, 33]. But the format of collusion with labor specialization, especially the information service, which is universal in the hacker community, has not been fully explored. More specifically, the questions are not studied yet about how the attackers would benefit from information assistance, and what is the bargaining process that determines their reward allocations. The answers to these questions are crucial to investigate why and how information service is provided in the hacker community, as well as when such a cooperation is formed among profit-driven attackers. Besides, the competition among attackers for the limited resource pool is another factor that impacts the attacking decisions and rewards, while it is usually ignored in the existing research, except in [34]. In an attempt to fill the gap in the current literature on the incentives of complex behaviors in the hacker society, this research takes into consideration both the cooperation among attackers specialized in different tasks and competition among similar attackers. Specifically, we analyze the interactions between a data broker and two competing and/or cooperative attackers through a multi-stage game approach. The value of information is derived and the impacts of such information service are evaluated.

2.3 Incentive Mechanisms for Crowdsourcing

With the proliferation of crowdsourcing platforms and applications, numerous results have been presented on the benefits of and the incentive mechanism for crowdsourcing or crowdsensing [35, 36]. Most of the existing research consider the interactions between one crowdsourcer and multiple potential contributors. The contributors, usually regarded as a group of users different from the service consumers, make the decisions regarding to crowdsourcing participation. The objective of the crowdsourcer (either a crowdsourcing platform or a service provider) is scenario dependent. It ranges from attracting more user participation[37], ensuring trustworthy data [38, 39], minimizing the total cost of compensating participants[40], up to maximizing the number of tasks performed [41]. Incentives used for user participation include financial incentive, entertainment, social/ethical [42, 38], information, and educational opportunities [41, 43]. A large body of research work has been devoted to studying the effectiveness and performance of using financial reward in crowdsourcing incentive mechanisms.

Papers that are closer in spirit to ours are those work about service as incentives [44], where the service is provided to the contributor as an incentive. Hoh *et al.* [45] introduce system credits to stimulate the participation of mobile users for contributing parking information. In [46], a service provider grants each user a service quota based on the user's contribution for maximizing fairness and social welfare [46]. Chou *et al.* [47] propose a virtual-credit-based incentive requiring that a participant can only obtain data uploaded by the others by paying credits, which can be earned by uploading its own sensor data. The basic idea of these work is that the amount of service each user can consume is dependent on its contributions. While this service-as-incentive framework works in many applications, in other scenarios when a service provider make a profit by selling its services, it is not desirable to limit the quantity of service supplied. Instead, our work considers the case when the service quality experienced by all the users are dependent on the total contributions. This situation is suitable for many crowdsourcing tasks, such as data sensing and content creation.

Different from the existing research that only discusses the benefits of crowdsourcing to the service, our research focuses on the impacts of consumer-contributed information on the service and further on the market. The incentive mechanism for crowdsourcing is proposed with the consideration of the consumers' intrinsic demand. Specifically, our study differs from the existing research in the following three profiles.

- *User profile:* By exploring the dual role of the users, we add the service quality into the users' utility function. And we consider user heterogeneity in two dimensions: privacy concern and service usage level. Each user makes decisions of whether to subscribe to and contribute to the service. This is different from most previous work which considers the users' heterogeneity and decisions only related to participation in crowdsourcing, such as cost [48, 49, 39], locations [50], and the number of tasks they can compete [41].

- *Service profile:* One central feature of our model is the endogenous nature of the service quality; once this endogeneity is taken into account, we show that, the consumers will have an intrinsic incentive for crowdsourcing due to the effect of positive network externality. This feature is consistent with many applications that rely on crowdsourcing to provide services, such as Waze [51].

- *Crowdsourcer profile:* Most crowdsourcing incentive models assume that the utility function of the crowdsourcer is a non-decreasing function of the participants' aggregated contributions minus the total reward given to the contributors ([37], [52]). Whereas in our study, the crowdsourcer is also a service provider, whose aim is to maximize its expected profit from selling its services. This formulation better fits the practice of a profit-seeking service provider who relies on crowdsourcing to reduce cost or improve service quality. Besides, none of the aforementioned studies have considered the competition between service providers, the impact of which is evaluated in our work.

In summary, the main contributions of this research are that it exploits the users' dual roles and combines both a financial reward and users' intrinsic demand for better services into the incentive mechanism. We incorporate the endogenous nature of the service quality and prove the convergence of the market dynamics under some mild conditions. Then the optimal reward to maximize the service provider's expected profit is analyzed. Finally, the impact of competition between the service providers is evaluated.

Chapter 3

Backup-reservation-based Network Infrastructure Sharing

3.1 Introduction

Network infrastructure sharing (NIS) is commonly believed to be a cost-effective solution for mobile network operators (MNOs) to quickly roll out new services, increase coverage, deploy new technologies, and improve resource utilization in a dynamic and uncertain network environment. NIS has been commonly referred to as the sharing of network infrastructure, resources, and functions among operators, such as supporting infrastructure including site locations, power supply, shelters, antenna masts, as well as spectrum and processing capacities of base stations [53], etc. The potential benefits of NIS have been well recognized by both MNOs and major standards developing organizations (SDOs) such as 3GPP and ITU-T, and its standardization and deployment are underway. For example, 3GPP Release 13 specifies several RAN (radio access network) sharing architectures to expedite new service rollout and reduce system upgrade cost [54]. MNOs throughout the world, such as AT&T, T-Mobile, and Verizon, have participated in network sharing. The FCC has adopted new rules for millimeter wave spectrum among multiple MNOs [55]. The trend of NIS is accelerating in the era of 5G due to its strong potential for cost saving [56, 57], e.g., China Telecom and China Unicom have recently reached an agreement in deploying a 5G network by sharing their network infrastructure [58].

Despite the obvious motivation to deploy NIS, the benefits of NIS have been hindered by various strategic and operational factors. The operators need to consider the following challenges that will affect the economic incentives to implement NIS. Firstly, conventional models of resource sharing, especially spectrum sharing in buy-in situations, are typically based on auction mechanisms. The organization and execution of auctions, however, are complicated, time-consuming, and may incur high overhead due to the amount of exchanged information and coordination required between the auctioneer and bidders. As a result, auction is often considered impractical, if not infeasible, for resource sharing on a short timescale, which could have been more desirable and useful sharing mechanisms for operators dealing with highly dynamic and uncertain user demand. To reduce sharing overhead and ensure rapid reaction to network dynamics, novel sharing models need to be explored.

Furthermore, due to the profit-driven nature of MNOs, the optimal amount of resources that should be shared among operators is not only an engineering decision problem depending on QoS requirements of end users, but also an economic one that targets at offering mutual benefits among all participating operators. Such a problem is difficult to solve beforehand due to the uncertainties and dynamics of users' demand and resource availability. In particular, a priori over-investment in sharing would result in resource wasting and reduced profit, while under-investment cannot guarantee a satisfactory QoS especially in high traffic demand. As such, instead of pursuing a conventional performance-oriented stochastic resource optimization framework, as has been well-investigated in the literature [59, 60], a novel network-economic sharing model taking into account traffic and network uncertainties from both engineering and economic perspectives becomes indispensable.

So far, the issue of modeling and optimizing competition among operators in an NIS framework has not been well addressed. Most existing sharing models implicitly assume that operators serve independent markets (i.e., user populations). In reality, however, competition for resources and users among MNOs is complex and how to analyze and incentivize the resource sharing is still an open problem [55]. When competition exists, it is not yet clear whether the resource owner always has the incentive to share its resources with the competitors or not, due to the risk of damaging its own market share and profit. Furthermore, even when sharing takes place, how the profit, if any, should be split among the competing operators is also a key question that is yet to be answered.

In an attempt to address the above challenges, in this section, we propose a novel contract-based backup reservation model for NIS in an uncertain and competitive market. In our model, the MNO with overloaded traffic demand (for example, a virtual network operator, VNO) can request another MNO (i.e., owner) with excessive resources to reserve a certain amount of resources for future sharing. For the resources that have been reserved in advance, the VNO can use as much as it wants to meet the need of its users, while for the capacity that has not been reserved, the owner offers no guarantee of sharing. Depending on the actual demand, the VNO may not use all the reserved capacity [61] (hence, the term “backup"). In this case, the owner MNO can use the leftover capacity if needed. Such a resource sharing strategy improves the flexibility in handling uncertain demand, resulting in improved capacity and resource utilization efficiency [62], [63], with reduced operational overhead. We study such an NIS scheme when adopted by one VNO with uncertain demand and one owner with resource surplus. One potential application of our model corresponds to the scenario where a VNO with relatively limited resource supply, wishes to access the resources and infrastructure of a major MNO by signing a backup reservation contract. To reduce the complexity in designing service level agreement (SLA) and implementation, pairwise contract between two operators, has already been commonly adopted in the wireless industry. For example, the Ultra Mobile (an VNO) signed a sharing contract with T-Mobile [64], and the VNO AirVoice Wireless has made an agreement with AT&T [65]. In fact, contracts between two entities in supply chains have been adopted in a wide range of industries, while those involving three or more agents are not so popular because of the difficulty of designing the multi-operator contract and/or implementing it. The contract for a multi-echelon supply chain or between multiple operators with more complex business relationships and bargaining power is beyond the scope of this work. We will discuss issues when extending the proposed solution into more general scenarios in Section 3.9.

Our main contributions are briefly summarized as follows:

First, to provide more flexibility in NIS and adaptation to demand/supply uncertainty, we introduce the concept of backup reservation into the sharing game and propose a novel contract mechanism to support efficient and profitable NIS. The equilibrium contract parameters related to the operators' sharing decisions under demand uncertainties are derived.

Second, we tackle the issue of incentivizing multi-operator cooperation for NIS in competitive market environments. The conditions under which operators will benefit from such a resource sharing scheme are examined, which include the competition intensity between operators, the service price and level, and demand variation faced by different operators. Our work is among the first to compare NIS decisions under independent and competitive market scenarios.

Third, we propose a comprehensive solution to optimize the operator's strategic long-term market planning related to service positioning together with the operational planning related to the NIS decisions for maximum profits. Though most researchers study purely operational decisions related to NIS scheme, the operator's long-term marketing strategy and its resource sharing decisions influence each other. To study the impact of such an NIS scheme, we propose a three-stage Stackelberg game approach to model the operators' strategic and operational decisions and incorporate the outcome of bargaining in backup reservation NIS into the marketing planning process. This is different from existing works that view the network infrastructure market separately from the wireless service market. To the best of our knowledge, this work is the first to quantitatively analyze how the sharing scheme in resource market affects the operator's marketing strategy and how competition in service markets affects the resource sharing decisions.

Finally, we show that under a backup reservation contract, if the VNO's service price is higher than that of the owner's, the owner is willing to take a risk of losing its own users to share the resources. This is in contrast to the finding in [66] in which the authors state that setting aside resources (e.g., spectrum) exclusively for secondary users will likely impinge on the current users of the primary system and may not be in the interest of its business model. Our numerical results also reveal that NIS through backup reservation leads to both increased resource utilization and improved target service level for

VNO users. These benefits for the whole system can be further improved in a competitive scenario.

Our findings provide insights into both the engineering and the economic aspects of NIS. The framework proposes a guideline for operators to determine their NIS partners and contracts that lead to an efficient and profitable cooperation in a volatile and competitive market. Our work also provides insights on optimizing the market strategies when NIS is implemented. Note that to make our description more concrete, our presentation is based on the case of NIS. However, our model can be directly extended into more general context related to multi-MNO resource sharing.

3.2 System Model and Problem Formulation

3.2.1 System Overview

Our model applies to a network that consists of multiple cells/areas. We consider a VNO that has no or limited resources and will have to rely on renting resources from other operators to provide wireless services to end users in n service areas, $n \geq 1$. Throughout, we add the index i to quantities associated with service area i, $i = 1, ..., n$. In a given service area, the anticipated number of users as well as the average traffic demand is uncertain. For a given target service level, if the VNO's available resources cannot meet the demand of all the users, the unsatisfied users will be ignored or will migrate to the VNO's competitors, resulting smaller market share and reduced profit. The VNO can however obtain more of the resources by negotiating for a backup reservation contract with a wireless service provider that has sufficient resources (the resource-owning operator, ROO). In the contract, the VNO reserves a certain amount of resources capacity from the ROO at a reservation price, and then uses the resources to serve its users when necessary, as illustrated in Fig. 3.1. As such, the ROO needs to determine the reservation price to maximize its own expected profit and the VNO encounters a decision about whether it would be worthwhile to make a reservation or not and how much to reserve.

We consider a marketing-planning period of T selling cycles.[1] The model's timing proceeds as follows. At the beginning of the period, the VNO selects its target market by determining its target service level and the corresponding service price. The marketing strategy stays fixed in the whole period. Then in each selling cycle $t = 1, ..., T$, before the demand is known, the ROO sets the unit resource reservation price, and the VNO determines its reservation quantity as a response. After the demand at the VNO is realized, the VNO uses the reserved capacity to meet its own demand if needed. If the reserved part of the resources is not used by the VNO, the ROO can still use it to serve its own traffic demand. However, if the VNO needs the reserved resources, higher priority should be given to its users. That is, the VNO is guaranteed the share of the reserved capacity at the ROO. Finally, demands are satisfied, or the potential users are lost due to the lack of resources for supporting the target service level. It is notable that our model allows for dynamics taking places at different timescales: the market positioning or pricing process is performed at a relatively large time period (called the market-planning period); while the resource reservation and sharing processes are performed more frequently at a relatively small time period (called the selling cycle). The reservation decisions are made based on the demand distribution in each selling cycle. Although it is more desirable to consider

[1]The duration of one marketing-planning period is decided by the VNO's long-term plan for the market strategy, e.g. one year. A selling cycle is the minimum time scale, e.g., one month per cycle. The user demand remains unchanged in one selling cycle, but may change across cycles.

how past reservation decisions may affect future user behaviors and the long-term profits, in this work we assume that the decision of $R_{i,t}$ is optimized to maximize the expected profit in each selling cycle t. In other words, we assume that a decision made in the current selling cycle will not impact the demand of future cycles. This treatment allows us to focus on the impacts of competition and makes the problem tractable. The more challenging scenario of time-dependent decision making with the consideration of long-term consequences of reservation decisions will be studied in a future work.

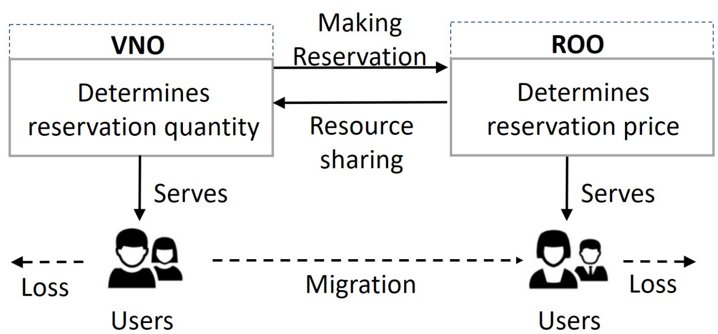


Figure 3.1: Backup reservation scheme between VNO and ROO.

3.2.2 Operators' Resource Capacity

Considering the heterogeneous demand in different areas, our model assumes that the operators' capacities differ across different areas. Therefore, the optimal reservation decisions are made differently in specific to each servicing area, while the target service level is made for the whole network as a marketing strategy. We are assuming each service area is sufficiently large (for example, a service area covers a whole city), so that an arbitrary location in area A is sufficiently far away from an arbitrary location in area B. Under such a setting, the assumption of independent service areas is reasonable in general, because

Table 3.1: List of Notation for Chapter 3.

Notation	**Explanation**
$M_i^{(V)}$	VNO's resource capacity
$M_i^{(R)}$	ROO's resource capacity
S_i	ROO's idle capacity
$l^{(V)}$	Avg. # of users sharing one unit resource at the VNO
$l^{(R)}$	Avg. # of users sharing one unit resource at the ROO
$p_c^{(V)}$	VNO's service price per user
$p_c^{(R)}$	ROO's service price per user
$p_r^{(V)}$	VNO's service price per unit resource
$p_r^{(R)}$	ROO's service price per unit resource
$x_i^{(c)}$	Potential number of users at the VNO
x_i	Potential resource demand at the VNO, with mean u_i
R_i	Reserved capacity level (a decision variable for the VNO)
w_{ri}	Unit reservation price (a decision variable for the ROO)
w	Unit capacity usage fee paid by the VNO
θ	competition intensity between operators
$\Pi_1^{(V)}$	VNO's expected profit without backup reservation
$\Pi_1^{(R)}$	ROO's expected profit without backup reservation
$\Pi_2^{(V)}$	VNO's expected profit with backup reservation
$\Pi_2^{(R)}$	ROO's expected profit with backup reservation
$\Pi_{CM}^{(V)}$	VNO's expected profit under coordination mechanism
$\Pi_{CM}^{(R)}$	ROO's expected profit under coordination mechanism
$\Delta\Pi$	Total improved profit as the benefit of backup reservation
β	Two operators' target service level difference, $\beta = l^{(V)}/l^{(R)}$
P	Average received power
n_0	Noise power
h	Steepness of the users' satisfactory curve

network infrastructures and resources only have a local scope. For example, the occupied/idle status of a given frequency channel in two geographically distant areas are likely to be independent from each other, and therefore the idle capacity in one area may not be idle in a different service area. There are some special/corner cases in which service areas are not independent. For instance, two adjacent areas may have correlated traffic, if most of the traffic originating from one area terminates in the other. We do not consider such corner cases in this work and leave it for future research.

In a given service area, the VNO's own network resource capacity is denoted by $M_i^{(V)}$, $i = 1, \cdots, n$. For the ROO, we consider a wireless service provider that operates in the service areas and that has developed a group of loyal customers or comparatively stable expected demand. This ROO has sufficient resource capacity to meet its own constant demand. Denote its total resource capacity as $M_i^{(R)}$ and the redundant capacity as $S_{i,t}$. The ROO typically allocate more resources in network planning than what are needed by the actual traffic load for two reasons. Firstly, over-provisioning is a commonly used strategy to ensure acceptable network performance against traffic variations caused by equipment failures or transient traffic surges [67]. In addition, as part of the service provider's expansion and growth plan, over-provisioning is a conservative approach to meet the needs of future growing demand in the long-term.

In the backup reservation contract, the VNO and its users have a higher priority in using the reserved capacity over the ROO's own users. If the ROO overly commits its capacity to the reservation (i.e., a reservation quantity larger than $S_{i,t}$) with not enough capacity left (i.e., less than $M_i^{(R)} - S_{i,t}$ units) for its own users, then the ROO may experience capacity starvation and partial user loss. This requires the two operators to carefully decide their reservation contract parameters to ensure mutual benefits in resource sharing. As in [68], the notion of infrastructure is quite general. It can be composed of resources such as links, servers, and buffers. Its quantity might be the bandwidth of a communication link or the cycles available in a computational grid.

3.2.3 Target Service Level

In the VNO's long-term market planning, it needs to determine a target service level that is related to its quality of service. A higher target service level indicates a higher service quality, and vice versa. Note that the concept of service level is general. It is not affected by any particular medium access control (MAC) protocol, but instead captures the basic behavior of all MAC mechanisms. This target service level is announced by the VNO to the market as an average service level, and is not necessarily equivalent to the instantaneous level experienced by every user at any given time. So it is reasonable to expect that when there is a traffic demand burst from users, the instant service level may deteriorate.

Intuitively, the service level or QoS enjoyed by a user deteriorates with the number of users accessing the same resource. Therefore, we represent the VNO's target service level as the average number of its own users, $l^{(V)}$, that will consume one unit of resource capacity. For example, the VNO may decide that an average traffic demand of five users can be satisfied by one unit of network resource ($l^{(V)} = 5$). The users' satisfaction level is closely related to $l^{(V)}$. The smaller the value $l^{(V)}$, the higher service level VNO offers, and the higher satisfaction level users would have.

Let $U(l^{(V)})$ be a measure of the users' average satisfaction level. We assume $\frac{dU}{dl^{(V)}} < 0$, $\frac{d^2U}{dl^{(V)2}} < 0$. For example, if one band of spectrum is shared among $l^{(V)}$ users, the average data rate is $\ln(1+\frac{P}{n_0+P(l^{(V)}-1)})$, where P is the average received power and n_0 is the noise level. The satisfaction level U can be defined as an increasing function of the difference between the achievable data rate and the users' data rate requirement. In line with [69, 70], we use the sigmoid function, which has been widely used to approximate the user's satisfaction with respect to service qualities [71]. Specifically,

$$U(l^{(V)}) = \frac{1}{1+e^{-h[\ln(1+\frac{P}{n_0+P(l^{(V)}-1)})-\ln(1+\frac{P}{n_0+P(l^{(V)}_{\max}-1)})]}} \tag{3.1}$$

where $l^{(V)}_{\max}$ is the maximum value of $l^{(V)}$, which specifies the lowest service level users can accept; and h is the steepness of the satisfactory curve [69]. We suppose $l^{(R)}$, a constant, is the average number of users sharing one unit of resource at the ROO, which has already been strategically determined [2].

3.2.4 Service Pricing

For each user, the VNO charges price $p_c^{(V)}$ as the service fee. This is equivalent to an average price per resource $p_r^{(V)} = l^{(V)}p_c^{(V)}$. Our model allows the VNO to adjust its price $p_c^{(V)}$ but it also needs to change the corresponding service level at the same time. That is, in maximizing the VNO's expected profit, the price $p_c^{(V)}$ is supposed to match its service level. The reason is that: it is the market that determines that the service price should reflect the quality of the service. That means an operator cannot arbitrarily increase the price without improving the service. Otherwise, the operator would be at the risk of losing its potential users. Therefore, the service price can be set as the users' willingness

[2]Our model allows the possibility of changing the ROO's target service level by taking into account a longer marketing period than that of the VNO.

to pay for the service, and normally increases with the service level. It is reasonable to assume that $p_c^{(V)} = \alpha U(l^{(V)})$ where α is the equivalent service value per degree of users' satisfaction. There is a tradeoff for the VNO's decisions: a higher price indicates a higher target service level and thus more resource consumption, which implies a larger user loss rate with limited resource capacity, but it also means a higher marginal revenue. The ROO's service price is $p_r^{(R)}$ per unit resource, and on average the traffic of $l^{(R)}$ users consume one unit of resource capacity.

3.2.5 Demand Uncertainty and Lost Demand

On the VNO's side, the number of its users during a selling cycle t is denoted by $x_{i,t}^{(c)}$. We assume $x_{i,t}^{(c)}$ is a random variable, and its distribution can be obtained through either an extensive market investigation based on publicly available data and customer surveys or data mining on sale history. Under a service level $l^{(V)}$, the total resource demand can therefore be written as $x_{i,t} = x_{i,t}^{(c)}/l^{(V)}$ units per cycle. Let $F_{i,t}(\cdot)$ and $f_{i,t}(\cdot)$ be the cumulative distribution function and probability density function of $x_{i,t}$, respectively, with $\mu_{i,t}$ be its expected value. The distribution depends on both the distribution of $x_{i,t}^{(c)}$ and the service level. For instance, if $x_{i,t}^{(c)}$ follows a uniform distribution $U(a_0, b_0)$, then $x_{i,t}$ follows $U(a_0/l^{(V)}, b_0/l^{(V)})$. To accommodate the difference in the demand distributions, we do not assume any specific function for the demand distribution and it can differ across different selling cycles. That is, the analysis in this work applies to any general functions of $F_{i,t}(\cdot)$ and $f_{i,t}(\cdot)$.

If the VNO's limited resources cannot serve all incoming users at the target service level, then some users will be lost to a competitor, possibly the ROO itself. In our model the user capacity is defined as the maximum number of users whose QoS can be satisfied statistically in a selling cycle (i.e., a soft QoS guarantee bound), rather than the maximum instantaneous number of users that can be served at the same time. In practice, the realization of this user capacity is related to the equilibrium of the dynamic process of users (or subscribers) coming and leaving. In this dynamic process, new users keep coming and they are all accepted by the operator. However, as the amount of demand exceeds the operator's capacity, the actual service level cannot meet the committed service guarantee (i.e., the prescribed target service level that corresponds to the service price). So existing users would start to look for another service provider and leave the current operator. Many wireless operators offer various satisfaction guarantee policies so that users have great flexibility in choosing the service providers. For example, both Sprint and T-mobile provide a free 30-day trial of their wireless service; the subscribers of Verizon may terminate service for any reason within 14 days of activation; and operators like AT&T provide no-contract based service to users. Under these policies, unsatisfied users have the freedom to stop using the current service and to switch to other service providers. As in many marketing literature (see, e.g., [72]), instead of focusing on the dynamics of the demand throughout the selling cycle, we are more interested in the equilibrium outcome when the demand matches the supply at the target service level. That is, the final satisfied demand is limited by both the capacity with respect to the target service level and the potential demand.

We define lost demand as the corresponding demand of users that cannot be satisfied at the claimed service level. The loss rate can be expressed as $1-\frac{\text{number of satisfied users}}{\text{number of arriving users}}$. Similar to [18], we consider a long-term average demand that is independent of short-term wireless characteristics. That is, a burst of users' demand in a selling cycle may cause a larger user loss rate; while a burst of data traffic demand among the users would not incur any demand loss.

Since the reservation decisions are repeated for every selling cycle, to simplify our presentation, but without loss of generality, we consider only one selling cycle (i.e. $T = 1$) and ignore the time-associated subscript (t) in the following model. An extension to a general case of multi-cycle model is straightforward, where the sum of the VNO' expected profits across all selling cycles is maximized for the whole market-planning period. As for the ROO who has built a stable user base, the number of potential users is assumed to be constant and can be calculated as $(M_i^{(R)} - S_i)l^{(R)}$. The notation used throughout this chapter is listed in Table 5.1[3].

3.2.6 Backup Reservation Scheme and Problem Formulation

As a condition of the backup reservation contract, the VNO pays a fixed monetary value $w_{ri}R_i$, regardless of the actual amount of resources used in the cycle. R_i is the reserved capacity level chosen by the VNO. If the VNO's current resource capacity can meet the actual resource demand, no backup resource from the ROO is needed; otherwise, the VNO should use the reserved resources at the ROO. Note that the ROO may or may not require the VNO to reserve the redundant capacity S_i before using it, depending on the market situations as will be specified later.

The interactions between two operators are formulated in a three-stage hierarchical order of decision making. Stage 1: the VNO makes the decision of its target service

[3] To be general, we assume the quantities throughout the chapter take normalized values with the metric units chosen depending on the application. Therefore, all the quantities are unit-less.

level $l^{(V)}$ as a high-level market positioning strategy. Stage 2: knowing the VNO's target market, the ROO sets the unit reservation price w_{ri} at each selling cycle by considering their possible competition in that market segment. ROO announces w_{ri} to the VNO. In stage 3, the VNO determines its reservation quantity R_i and pays the ROO a monetary amount $w_{ri}R_i$. After the demand at the VNO in each selling cycle is realized, the VNO determines the quantity of reserved resources to be used in the cycle and pays the capacity usage fee to the ROO. The unit capacity usage fee is denoted by w, $w < \min\{p_r^{(R)}, p_r^{(V)}\}$. In order to concentrate our analysis on the resource reservation problem, we consider a fixed resource sharing model, that is, w is exogenously determined by the resource market. The decoupling of the reservation price from the sharing price (usage fee), without altering the existing process of NIS, would allow us to better focus on the benefits of the backup reservation scheme.

We assume both the demand and the capacity can be estimated by the operators through an analysis of the supply market and the users market. In the case that this information is considered private and sensitive, one possible way to implement the proposed mechanism is to delegate the computation of the proposed mechanism to a trusted third party, who is a central controller in charge of the trading process and supports decision-making for the operators based on their inputs, but does not disclose the information submitted by one operator to the other operator. In such a way the proposed mechanism can be performed without disclosing the private/sensitive information to the other operator. Such a trusted third-party solution has been widely used in the literature, e.g., an auctioneer in online spectrum auctions. The problem under asymmetric information is normally solved through contract design with different sets of prices and reservation quantities. By satisfying both individual rationality and incentive compatibility constraints in moral hazard models, the operators would truthfully reveal their private demand information, as studied in [68]. In order to concentrate on the benefits of reservation-based NIS, we will focus on the contract design under symmetric information, which is widely assumed as in the literature (see, for example, [19, 18, 66]).

In the rest of this chapter, we aim to answer all of the following questions:

1. How would the reservation-based sharing scheme affect the VNO's determination of its target service level and the two players' expected profits?

2. Can both operators benefit from the backup reservation? When does the VNO prefer not to make any reservation even at the risk of losing some users? When does the ROO reject the reservation request?

3. How do the redundant capacity at the ROO and the service prices at the two operators affect the reservation price and the reservation quantity?

4. If the service markets are competitive, do the operators still benefit from backup reservation? How does the market competition intensity affect both operators' decisions and expected profits?

3.3 Sharing Decisions with Independent Markets

As a benchmark, we first consider the scenario in which two operators serve two completely independent markets. That is, the two operators provide totally different services that are irreplaceable to each other but are using the same kind of resources. A good example is the wireless TV broadcast service and the wireless data communication service over the TV white space. In this case the two operators are targeting different user populations, i.e., there is no competition between the VNO and ROO. To analyze the decisions of backup reservation, we consider two cases: i) the VNO does not request any reservation, and the amount of shared resources from the ROO does not exceed S_i, and ii) in case the required amount of resources from the VNO exceeds S_i, it needs to reserve in advance the extra part; and when the resource demand is larger than $M_i^{(V)}$, the total amount of $R_i + S_i$ resources can be shared if needed.

3.3.1 Utility Functions

Without Backup Reservation

In the case of no backup reservation, VNO users that cannot be served with $M_i^{(V)} + S_i$ amount of resources are simply ignored. With an average number of $l^{(V)} \int_{M_i^{(V)}+S_i}^{+\infty} (x_i - M_i^{(V)} - S_i) f_i(x_i) dx_i$ users lost due to resource scarcity, the VNO's expected profit can be written as

$$\begin{aligned}
\Pi_1^{(V)} = \sum_{i=1}^{n} [&-w \int_{M_i^{(V)}}^{M_i^{(V)}+S_i} (x_i - M_i^{(V)}) f_i(x_i) dx_i - wS_i \int_{M_i^{(V)}+S_i}^{+\infty} f_i(x_i) dx_i \\
&+ \mu_i p_r^{(V)} - p_r^{(V)} \int_{M_i^{(V)}+S_i}^{+\infty} (x_i - M_i^{(V)} - S_i) f_i(x_i) dx_i]
\end{aligned} \tag{3.2}$$

where the first two terms inside the summation represent the expected capacity usage fee when the VNO's own resources cannot satisfy the actual demand and the last two terms represent the revenue from the users. The VNO's decision is represented as $l_1^{(V)*} = \arg\max_{l_1^{(V)} \le l_{\max}^{(V)}} \Pi_1^{(V)}$.

Similarly, the ROO's expected profit is

$$\begin{aligned}\Pi_1^{(R)} = \sum_{i=1}^{n} [(M_i^{(R)} - S_i) p_r^{(R)} + w S_i \int_{M_i^{(V)}+S_i}^{+\infty} f_i(x_i) dx_i \\ + w \int_{M_i^{(V)}}^{M_i^{(V)}+S_i} (x_i - M_i^{(V)}) f_i(x_i) dx_i].\end{aligned} \tag{3.3}$$

With Backup Reservation

We now consider the case in which the VNO reserves R_i units of resources in advance and the ROO accepts the reservation request. For the VNO, the actual number of lost users in one selling cycle can be written as $l^{(V)}(x_i - M_i^{(V)} - S_i - R_i)^+$, where $(\cdot)^+ = \max(\cdot, 0)$, and the actual amount of used resources shared from the ROO is $\min\{(x_i - M_i^{(V)})^+, S_i + R_i\}$. The VNO's expected profit is

$$\begin{aligned}\Pi_2^{(V)} = \sum_{i=1}^{n} [-w_{ri} R_i + \mu_i p_r^{(V)} - p_r^{(V)} \int_{M_i^{(V)}+S_i+R_i}^{+\infty} (x_i - M_i^{(V)} - S_i - R_i) f_i(x_i) dx_i \\ - w(S_i + R_i) \int_{M_i^{(V)}+S_i+R_i}^{+\infty} f_i(x_i) dx_i - w \int_{M_i^{(V)}}^{M_i^{(V)}+S_i+R_i} (x_i - M_i^{(V)}) f_i(x_i) dx_i]\end{aligned} \tag{3.4}$$

where $w_{ri} R_i$ is the reservation cost irrespective of the usage amount, and the last two terms indicate the expected capacity usage fee under high demand. In the following analysis, we do not consider the trivial case when the highest possible demand is smaller than $M_i^{(V)} + S_i$ and no backup reservation is needed.

The ROO's expected profit $\Pi_2^{(R)}$ consists of three parts: the reservation cost paid by the VNO, the revenue earned through sharing resources with the VNO, and the revenue of offering services to its own users. Note that more shared resource with the VNO would also indicate a larger user loss rate, and thus less revenue from users. Accordingly,

$$\begin{aligned}\Pi_2^{(R)} = &\sum_{i=1}^{n}[w_{ri}R_i + w\int_{M_i^{(V)}}^{M_i^{(V)}+S_i+R_i}(x_i - M_i^{(V)})f_i(x_i)dx_i + w(S_i+R_i)\int_{M_i^{(V)}+S_i+R_i}^{+\infty} f_i(x_i)dx_i \\ &+ p_r^{(R)}\int_{M_i^{(V)}+S_i}^{M_i^{(V)}+S_i+R_i}(R_i - (x_i - M_i^{(V)} - S_i))f_i(x_i)dx_i + R_i p_r^{(R)}\int_{0}^{M_i^{(V)}+S_i} f_i(x_i)dx_i \\ &+ p_r^{(R)}(M_i^{(R)} - S_i - R_i)].\end{aligned} \tag{3.5}$$

3.3.2 Stackelberg Game and Equilibrium Solution Analysis

It is straightforward to see that the amount of shared resources to be used by the VNO in a selling cycle is $\min\{(x_i - M_i^{(V)})^+, S_i + R_i\}$. To make optimal decisions on the reservation quantity and price, as well as the VNO's target service level, a three-stage Stackelberg game is employed to describe the bargaining process. In the following analysis, we adopt backward induction to derive the Stackelberg equilibrium solution. First, in stage 3: for a given target service level and reservation price, we derive the VNO's optimal reservation quantity; second, with the prediction of the VNO's response, the ROO's optimal reservation price is analyzed in stage 2; finally, in stage 1, the VNO determines the target service level. These three stages are now described in detail.

Stage 3 (VNO sets the reservation quantity): Given the target service level and the reservation price, by maximizing the VNO's expected profit in (3.4), we have:

Lemma 3.1. In independent markets, for a given reservation price w_{ri} and the target service level, the optimal reservation quantity must satisfy:

$$R_i^* = \max\{\min\{F_i^{-1}(1 - \frac{w_{ri}}{p_r^{(V)} - w}) - S_i - M_i^{(V)}, M_i^{(R)} - S_i\}, 0\}. \tag{3.6}$$

Note that the VNO does not need to reserve any of the ROO's redundant capacity S_i. In fact, the maximum reserved quantity cannot exceed $M_i^{(R)} - S_i$. For $R_i > 0$, the unit reservation price should be low enough so that $w_{ri} < (p_r^{(V)} - w) \int_{M_i^{(V)}+S_i}^{+\infty} f_i(x_i) dx_i$. The explanation is intuitive: only if the reservation price is lower than the expected revenue (i.e., price minus usage fee), will the VNO make a reservation.

Stage 2 (ROO sets the reservation price): The ROO needs to decide the optimal reservation price that maximizes its expected profit. If it does not expect to share more resources than S_i, it will set $w_{ri} = (p_r^{(V)} - w) \int_{M_i^{(V)}+S_i}^{+\infty} f_i(x_i) dx_i$; otherwise, an optimal value of w_{ri} will be given with the consideration of the VNO's response in (3.6).

Proposition 3.1. In independent markets, if the optimal reservation quantity is smaller than $M_i^{(R)} - S_i$, then the optimal unit reservation price w_{ri} should satisfy:

$$R_i(w_{ri})(p_r^{(V)} - w)^2 f_i(M_i^{(V)} + S_i + R_i(w_{ri})) = w_{ri}(p_r^{(V)} - p_r^{(R)}) \tag{3.7}$$

where $R_i(w_{ri}) = F_i^{-1}(1 - \frac{w_{ri}}{p_r^{(V)} - w}) - M_i^{(V)} - S_i$.

From (3.7), for the reservation quantity to be positive, we must have $p_r^{(V)} > p_r^{(R)}$. That is, if the VNO sets a service price per unit resource lower than that of the ROO, a high reservation price would be set by the ROO so that the VNO would opt for no reservation. This way, the resources at the VNO would be sold at the higher price, $p_r^{(R)}$. On the other hand, if the ROO sets a price higher than $p_r^{(R)}$, the ROO would set a low reservation price to encourage backup reservation. Therefore, the shared resources are always used by the operator with a higher price. In other words, the ROO is always better off under backup reservation as long as $p_r^{(V)} > p_r^{(R)}$.

Stage 1 (VNO sets the target service level): When the predictions of R and w_r are made, the VNO needs to determine the optimal value of $l^{(V)}$ or $p_r^{(V)}$, denoted by

$$l_2^{(V)*} = \arg\max \Pi_2^{(V)} \quad s.t. \quad l_2^{(V)} \leq l_{\max}^{(V)}. \tag{3.8}$$

Because $p_r^{(V)} = l^{(V)} p_c^{(V)}(l^{(V)}) = l^{(V)} \alpha U(l^{(V)})$, $\frac{dp_r^{(V)}}{dl^{(V)}} = p_c^{(V)}(l^{(V)}) + l^{(V)} \frac{dp_c^{(V)}(l^{(V)})}{dl^{(V)}}$, $\frac{d^2 p_r^{(V)}}{dl^{(V)2}} = l^{(V)} \frac{d^2 p_c^{(V)}}{dl^{(V)2}} + 2\frac{dp_r^{(V)}}{dl^{(V)}} < 0$, we can find a range of $[l_A^{(V)}, l_B^{(V)}]$ within which $p_r^{(V)} \geq p_r^{(R)}$ can be satisfied, where $l_A^{(V)}$ or $l_B^{(V)}$ satisfies $p_r^{(V)} = p_r^{(R)}$. If $l_A^{(V)} = l_B^{(V)}$, then no backup reservation is made.

It is intuitive to see that with the backup reservation scheme, more resources are available to the VNO, which encourages the VNO to offer a higher level of service. Due to the complexity of solving problem (8), it is hard to provide a closed-form expression of $l^{(V)*}$. A heuristic algorithm for calculating the optimal value of $l^{(V)}$ is as follows:

Step 1) Calculate the thresholds of making backup reservation $p_r^{(V)} \geq p_r^{(R)}$, where $l^{(V)} \in [l_A^{(V)}, l_B^{(V)}]$;

Step 2) For $l^{(V)} \notin [l_A^{(V)}, l_B^{(V)}]$, find $l_{21}^{(V)*} = \arg\max \Pi_1^{(V)}$; if $l_{21}^{(V)*} > l_{\max}^{(V)}$, then $l_{21}^{(V)*} = l_{\max}^{(V)}$;

Step 3) For $l^{(V)} \in [l_A^{(V)}, l_B^{(V)}]$, first calculate the value of R_i and w_{ri} using (3.6) and (3.7), then combine them with (3.4) and find $l_{22}^{(V)*} = \arg\max \Pi_2^{(V)}$; if $l_{22}^{(V)*} > l_{\max}^{(V)}$, then $l_{22}^{(V)*} = l_{\max}^{(V)}$;

Step 4) Compare $\Pi_1^{(V)}|_{l^{(V)}=l_{21}^{(V)*}}$ and $\Pi_2^{(V)}|_{l^{(V)}=l_{22}^{(V)*}}$; select the one with a larger value for the final expected profit, and the corresponding $l^{(V)}$ as the optimal solution.

The basic idea is to first find the service level interval that $R_i^* > 0$. Within this interval, the optimal reservation decisions in (3.6) and (3.7) are incorporated into the VNO's expected profit function (3.4). Outside the interval, the VNO's expected profit function is written as (3.2). Then we find the optimal target service level within or outside of the interval that maximizes the VNO's expected profit respectively. Thus we can find the optimal target service level with the maximum expected profit. We will rely on numerical analysis to further investigate the equilibrium outcome of the target service level.

3.4 Sharing Decisions Between Competitive Operators

In this section, we are considering that the two operators are indirect competitors, whose services are different but substitutable (such as mobile and fixed broadband services). In this situation, we suppose the users always prefer to subscribe to their first-choice service provider, and a user chooses an alternative service only when his or her primary service provider is “out of stock". That is, the VNO's potential users would switch to the ROO only when their demand cannot be satisfied by the VNO [73]. Let θ be the fraction of unsatisfied users that switch to the ROO, where $0 < \theta \leq 1$, which measures the competition intensity between operators [74], with θ = 1 indicating the highest competition intensity[4]. Note that

[4]The user's switching rate θ can be estimated by studying user switching behavior and the market share dynamics. For example, factor analysis and logistic regression can be used to investigate the impacts of various factors such as the switching cost and users' evaluation of the service quality. User utility model that takes the influence factors into account can be built to help determine the value of θ. In particular, for the case of two competing operators, a hotelling model can be used to capture the users' preference and further determine the users switching rate.

in our model the competition level θ is a statistical index that measures user's VNO-to-ROO switching rate over the time period of one selling cycle. It is an input argument that is used in our model to support the service providers' NIS decision making. Its value is fixed in one selling cycle, but can change from cycle to cycle. In our model, θ largely depends on the ROO's service level: A higher target service level at the ROO side (under the same price) is expected to attract more users from its competitor, i.e., a larger θ value. However, since in our model we assume that the ROO's target service level has already been strategically determined, the value of θ is given exogenously. This value could change with the ROO's target service level in a different decision cycle.

We assume that the ROO has a redundant capacity S_i after satisfying its own users. This redundant capacity can be used later to satisfy the new demand coming from the VNO's market. If the redundant capacity is shared with the VNO, then it is possible that the ROO would lose some of the new users. Therefore, the ROO becomes conservative in sharing its redundant resources with the VNO. In this case, we consider the situation when the ROO requires the VNO to make a backup reservation first before sharing any amount of the resources, and examine the conditions under which a backup reservation benefits both operators.

3.4.1 Utility Functions

Without Backup Reservation

If no backup reservation is made at the ROO, the VNO only serves its users with its own resources. The VNO's decision is $l_1^{(V)*} = \arg\max_{l_1^{(V)} \leq l_{\max}^{(V)}} \Pi_1^{(V)}$, where

$$\Pi_1^{(V)} = p_r^{(V)} \sum_{i=1}^{n} [\int_0^{M_i^{(V)}} x f_i(x_i) dx_i + M_i^{(V)} \int_{M_i^{(V)}}^{+\infty} f_i(x_i) dx_i]. \tag{3.9}$$

The ROO's expected profit is

$$\Pi_1^{(R)} = \sum_{i=1}^{n} [(M_i^{(R)} - S_i) p_r^{(R)} + p_r^{(R)} S_i \int_{M_i^{(V)} + \frac{S_i}{\theta\beta}}^{+\infty} f_i(x_i) dx_i + \theta\beta p_r^{(R)} \int_{M_i^{(V)}}^{M_i^{(V)} + \frac{S_i}{\theta\beta}} (x_i - M_i^{(V)}) f_i(x_i) dx_i] \tag{3.10}$$

where $\beta = l^{(V)}/l^{(R)}$ indicates the difference between the two operators' service levels. The first term inside the summation in (3.10) is the revenue from satisfying the ROO's own users, and the following two are from satisfying new users switching from the VNO's market.

With Backup Reservation

For the VNO, the actual number of unsatisfied users in one selling cycle is $l^{(V)}(x_i - M_i^{(V)} - R_i)^+$, and the actual amount of resources shared with the ROO is $\min\{(x_i - M_i^{(V)})^+, R_i\}$.

Then the VNO's expected profit can be written as

$$\Pi_2^{(V)} = \sum_{i=1}^{n}[p_r^{(V)} \int_0^{M_i^{(V)}+R_i} x_i f_i(x_i)dx_i + p_r^{(V)} \int_{M_i^{(V)}+R_i}^{+\infty} (M_i^{(V)} + R_i) f_i(x_i)dx_i - w_r R_i$$
$$- wR_i \int_{M_i^{(V)}+R_i}^{+\infty} f_i(x_i)dx_i - w \int_{M_i^{(V)}}^{M_i^{(V)}+R_i} (x_i - M_i^{(V)}) f_i(x_i)dx_i]. \tag{3.11}$$

As for the ROO, two cases need to be considered:

Case 1: $R_i \leq S_i$. All of ROO's own users can be satisfied, and the ROO may still have capacity to satisfy new demands coming from the other market. The ROO's expected profit in each service area i is

$$\Pi_{2i}^{(R)} = (M_i^{(R)} - S_i)p_r^{(R)} + w_{ri}R_i + wR_i \int_{M_i^{(V)}+R_i}^{+\infty} f_i(x_i)dx_i$$
$$+ w \int_{M_i^{(V)}}^{M_i^{(V)}+R_i} (x_i - M_i^{(V)}) f_i(x_i)dx_i + \theta\beta p_r^{(R)} \int_{M_i^{(V)}+R_i}^{M_i^{(V)}+R_i+\frac{S_i-R_i}{\theta\beta}} (x_i - M_i^{(V)} - R_i) f_i(x_i)dx_i$$
$$+ p_r^{(R)} \int_{M_i^{(V)}+R_i+\frac{S_i-R_i}{\theta\beta}}^{+\infty} (S_i - R_i) f_i(x_i)dx_i. \tag{3.12}$$

The first term is the revenue obtained from the ROO's own users. The second term is the reservation price paid by the VNO. The third item is the capacity usage fee paid by the VNO when the actual demand at the VNO x_i is larger than $M_i^{(V)} + R_i$ and all the reservation quantity R_i is shared. The fourth item is the capacity usage fee paid by the VNO when $x_i \leq M_i^{(V)} + R_i$ and only an amount of $x_i - M_i^{(V)}$ resources is shared. The last two items are the revenue from satisfying the new demand, with an average number of

$\theta l^{(V)} \int_{M_i^{(V)}+R_i}^{M_i^{(V)}+R_i+\frac{S_i-R_i}{\theta\beta}} (x_i - M_i^{(V)} - R_i) f_i(x_i) dx_i + l^{(R)} \int_{M_i^{(V)}+R_i+\frac{S_i-R_i}{\theta\beta}}^{+\infty} (S_i - R_i) f_i(x_i) dx_i$ new users.

Case 2: $R_i > S_i$. In this case, some of the ROO's own users may not be satisfied when VNO uses more resources than S_i, and clearly the ROO has no redundant capacity to satisfy any new demand. Thus, the ROO's expected profit in a given service area is

$$\begin{aligned}
\Pi_{2i}^{(R)} &= (M_i^{(R)} - R_i) p_r^{(R)} + w_{ri} R_i + w R_i \int_{M_i^{(V)}+R_i}^{+\infty} f_i(x_i) dx_i \\
&+ w \int_{M_i^{(V)}}^{M_i^{(V)}+R_i} (x_i - M_i^{(V)}) f_i(x_i) dx_i \\
&+ p_r^{(R)} \int_0^{M_i^{(V)}+S_i} (R_i - S_i) f_i(x_i) dx_i + p_r^{(R)} \int_{M_i^{(V)}+S_i}^{M_i^{(V)}+R_i} (R_i - x_i + M_i^{(V)}) f_i(x_i) dx_i.
\end{aligned} \tag{3.13}$$

The first term is the revenue from the ROO's own users using the resources that are not reserved by the VNO. The second term is the reservation price paid by the VNO. The third item is the capacity usage fee paid by the VNO when the actual demand at the VNO x_i is larger than $M_i^{(V)} + R_i$ and all the reservation quantity R_i is shared. The fourth item is the capacity usage fee paid by the VNO when $x_i \leq M_i^{(V)} + R_i$ and only an amount of $x_i - M_i^{(V)}$ resources is shared. The last two items are the revenue from satisfying the ROO's own users using the reserved resources that are not shared with the VNO. The ROO's total expected profit is $\Pi_2^{(R)} = \sum_{i=1}^{n} \Pi_{2i}^{(R)}$.

3.4.2 Stackelberg Game and Equilibrium Analysis

Similar to the procedures of decision-making under the independent scenario, in the following models, we first derive the Stackelberg equilibrium solution in stages 3 and 2, and then analyze the VNO's optimal service level in stage 1.

Stage 3 (VNO sets the reservation quantity): By maximizing the VNO's expected function of (3.11), the optimal reservation quantity is given by the following lemma:

Lemma 3.2. In a competitive market, given the VNO's target service level and the reservation price w_{ri}, the optimal reservation quantity is given by:

$$R_i^* = \max\{\min\{F_i^{-1}(1 - \frac{w_{ri}}{p_r^{(V)} - w}) - M_i^{(V)}, M_i^{(R)}\}, 0\}. \tag{3.14}$$

Lemma 3.2 shows that the VNO would cooperate with the ROO through backup reservation as long as the reservation price is low enough such that $w_{ri} < (p_r^{(V)} - w) \int_{M_i^{(V)}}^{+\infty} f_i(x_i) dx_i$. We observe that in a competitive market the ROO's potential redundant capacity S_i does not affect the VNO's reservation quantity. This result is intuitive because the ROO requires the VNO to make a reservation first before sharing any amount of resource, including the ROO's redundant capacity. However, the market competition does not only affect the reservation quantity, but also the reservation price. Therefore, we need to examine the change in the reservation price to evaluate the overall impact of competition.

Stage 2 (ROO sets the reservation price): By substituting R_i in (3.12), and (3.13) with (3.14), and by setting $\frac{d\Pi_{2i}^{(R)}}{dR_i} = 0$, the following result can be obtained.

Proposition 3.2. In a competitive market, the optimal value of unit reservation price w_{ri}^* takes one of five possible values: $(1 - F_i(M_i^{(V)}))(p_r^{(V)} - w)$, w_{ri1}, $(1 - F_i(S_i +$

$M_i^{(V)}))(p_r^{(V)}-w)$, w_{ri2}, and $(1-F_i(M_i^{(R)}+M_i^{(V)}))(p_r^{(V)}-w)$. The VNO makes no reservation when $w_{ri} \geq (1-F_i(M_i^{(V)}))(p_r^{(V)}-w)$, and makes maximum reservation when $w_{ri} \leq (1-F_i(M_i^{(R)}+M_i^{(V)}))(p_r^{(V)}-w)$, where w_{ri1} satisfies

$$\begin{aligned} & R_i(w_{ri})(p_r^{(V)}-w)f_i(M_i^{(V)}+R_i(w_{ri})) \\ & = w_{ri}(p_r^{(V)}-p_r^{(R)})/(p_r^{(V)}-w)+p_r^{(R)}(1-\theta\beta)\int_{M_i^{(V)}+R_i(w_{ri})}^{M_i^{(V)}+R_i(w_{ri})+\frac{S_i-R_i(w_{ri})}{\theta\beta}} f_i(x_i)dx_i \end{aligned} \tag{3.15}$$

w_{ri2} satisfies

$$w_{ri}(p_r^{(V)}-p_r^{(R)}) = R_i(w_{ri})(p_r^{(V)}-w)^2 f_i(M_i^{(V)}+R_i(w_{ri})) \tag{3.16}$$

and $R_i(w_{ri}) = F_i^{-1}(1-\frac{w_{ri}}{p_r^{(V)}-w})-M_i^{(V)}$.

The following results can be derived concerning the ROO's decision when the VNO's response is considered.

Corollary 3.1. In a competitive market, if $p_r^{(V)} < p_r^{(R)}$ and $\theta\beta > 1$, then the ROO will set a high reservation price that no reservation is made by the VNO.

From Corollary 3.1, if the VNO's service price per unit resource is lower than that of the ROO's, and their competition intensity is high enough, then the ROO benefits more from the competition than from cooperation, and it is not beneficial to cooperate with the VNO. This results can be explained by the fact that, the ROO has no incentive to share its resources with its competitor when it could make better use of the resources by selling at a higher price, and it could lose the opportunity of obtaining a considerable number of new users otherwise.

Stage 1 (VNO sets the target service level): Now we analyze the VNO's optimal value of $l^{(V)}$ or $p_r^{(V)}$ as in (3.8), but with a different profit function (3.11). The algorithm for calculating the optimal target service level is similar to that in Section 3.4.2. We rely on numerical analysis to further compare the results between independent and competitive markets.

3.5 Impacts of Market Competition

In this section, we investigate the following three questions: (1) How are the reservation decisions and the maximum amount of sharing resources under an independent market scenario different from those under a competitive market scenario? (2) How are the conditions of backup reservation affected when there is competition between operators? (3) Under the competitive scenario, how do the reservation decisions depend on the competition intensity?

From the optimal reservation quantities shown in Lemmas 3.1 and 3.2, we have: (1) if $w_{ri} > (p_r^{(V)} - w) \int_{M_i^{(V)}+S_i}^{+\infty} f_i(x_i)dx_i$, then $R_i^* > 0$ under the independent scenario, and $R_i^* > S_i$ under the competitive scenario; and (2) if $(p_r^{(V)} - w) \int_{M_i^{(V)}+S_i}^{+\infty} f_i(x_i)dx_i < w_{ri} < (p_r^{(V)} - w) \int_{M_i^{(V)}}^{+\infty} f_i(x_i)dx_i$, then $R_i^* = 0$ under the independent scenario, and $R_i^* > 0$ under the competitive scenario. By comparing (3.6) with (3.14), we get the following results.

Corollary 3.2. Under the same reservation price and the VNO's target service level, the need of the VNO to make a reservation under the competitive scenario is higher than that under the independent scenario; and the reservation quantity in competitive markets is S_i units larger than that in independent markets, resulting in the same amount of shared resources available under both scenarios.

It indicates that given a reservation price, the relationship between the operators' markets does not impact the maximum amount of sharing resources, but affects the reservation quantity. The result is based on the implicit assumption that, under the independent scenario, the VNO does not need to make any reservation before sharing the ROO's redundant capacity. To examine the impacts of market competition on the reservation price, we assume the demand follows a uniform distribution, and the following results can be derived.

Corollary 3.3. Given the target service level and uniformly distributed demand at the VNO, the optimal reservation price under the competitive scenario is not smaller than that under the independent scenario, and the maximum amount of resources to be shared under the competitive scenario is not larger than that under the independent scenario.

Corollary 3.3 shows that the relationship between the two operators does impact the final reservation decisions. Intuitively, the ROO would like to obtain more users from the VNO's market. Therefore, the ROO has less incentive to share resources with the VNO, and it raises the reservation price to lower the amount of shared resources when the two operators compete in the wireless service markets. As to the condition under which a backup reservation is made, the following corollary shows some interesting results.

Corollary 3.4. Under the independent scenario, the ROO agrees to cooperate with the VNO through backup reservation as long as $p_r^{(V)} > p_r^{(R)}$; while under the competitive scenario, the condition that the backup-reservation scheme benefits the ROO depends on various factors such as the competition intensity, the demand, the service prices and the resource capacities. Specifically, with a uniformly distributed number of users $U(a_i, b_i)$ at the VNO, if the difference between their service prices is large enough that

$$(p_r^{(V)} - p_r^{(R)}) > S_i p_r^{(R)}(1 - \frac{1}{\theta\beta})/(\frac{b_i}{l^{(V)}} - M_i^{(V)}) \tag{3.17}$$

or the competition intensity is low enough that

$$\theta < \frac{S_i p_r^{(R)}/\beta}{S_i p_r^{(R)} - (p_r^{(V)} - p_r^{(R)})(b_i/l^{(V)} - M_i^{(V)})} \tag{3.18}$$

then the ROO is willing to cooperate with the VNO through backup reservation.

Corollary 3.4 points out that the condition of backup reservation-based cooperation under the independent scenario is totally determined by the operators' service price difference, while the conditions under the competitive scenario are comprehensively affected by multiple factors.

Intuitively, under the competitive scenario, if the ROO would not get a substantial number of new users from the VNO's market, cooperation might bring more profits to the ROO than gaining additional market shares from its competitor would do. Corollary 3.4 shows that only when the user's switching rate is low enough that (5.12) is satisfied, the ROO gains less from the market competition than from the cooperation. This implies the threshold of the competition intensity below which the ROO is willing to cooperate with the VNO.

In contrast to the the results under the independent scenario, when $\theta\beta > 1$, even if $p_r^{(V)} > p_r^{(R)}$, the ROO would not cooperate with the VNO, unless their price difference is larger than the threshold shown in (5.9). On the other hand, the two operators can still benefit from the backup reservation when $p_r^{(V)} < p_r^{(R)}$, as long as (5.12) is satisfied.

The condition of cooperation is not straightforward due to the requirements on both the price difference and the competition intensity. Moreover, as the competition intensity θ increases, the VNO's price p_r^V needs to be higher to facilitate the cooperation. Intuitively, with more intense competition between the operators, it is expected that more users switch to the ROO when the VNO is "out of stock", and therefore the ROO benefits more from the competition than from the cooperation. Corollary 3.5 shows that the ROO would enhance the reservation price to decrease cooperation as a response to a larger competition intensity.

Corollary 3.5. With a uniformly distributed number of users $U(a_i, b_i)$ at the VNO, the reservation quantity decreases with the competition intensity, and the reservation price increases with the competition intensity.

3.6 Numerical Results

In this section, we numerically illustrate the equilibrium of the Stackelberg game, and evaluate the system performance at equilibrium to provide more insight. First, we are interested in comparing our scheme with counterpart schemes that are used in practice. The main goal is to shed light on the benefits of our proposed backup-reservation scheme in NIS. Then we evaluate the impacts of various parameters on the benefits of NIS through backup reservation. In all of the examples, we present the results in both the independent and competitive scenarios to show the impacts of market competition.

We consider an example of spectrum sharing between two operators. Channels are assumed to be orthogonal, thus relegating the problem of channel interference. We will focus on the basic model with only one service market of uncertain user demand that follows a uniform distribution $U(a_0, b_0)$, and so we omit the subscript i here. The base parameter set is provided as: $M^{(V)} = 200$, $M^{(R)} = 600$, $S = 100$, $w = 300$, $p_r^{(R)} = 400$, $P = 100$, $\alpha = 80$, $h = 120$, $n_0 = 50$, $l_{\max}^{(V)} = 10$, $\theta = 1$, $a_0 = 1500$, $b_0 = 4500$, where the VNO's service price has a nonlinear relationship with its service level. Since we are only interested in the optimal outcomes, we ignore the subscript (*) in the rest of this section.

3.6.1 Comparisons with Other Reservation Schemes

Firstly, we compare our proposed reservation scheme with the following two reservation schemes: a mean demand (MD) satisfaction scheme and a linear price-based scheme. The former one indicates that the reservation quantity is made so that the mean demand can always be satisfied, i.e. $R + M^{(V)} + S \geq \mu$ under the independent scenario, or $R + M^{(V)} \geq \mu$ under the competitive scenario, regardless of the reservation price. This scheme may be adopted by the operator for its simplicity. In the linear price-based scheme, the reservation quantity is linearly dependent on the reservation price as in $R = D - M^{(V)} - \alpha w_r$, where D is the highest possible resource demand (note in independent markets with uniform demand, $D = \frac{b_0}{l^{(V)}} - S$). The ROO would set the reservation price lower than $p_r^{(V)} - w$. The comparison results for the VNO are shown in Fig. 3.2 with different values of the reservation price under an arbitrary value set of $l^{(V)} = 8.0$, $a_0 = 1000$, and $b_0 = 5000$, which illustrates clearly the dominance of our reservation scheme over other two schemes with respect to the VNO's expected profits, in both independent and competitive markets. It is worth mentioning that, the ROO's expected profit may be larger when the VNO chooses the mean demand satisfaction or the linear price-based scheme. Because in these two strategies, the VNO might reserve more than its optimal reservation quantity.

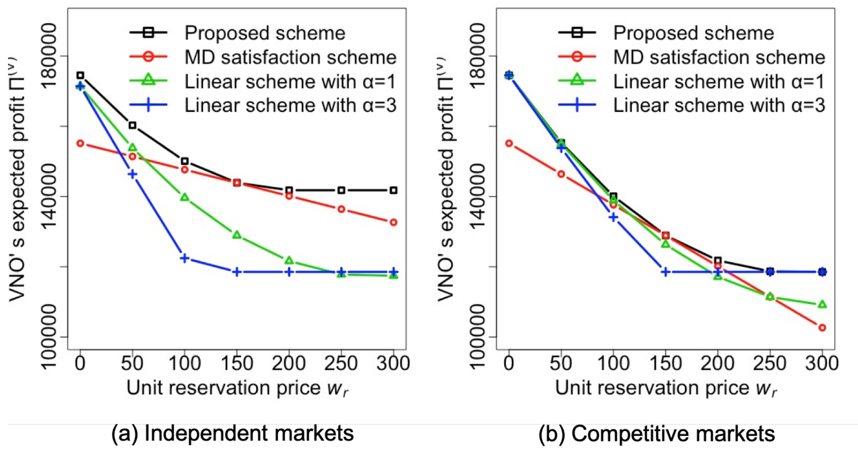


Figure 3.2: VNO's expected profit under three reservation schemes vs. w_r.

3.6.2 Sensitivity Analysis

Impact of Demand Variance

Let the ROO's improved profit from the backup reservation scheme $\Delta\Pi^{(R)} = \Pi_2^{(R)} - \Pi_1^{(R)}$ be the value of backup reservation for the ROO. The results under three levels of demand uncertainty (high ($U(1000, 5000)$), moderate ($U(1500, 4500)$) and low ($U(2000, 4000)$)) are: 8981, 7771, 6654 in independent markets and 15313, 14729, 13935 in competitive markets, respectively. It is revealed that the ROO is more willing to cooperate under higher demand risks. In addition, we can also observe that the ROO benefits more from the reservation under the competitive scenario than under the independent scenario.

Impact of Redundant Resource Capacity

To see how the ROO's potential redundant capacity affects the outcome performance, we change the value of S from 100 to 300, with the constant value of $M^{(R)} = 600$ and $\theta = 0.8$. Table 3.2 reports the equilibrium results, from which three important observations can be made.

First of all, under the independent scenario, both the reservation price and quantity decrease with the ROO's redundant capacity S. This is because there is less need for the VNO to make reservations with more redundant resources available at the ROO. While under the competitive scenario, the reservation quantity increases with S due to the ROO's stronger incentive to share its redundant capacity and thus the reducing reservation price. Besides, despite the different values of $l^{(V)}$ in two scenarios, compared to the independent scenario, the reservation price is higher and the maximum amount of resources to be shared is smaller under the competitive scenario, which confirms the results in Corollary 3.3.

Secondly, denote the total improved profit $\Delta\Pi = \Pi_2^{(R)} + \Pi_2^{(V)} - \Pi_1^{(R)} - \Pi_1^{(V)}$ as the benefit of backup reservation for the two operators, the value of which is always higher under the competitive scenario. $\Delta\Pi$ decreases with S under the independent scenario since less reservation is made, while it increases under the competitive scenario. This is because, larger redundant capacity indicates less need to compromise the ROO's own user demand when the VNO shares the spectrum, thereby increasing the benefit of the backup reservation. Hence, the implementation of a backup reservation scheme is most effective in improving profits under the competitive scenario or the independent scenario with small redundant capacity. Finally, if we use $\Delta l^{(V)} = l_1^{(V)} - l_2^{(V)}$ to denote the improvement in the target service level when implementing backup reservation scheme, it can be seen in our numerical examples that the users typically benefit more from the reservation-based NIS when the operators compete.

Table 3.2: Impact of redundant Capacity on the Reservation Decisions and the Value of Backup Reservation.

S	Independent markets				Competitive markets			
	w_r	R	$\Delta\Pi$	$\Delta l^{(V)}$	w_r	R	$\Delta\Pi$	$\Delta l^{(V)}$
100	129	104	12287	0.051	178	144	24680	0.069
200	80	66	4967	0.033	170	152	29466	0.088
300	34	28	1031	0.017	158	169	36519	0.123

Impact of Market Competition

NIS is supposed to benefit the collaborators through cost sharing and larger market share. However, previous studies have assumed the independence between the markets of the operators. Our theoretical analysis has shown us that the conditions of using the backup reservation scheme are different when the operators compete. In the present section, we further explore the impacts of competition intensity on the reservation decisions. Fig. 3.3 shows that the impact of the competition intensity θ is affected by the potential redundant capacity S. When S is small, the impact of competition intensity θ is negligible, because the VNO reserves all of its redundant capacity and no new users are expected to switch to the ROO; but when S is large, as the competition intensity increases, the ROO would increase the reservation price to inhibit the reservation and to keep more resources for new users.

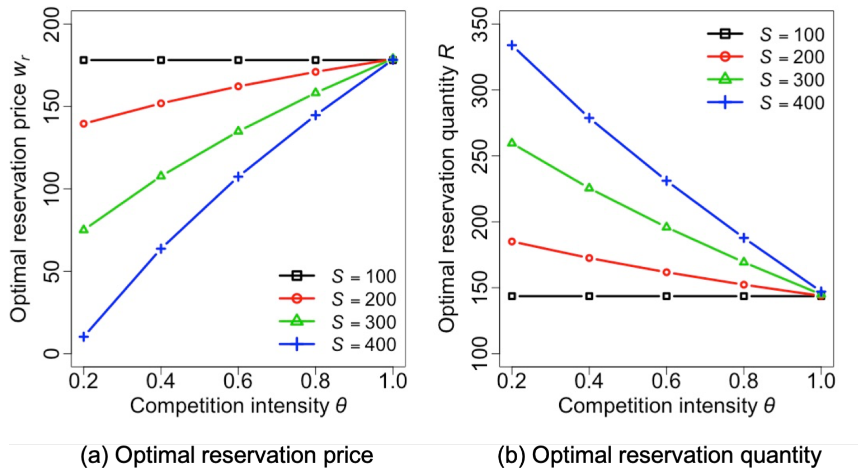


Figure 3.3: Impact of market competition on the reservation decisions.

Impact of the ROO's Market Strategy

Next we study the impacts of the ROO's market strategy, indicated by the price and the target service level, on the equilibrium results. As shown in Fig. 3.4, by increasing $p_r^{(R)}$ from 300 to 650, the total improved profit for the two operators decreases in either independent or competitive markets. This is because, it is more profitable to use the limited resources to satisfy the ROO's users than to satisfy the VNO's when $p_r^{(R)} - p_r^{(V)}$ is large enough. This observation actually adheres to the result in Proposition 3.1. Under the independent scenario, the optimal reservation decisions are independent on the ROO's target service level. However, Fig. 3.5 reveals that under the competitive scenario, when S is large, the optimal reservation quantity increases with $l^{(R)}$. This result complies with our result in Proposition 3.2: When S is small enough, $R > S$ and (3.16) is satisfied, the impact of $l^{(R)}$ is negligible since no spectrum is left at the ROO to serve new users. When

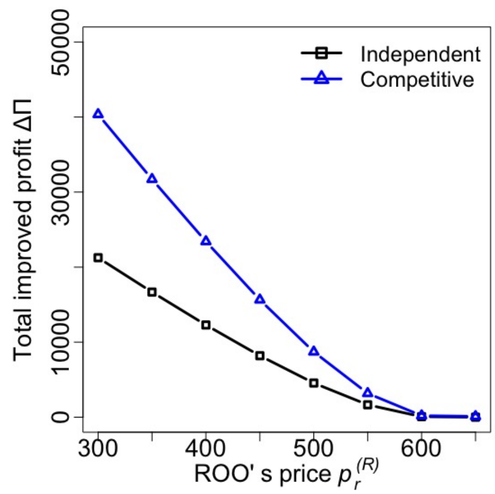


Figure 3.4: Impact of the ROO's price $p_r^{(R)}$ on the total improved profit.

S is large, the revenue from satisfying the new users decreases with $l^{(R)}$. Therefore, the ROO would lower the reservation price to induce larger reservation quantity for more revenue obtained from NIS. Moreover, we can use $E\{1 - \frac{\text{unused spectrum}}{\text{total spectrum}}\}$ to represent the spectrum efficiency. We plot the resource efficiency as a function of R (shown below as Fig. 3.6(a)). In addition, numerical results have been obtained to study how the resource efficiency changes with other parameters such as S and θ. These results are plotted in Fig. 3.6(b). From these figures, it can be observed that resource efficiency is increased by implementing the backup reservation scheme. The percentage of increase in resource efficiency increases with S and decreases with θ under the competitive scenario. This observation can be explained as follows: as a larger proportion of the VNO's unsatisfied users would turn to the ROO, there is less space to increase the resource efficiency through the backup reservation scheme. With more redundant capacity, the total resource efficiency is

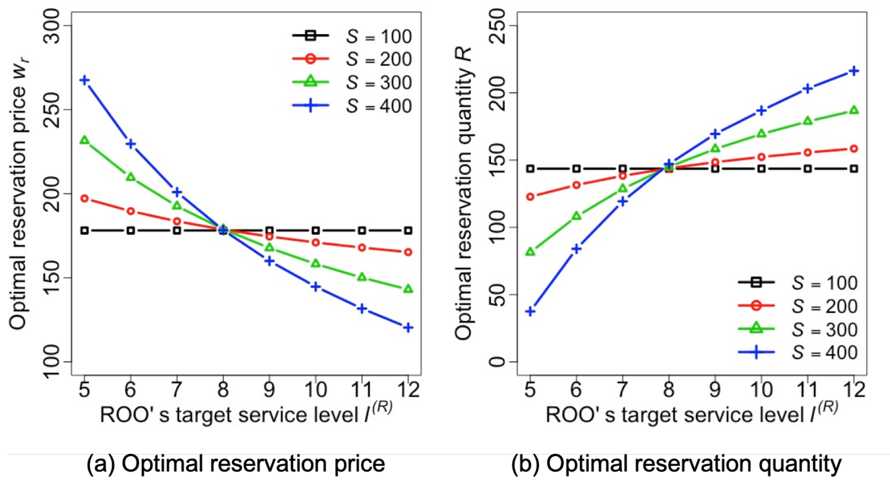


Figure 3.5: Impact of $l^{(R)}$ on the optimal reservation decisions.

expected to increase with a larger reservation quantity. This observation is also consistent with our results in Fig. 3.3(b) regarding the impact of R.

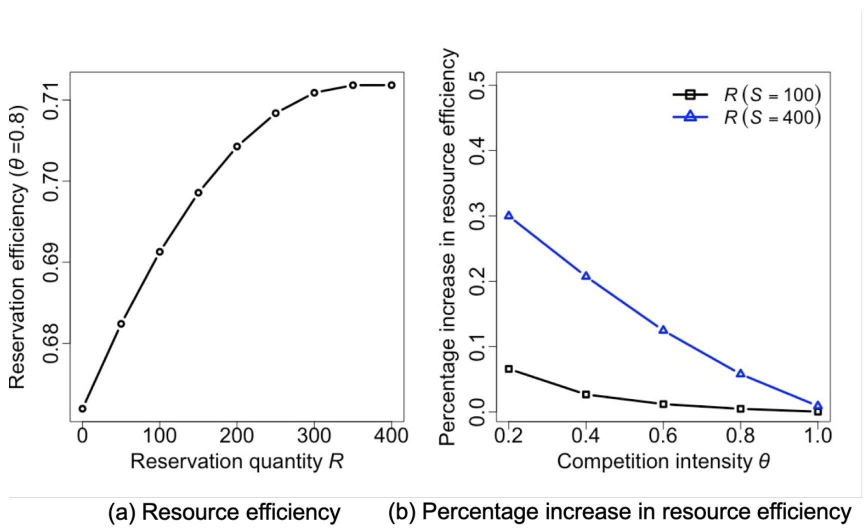


Figure 3.6: (a) Resource efficiency vs. R; and (b) Percentage increase in resource efficiency vs. θ.

Note that the benefit of backup reservation scheme is to significantly improve profits, and our results in Fig. 3.7 show that this benefit increases with S and decreases with θ under the competitive scenario. The reason is that, with more redundant capacity or a smaller value of competition intensity, the ROO faces a lower risk of hurting its own users or of losing new users through NIS, which indicates that their cooperation becomes more profitable for the ROO. Therefore, a lower price is set to encourage a larger reservation quantity, such that both parties benefit more from the backup reservation scheme.

Moreover, from Fig. 3.3, Fig. 3.6(b), and Fig. 3.7, it can be observed that under low to mild surplus capacity (e.g., $S = 100 \sim 200$), the estimation accuracy of θ only has minor impact on the decision making, as the optimal reservation price, reservation quantity, resource efficiency gain, and total profit gain do not change significantly for small perturbations of the θ value (i.e., all these decision-vs.-θ curves have small/flat slopes). Therefore it is clear that the proposed model is robust against estimation errors in θ.

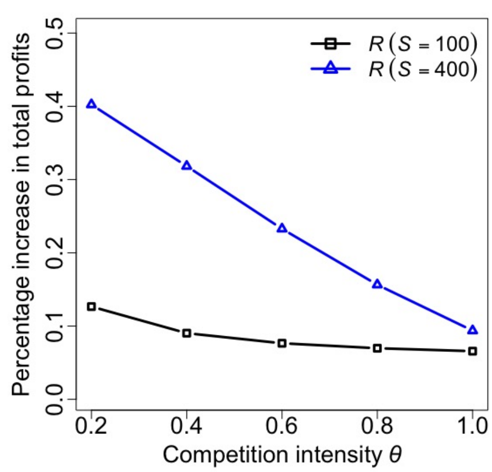


Figure 3.7: Percentage increase in total profits under competitive scenario.

3.7 Alternative Models and Assumptions

In this section, we discuss two alternative modeling assumptions and some limitations of our analysis. In particular, we examine (1) the impacts of the ROO's demand uncertainty, and (2) a different timing of moves in the Stackelberg game when the ROO makes the reservation price first.

3.7.1 Uncertain ROO's demand

In our base model, the ROO's demand is assumed to be fixed. In this section, we study the more general case in which the ROO's demand is uncertain. We assume that the true value of its demand is unknown, but it is publicly known to be either high, and the users consume all the ROO's resource capacity, or low, which results in a redundant capacity of S_i. The probability of a high market demand is denoted by q_i.

The three-stage decision process is similar to that depicted in Section 3.3. The analysis of stage 1 is the same as subsection 3.4.2. We now analyze the optimal backup reservation contract parameters for stages 2 and 3 in both independent and competitive scenarios. It is worth to note that, under the independent scenario, due to the demand uncertainty, the ROO does not always have redundant resources to be shared with the VNO. In this case, naturally we consider the situation when the VNO is required to make a reservation before sharing any amount of resource. In the Appendices in Section 3.9, we provide the service providers' expected profit functions for each scenario. Regarding the optimal unit reservation price, we can derive a similar result following the same steps of Proposition 3.2, but with a different value for w^*_{ri1}, given in the following Proposition.

Proposition 3.3. With uncertain demand at the ROO, the optimal unit reservation price w^*_{ri1} that leads to a reservation quantity $R \in (0, S)$ satisfies the following equations:

1) In a independent market,

$$R_i(w_{ri1})(p_r^{(V)}-w)f_i(M_i^{(V)}+R_i(w_{ri1})) = w_{ri1}-((1-q_i)p_r^{(R)}-w)\int_{M_i^{(V)}+R_i(w_{ri1})}^{+\infty} f_i(x_i)dx_i. \tag{3.19}$$

2) In a competitive market,

$$\begin{aligned} &R_i(w_{ri1})(p_r^{(V)} - w)f_i(M_i^{(V)} + R_i(w_{ri1})) \\ &= w_{ri1}(p_r^{(V)} - p_r^{(R)})/(p_r^{(V)} - w) + qp_r^{(R)}(1 - \theta\beta)\int_{M_i^{(V)}+R_i(w_{ri1})}^{M_i^{(V)}+R_i(w_{ri1})+\frac{S_i-R_i(w_{ri1})}{\theta\beta}} f_i(x_i)dx_i. \end{aligned} \tag{3.20}$$

By comparing the results under the independent and competitive scenarios, we observe that, under the same reservation price and the VNO's target service level, the relationship between the operators' markets does not impact the reservation quantity. The reason is that, both in independent and competitive scenarios, the ROO needs to be conservative in sharing resources since its own demand is uncertain. These results is different with the one stated in Corollary 3.2 in the case of fixed demand. Besides, under the independent scenario, different from the fixed demand case, the two operators may still benefit from the backup reservation scheme even if $p_r^{(V)} < p_r^{(R)}$ due to the demand uncertainty. In a competitive market, if $p_r^{(V)} < p_r^{(R)}$ and $\theta\beta > 1$, then the ROO will set a high reservation price that no reservation is made by the VNO. This is consistent with what we observed when the ROO's demand is fixed.

In the following numerical studies, we are interested in investigating the impacts of the demand uncertainty on the reservation decisions and the system performance. If we use the relative variation, i.e., the variance divided by the absolute value of the mean, to denote the demand variance, then a larger value of q_i indicates a larger demand uncertainty. Using the same problem setting for only one service market and input parameters values as in Section 3.7, we obtain the optimal reservation decisions with different values of q at $S = 300$ and $\theta = 1$. Fig. 3.8(a) illustrates that, as q increases, the ROO encourages a larger backup reservation quantity by setting a lower price under the independent scenario. That is, more risk from the demand side makes it more attractive to implement the backup reservation scheme. However, under the competitive scenario, the impacts of the demand uncertainty is negligible because the ROO would get new users from the VNO's market and the demand risk is mitigated. Fig. 3.8(b) shows that the total improved profit increases with the demand uncertainty, more apparently in an independent market.

3.7.2 ROO moves first

Our base model assumes that the ROO provides the backup reservation scheme after the VNO sets its target service level. This decision sequence is suitable for the situation when the VNO's market planning is strategic (i.e., performed at a relatively large time period) and the target service level needs to be made before the operational reservation decisions. An alternative decision sequence is allowing the ROO to be the first mover by declaring the reservation price, and then the VNO determines its target service level and the reservation

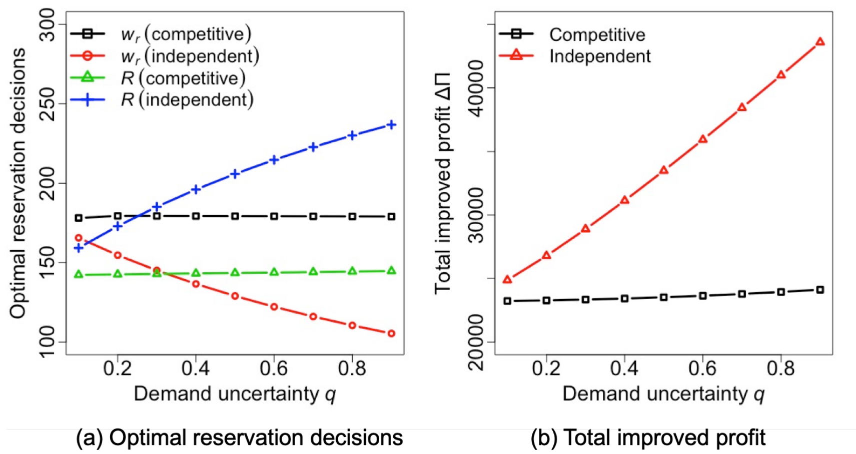


Figure 3.8: Impact of ROO's demand uncertainty.

quantity. This situation occurs when the VNO is a new entrant operator and the ROO would like to take the potential first-mover advantage. In this subsection, We discuss the robustness of our analytical results with respect to the above alternative decision sequence and examine whether there is a first mover advantage for the ROO in our Stackelberg game model.

In the alternative decision sequence, it is difficult to solve the optimal decisions in closed-form, but they can be solved easily using numerical methods. Table 3.3 shows that, consistent with what we observed in our base model, the backup reservation scheme benefits the two operators and the users especially when the ROO has small redundant capacity or the market is competitive. Besides, the impacts of the redundant capacity S or the competition intensity θ on the final reservation decisions when the ROO moves first are similar to those in our base model, as illustrated by Fig. 3.9(a).

Table 3.3: Impact of Redundant Capacity When the ROO Moves First.

S	Independent markets				Competitive markets ($\theta = 0.8$)			
	w_r	R	$\Delta\Pi$	$\Delta l^{(V)}$	w_r	R	$\Delta\Pi$	$\Delta l^{(V)}$
100	126	110	12801	0.09	175	148	25353	0.113
200	75	74	5701	0.08	166	159	30643	0.123
300	23	45	1847	0.06	151	178	37779	0.139

Fig. 3.9(b) plots the increased profit of the ROO ($\Delta\Pi^{(R)} = \Pi^{(R)}$(ROO moves first)-$\Pi^{(R)}$(VNO moves first)) by letting it making decision first. It shows that the ROO does not always enjoy the first-mover advantage in our reservation model. Whether or not the ROO benefits from making the decision first is determined by the values of its redundant capacity and the competition intensity. Specifically, if the ROO has large redundant capacity and the competition is intense enough, it is beneficial for the ROO to delay its reservation price decision after the VNO has decided its target service level. This is due to the fact that the ROO can adjust optimally its reservation price with more information of the VNO's demand.

3.8 Appendices

This section provides the proofs for Chapter 3.

1. Proof for Lemma 3.1.

Proof: Because $\frac{d\Pi_2^{(V)}}{dR_i} = -w_{ri} - (w - p_r^{(V)}) \int_{M_i^{(V)}+S_i+R_i}^{+\infty} f_i(x_i)dx_i$, and $\frac{d^2\Pi_2^{(V)}}{dR_i^2} = (w - p_r^{(V)})f_i(M_i^{(V)} + S_i + R_i) < 0$, the optimal reservation quantity satisfies $\frac{d\Pi_2^{(V)}}{dR_i} = 0$, i.e., $\int_{M_i^{(V)}+S_i+R_i}^{+\infty} f_i(x_i)dx_i = \frac{w_{ri}}{p_r^{(V)}-w}$. □

2. Proof for Proposition 3.1.

Proof: If $0 < R_i < M_i^{(R)} - S_i$, substitute R_i in $\Pi_2^{(R)}$ with (3.6), we can re-write the ROO's

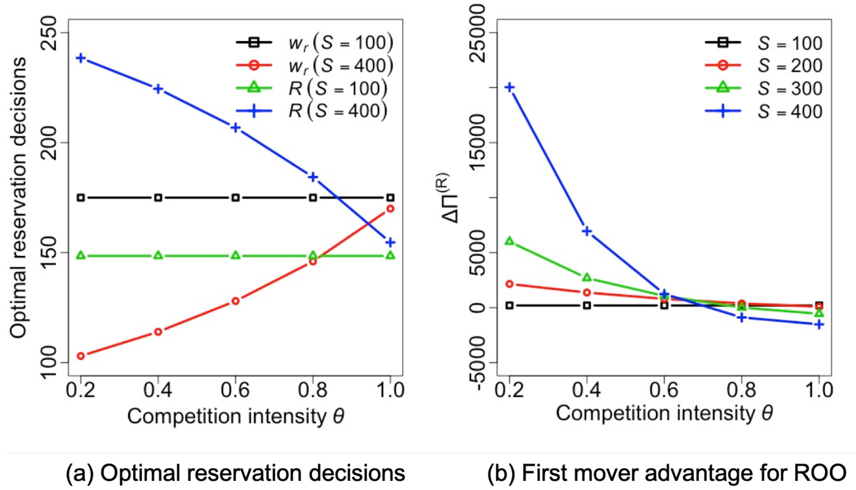


Figure 3.9: Results when ROO moves first.

profit as a function of R_i. From $\frac{d\Pi_2^{(R)}}{dR_i} = p_r^{(V)} - p_r^{(R)} - (p_r^{(V)} - p_r^{(R)}) \int_0^{M_i^{(V)}+S_i+R_i} f_i(x_i)dx_i + R_i(w - p_r^{(V)}) f_i(M_i^{(V)} + S_i + R_i) = 0$, we can get the results shown in Proposition 3.1. $\square$

3. Proof for Lemma 3.2.

Proof: Because $\frac{d\Pi_2^{(V)}}{dR_i} = -w_{ri} - (w - p_r^{(V)}) \int_{M_i^{(V)}+R_i}^{+\infty} f_i(x_i)dx_i$, $\frac{d^2\Pi_2^{(V)}}{dR_i^2} = (w - p_r^{(V)}) f_i(M_i^{(V)} + R_i) < 0$, the optimal reservation quantity satisfies $\int_{M_i^{(V)}+R_i}^{+\infty} f_i(x_i)dx_i = \frac{w_{ri}}{p_r^{(V)} - w}$, from which we can derive the result in Lemma 3.2. $\square$

4. Proof for Corollary 3.1.

Proof: If $p_r^{(V)} < p_r^{(R)}$ and $\theta\beta > 1$, the value of R_i that satisfies (3.15) or (3.16) is negative. Because $\frac{d\Pi_{2i}^{(R)}}{dR_i}|_{R_i=S_i} = (p_r^{(V)} - p_r^{(R)}) \int_{M_i^{(V)}+S_i}^{+\infty} f_i(x_i)dx_i - (p_r^{(V)} - w) S_i f_i(M_i^{(V)} + S_i) < 0$, the optimal value of R_i is 0 for the ROO. $\square$

5. Proof for Corollary 3.3.

Proof: Under the independent scenario, we can derive that the optimal reservation quantity with a uniformly distributed number of users $U(a_i, b_i)$ is $(b_i/l^{(V)} - S_i - M_i^{(V)})\frac{p_r^{(R)} - p_r^{(V)}}{p_r^{(R)} - 2p_r^{(V)} + w}$. That is, the maximum amount of resources to be shared between the two operators is $(b_i/l^{(V)} - S_i - M_i^{(V)})\frac{p_r^{(R)} - p_r^{(V)}}{p_r^{(R)} - 2p_r^{(V)} + w} + S_i$.

Under the competitive scenario, since $\frac{d^2\Pi_{2i}^{(R)}}{dR_i^2} = \frac{(2 - \frac{1}{\theta\beta})p_r^{(R)} - 2p_r^{(V)} + w}{b_i/l^{(V)} - a_i/l^{(V)}}$, $\frac{d^2\Pi_{2i}^{(R)}}{dR_i^2} = (p_r^{(R)} - 2p_r^{(V)} + w)\frac{1}{b_i/l^{(V)} - a_i/l^{(V)}}$, $\Pi_{2i}^{(R)}$ is either convex or concave functions of R_i.

We denote $A = \frac{d\Pi_{2i}^{(R)}}{dR_i}|_{R_i=0}$, $B = \frac{d\Pi_{2i}^{(R)}}{dR_i}|_{R_i=S_i}$, and $C = \frac{d\Pi_{2i}^{(R)}}{dR_i}|_{R_i=M_i^{(R)}}$. If the user number at the VNO follows uniform distribution $U(a_i, b_i)$, we have

$$A = (p_r^{(V)} - p_r^{(R)})(b_i/l^{(V)} - M_i^{(V)}) - S_i p_r^{(R)}(1 - \frac{1}{\theta\beta}) \tag{3.21}$$

$$B = (p_r^{(V)} - p_r^{(R)})(b_i/l^{(V)} - M_i^{(V)}) - S_i(2p_r^{(V)} - p_r^{(R)} - w) \tag{3.22}$$

$$C = B + (S_i - M_i^{(R)})(2p_r^{(V)} - p_r^{(R)} - w). \tag{3.23}$$

At equilibrium, where $p_r^{(V)} = l^{(V)}p_c^{(V)}$,

(i) if $A < 0$, $B < 0$, and $C < 0$, then the VNO does not make any reservation;

(ii) if $A > 0$, $B < 0$, and $C < 0$, then

$$w_{ri}^* = (p_r^{(V)} - w)\frac{(b_i - M_i^{(V)}l^{(V)})(p_r^{(V)} - w - p_r^{(R)} + \frac{p_r^{(R)}}{\theta\beta}) + (S_i - \frac{S_i}{\theta\beta})p_r^{(R)}}{(2p_r^{(V)} - w - 2p_r^{(R)} + \frac{p_r^{(R)}}{\theta\beta})(b_i - a_i)} \tag{3.24}$$

$$R_i^* = \frac{(b_i/l^{(V)} - M_i^{(V)})p_r^{(V)} - (S_i + b_i/l^{(V)} - M_i^{(V)} - \frac{S_i}{\theta\beta})p_r^{(R)}}{2p_r^{(V)} - w - 2p_r^{(R)} + \frac{p_r^{(R)}}{\theta\beta}}; \tag{3.25}$$

(iii) if $A > 0$, $B > 0$, and $C < 0$, then

$$w_{ri}^* = \frac{(l^{(V)}M_i^{(V)} - b_i)(p_r^{(V)} - w)^2}{(p_r^{(R)} - 2p_r^{(V)} + w)(b_i - a_i)} \tag{3.26}$$

$$R_i^* = (b_i/l^{(V)} - M_i^{(V)})\frac{p_r^{(R)} - p_r^{(V)}}{p_r^{(R)} - 2p_r^{(V)} + w}; \tag{3.27}$$

(iv) if $A > 0$, $B > 0$, and $C > 0$, then the ROO makes the maximum reservation $R_i^* = M_i^{(R)}$ and

$$w_{ri}^* = \frac{l^{(V)}(M_i^{(R)} + M_i^{(V)} - b_i/l^{(V)})(w - p_r^{(V)})}{b_i - a_i}; \tag{3.28}$$

(v) if $A < 0$, and $C > 0$, then the ROO either makes maximum reservation or no reservation with

$$w_{ri}^* = (p_r^{(V)} - w)\frac{b_i - l^{(V)}M_i^{(V)}}{b_i - a_i}; \tag{3.29}$$

(vi) if $A < 0$, $B > 0$, $C < 0$, then either R_i^* satisfies (3.27) or $R_i^* = 0$;

(vii) if $A > 0$, $B < 0$, $C > 0$, then either R_i^* satisfies (3.25) or $R_i^* = M_i^{(R)}$.

The two possible positive reservation quantities smaller than the maximum reservation quantity under the competitive scenario are those in (3.25) and (3.27). It is easy to prove that both values are smaller than $(b_i/l^{(V)} - S_i - M_i^{(V)})\frac{p_r^{(R)} - p_r^{(V)}}{p_r^{(R)} - 2p_r^{(V)} + w} + S_i$. If the ROO makes maximum reservation $M_i^{(R)}$ under the competitive scenario, we have $(p_r^{(V)} - p_r^{(R)})(\frac{b_0}{l^{(V)}} - M_i^{(V)}) - M_i^{(R)}(2p_r^{(V)} - p_r^{(R)} - w) > 0$, then $(\frac{b_0}{l^{(V)}} - S_i - M_i^{(V)})(p_r^{(V)} - p_r^{(R)}) > (M_i^{(R)} - S_i)(2p_r^{(V)} - p_r^{(R)} - w)$, which means that the ROO also makes maximum reservation $R_i^* = M_i^{(R)} - S_i$ under the independent scenario. In this case, the amount of resources that can be shared are the same for the two situations. Since the maximum amount of shared resources under the competitive scenario is smaller or equals to that under the independent scenario, according to Corollary 3.2, the optimal reservation price is not smaller under the competitive scenario. $\square$

6. Proof for Corollary 3.4.

Proof: Under the independent scenario, the condition of a backup reservation scheme can be derived from (3.7). Under the competitive scenario, from the proof for Corollary 3.3, we can find that if $A > 0$, $R_i^* > 0$. By solving $A > 0$, we can therefore deduce the results in Corollary 3.4. $\square$

7. Proof for Corollary 3.5.

Proof: Firstly, it is easy to prove that either B in (3.22) or C in (3.23) does not change with θ, while A in (3.21) decreases in θ. Therefore, as θ increases, the value of A either stays positive or negative, or changes from positive to negative.

Case 1: when A stays negative, (1) if $B < 0$ and $C < 0$, then the VNO does not make any reservation and the ROO's reservation price stays at $w_{ri}^* = (p_r^{(V)} - w)\frac{b_i - l^{(V)}M_i^{(V)}}{b_i - a_i}$; (2) if $B > 0$ and $C < 0$, then either R_i^* satisfies (3.27) which is not impacted by θ or $R_i^* = 0$; similarly, w_{ri}^* satisfies either (3.26) or (3.29), with the latter value larger than the former one. (3) if $C > 0$, then the ROO either makes maximum reservation or no reservation, and w_{ri}^* satisfies either (3.28) or (3.29), with the latter value larger than the former one. As θ increases, $\Pi_{2i}^{(R)}|_{R_i=0}$ increases but the value of $\Pi_{2i}^{(R)}$ when R_i^* satisfies (3.27) or $\Pi_{2i}^{(R)}|_{R_i=M_i^{(R)}}$ stays unchanged. Therefore R^* is non-increasing with θ, and w_{ri}^* is non-decreasing with θ.

Case 2: when A stays positive, then the optimal reservation quantity takes one of the three values: $\frac{(b_i/l^{(V)}-M_i^{(V)})p_r^{(V)}-(S_i+b_i/l^{(V)}-M_i^{(V)}-\frac{S_i}{\theta\beta})p_r^{(R)}}{2p_r^{(V)}-w-2p_r^{(R)}+\frac{p_r^{(R)}}{\theta\beta}}$, $(b_i/l^{(V)} - M_i^{(V)})\frac{p_r^{(R)}-p_r^{(V)}}{p_r^{(R)}-2p_r^{(V)}+w}$ and $M_i^{(R)}$. Because the latter two values are independent of θ, we only needs to prove that $R_i^*(\theta) = \frac{(b_i/l^{(V)}-M_i^{(V)})p_r^{(V)}-(S_i+b_i/l^{(V)}-M_i^{(V)}-\frac{S_i}{\theta\beta})p_r^{(R)}}{2p_r^{(V)}-w-2p_r^{(R)}+\frac{p_r^{(R)}}{\theta\beta}}$ is decreasing in θ. First, we can derive that $\frac{dR_i^*}{d\frac{1}{\theta\beta}} = \frac{-Bp_r^{(R)}}{(2p_r^{(V)}-w-2p_r^{(R)}+\frac{p_r^{(R)}}{\theta\beta})^2} > 0$, where $B = (p_r^{(V)} - p_r^{(R)})(b_i/l^{(V)} - M_i^{(V)}) - S_i(2p_r^{(V)} - p_r^{(R)} - w) < 0$. Because $\frac{d\frac{1}{\theta\beta}}{d\theta} < 0$, we have $\frac{dR_i^*}{d\theta} = \frac{dR_i^*}{d\frac{1}{\theta\beta}}\frac{d\frac{1}{\theta\beta}}{d\theta} < 0$. As for the value of w_{ri}^*, because $\frac{dw_{ri}^*}{d\frac{1}{\theta\beta}} = Bl^{(V)}p_r^{(R)}\frac{p_r^{(V)}-w}{(b_i-a_i)(2p_r^{(V)}-w-2p_r^{(R)}+\frac{p_r^{(R)}}{\theta\beta})^2} < 0$ when w_{ri}^* satisfies (3.24), we can derive that $\frac{dw_{ri}^*}{d\theta} > 0$.

Case 3: when A changes from positive to negative, because R^* either stays unchanged or decreases to zero, we conclude that R^* is non-increasing with θ. Similarly, w_{ri}^* increases with θ. $\square$

8. This appendix contains the operators' profit analysis for Subsection 3.8.1.

In independent scenario, without backup reservation, the VNO's expected profit is written as in (3.9). The ROO's expected profit in each service area i i

$$\Pi_1^{(R)} = q((M_i^{(R)} - S_i)p_r^{(R)} + wS_i\int_{M_i^{(V)}+S_i}^{+\infty} f_i(x_i)dx_i + w\int_{M_i^{(V)}}^{M_i^{(V)}+S_i}(x_i - M_i^{(V)})f_i(x_i)dx_i)$$
$$+ (1-q)p_r^{(R)}M_i^{(R)}. \tag{3.30}$$

In independent scenario, with backup reservation, the VNO's expected profit is written as in (3.11). As for the ROO, three cases need to be considered:

Case 1: With low demand, and $R_i \leq S_i$. The ROO's expected profit is

$$\Pi_{2Ai}^{(R)} = w \int_{M_i^{(V)}}^{M_i^{(V)}+R_i} (x_i - M_i^{(V)}) f_i(x_i) dx_i + wR_i \int_{M_i^{(V)}+R_i}^{+\infty} f_i(x_i) dx_i + w_r R_i + p_r^{(R)} (M_i^{(R)} - S_i). \tag{3.31}$$

Case 2: WIth low demand, and $R_i > S_i$. The ROO's expected profit is

$$\begin{aligned}\Pi_{2Bi}^{(R)} = w &\int_{M_i^{(V)}}^{M_i^{(V)}+R_i} (x_i - M_i^{(V)}) f_i(x_i) dx_i + wR_i \int_{M_i^{(V)}+R_i}^{+\infty} f_i(x_i) dx_i + w_r R_i \\ &+ p_r^{(R)} \int_{0}^{M_i^{(V)}+S_i} (M_i^{(R)} - S_i) f_i(x_i) dx_i + p_r^{(R)} \int_{M_i^{(V)}+S_i}^{M_i^{(V)}+R_i} (M_i^{(V)} + M_i^{(R)} - x_i) f_i(x_i) dx_i \\ &+ p_r^{(R)} \int_{M_i^{(V)}+R_i}^{+\infty} (M_i^{(R)} - R_i) f_i(x_i) dx_i.\end{aligned} \tag{3.32}$$

Case 3: With high demand. In the case of no surplus capacity, the ROO's expected profit in each service area i is

$$\begin{aligned}\Pi_{2Ci}^{(R)} = w &\int_{M_i^{(V)}}^{M_i^{(V)}+R_i} (x_i - M_i^{(V)}) f_i(x_i) dx_i + wR_i \int_{M_i^{(V)}+R_i}^{+\infty} f_i(x_i) dx_i + w_r R_i \\ &+ p_r^{(R)} M_i^{(R)} \int_{0}^{M_i^{(V)}} f_i(x_i) dx_i + p_r^{(R)} \int_{M_i^{(V)}}^{M_i^{(V)}+R_i} (M_i^{(V)} + M_i^{(R)} - x_i) f_i(x_i) dx_i \\ &+ p_r^{(R)} \int_{M_i^{(V)}+R_i}^{+\infty} (M_i^{(R)} - R_i) f_i(x_i) dx_i.\end{aligned} \tag{3.33}$$

By taking the demand uncertainty into consideration, the ROO's expected profit in service area i can be written as

$$\Pi_{2i}^{(R)} = \begin{cases} (1-q_i)\Pi_{2Ci}^{(R)} + q_i\Pi_{2Ai}^{(R)} & \text{if } R_i \le S_i, \\ (1-q_i)\Pi_{2Ci}^{(R)} + q_i\Pi_{2Bi}^{(R)} & \text{if } R_i > S_i. \end{cases} \tag{3.34}$$

The ROO's total expected profit is $\Pi_2^{(R)} = \sum_{i=1}^n \Pi_{2i}^{(R)}$.

In competitive scenario, without backup reservation, the VNO's expected profit is written as in (3.9). The ROO's expected profit in each service area i is

$$\begin{aligned} \Pi_1^{(R)} &= q_i[(M_i^{(R)} - S_i)p_r^{(R)} + p_r^{(R)} S_i \int_{M_i^{(V)}+\frac{S_i}{\theta\beta}}^{+\infty} f_i(x_i)dx_i \\ &+ \theta\beta p_r^{(R)} \int_{M_i^{(V)}}^{M_i^{(V)}+\frac{S_i}{\theta\beta}} (x_i - M_i^{(V)}) f_i(x_i)dx_i] + (1-q_i)p_r^{(R)} M_i^{(R)}. \end{aligned} \tag{3.35}$$

In competitive scenario, with backup reservation, the VNO's expected profit is written as in (3.11). As for the ROO, three cases need to be considered:

Case 1: With low demand, and $R_i \le S_i$. The ROO's expected profit is

$$\begin{aligned} \Pi_{2Ai}^{(R)} &= (M_i^{(R)} - S_i)p_r^{(R)} + w_{ri}R_i + wR_i \int_{M_i^{(V)}+R_i}^{+\infty} f_i(x_i)dx_i + w\int_{M_i^{(V)}}^{M_i^{(V)}+R_i} (x_i - M_i^{(V)}) f_i(x_i)dx_i \\ &+ \theta\beta p_r^{(R)} \int_{M_i^{(V)}+R_i}^{M_i^{(V)}+R_i+\frac{S_i-R_i}{\theta\beta}} (x_i - M_i^{(V)} - R_i) f_i(x_i)dx_i + p_r^{(R)} \int_{M_i^{(V)}+R_i+\frac{S_i-R_i}{\theta\beta}}^{+\infty} (S_i - R_i) f_i(x_i)dx_i. \end{aligned} \tag{3.36}$$

Case 2: With low demand, and $R_i > S_i$. The ROO's expected profit is

$$\Pi_{2Bi}^{(R)} = (M_i^{(R)} - R_i)p_r^{(R)} + w_{ri}R_i + wR_i \int_{M_i^{(V)}+R_i}^{+\infty} f_i(x_i)dx_i + w \int_{M_i^{(V)}}^{M_i^{(V)}+R_i} (x_i - M_i^{(V)})f_i(x_i)dx_i$$
$$+ p_r^{(R)} \int_0^{M_i^{(V)}+S_i} (R_i - S_i)f_i(x_i)dx_i + p_r^{(R)} \int_{M_i^{(V)}+S_i}^{M_i^{(V)}+R_i} (R_i - x_i + M_i^{(V)})f_i(x_i)dx_i. \tag{3.37}$$

Case 3: With high demand. The ROO's expected profit in each service area i is

$$\Pi_{2Ci}^{(R)} = w_{ri}R_i + wR_i \int_{M_i^{(V)}+R_i}^{+\infty} f_i(x_i)dx_i + w \int_{M_i^{(V)}}^{M_i^{(V)}+R_i} (x_i - M_i^{(V)})f_i(x_i)dx_i$$
$$+ M_i^{(R)}p_r^{(R)} \int_0^{M_i^{(V)}} f_i(x_i)dx_i + p_r^{(R)} \int_{M_i^{(V)}}^{M_i^{(V)}+R_i} (M_i^{(R)} - x_i + M_i^{(V)})f_i(x_i)dx_i$$
$$+ p_r^{(R)}(M_i^{(R)} - R_i) \int_{M_i^{(V)}+R_i}^{+\infty} f_i(x_i)dx_i. \tag{3.38}$$

By taking the demand uncertainty into consideration, the ROO's expected profit in service area i can be written as

$$\Pi_{2i}^{(R)} = \begin{cases} (1-q)\Pi_{2Ci}^{(R)} + q\Pi_{2Ai}^{(R)} & \text{if } R_i \le S_i, \\ (1-q)\Pi_{2Ci}^{(R)} + q\Pi_{2Bi}^{(R)} & \text{if } R_i > S_i. \end{cases} \tag{3.39}$$

Chapter 4

Collaboration among Attackers with Security Information Trading

4.1 Introduction

Target information is undoubtedly a crucial factor of security problems in various attacks against critical infrastructures such as transportation and computer networks. Attackers conduct surveillance to gain awareness of targets' vulnerabilities and security operations, based on which to decide where to attack and how much effort to take in attacking [75, 76]. In reality, most often attackers have limited observation capabilities such that they may have only little or partial information about the target's vulnerability [77]. However, in some situations, the attackers do not necessarily need to observe by themselves to gain the information. The widespread use of and thus an immense demand for potential target information in hacker communities has spawned a *data brokers* industry [27]. The data brokers in crime society are specialists in collecting target information (e.g., software vulnerabilities, snippets of code, credit card numbers, and compromised accounts) and sell them in black markets in exchange for financial gain [28]. For example, users of the underground forums regularly engage in the buying, selling and trading of illegally obtained information to support criminal activities [78]. As a report [79] published by

TrendMicro states: “Underground hackers are monetizing every piece of data they can steal or buy and are continually adding services so other scammers can successfully carry out online and in-person fraud.” The Shadow Brokers, which trades in compromised network data and exploits, is a representative of such a data broker as a hacker group. In June 2017, the computer virus NotPetya was able to spread by leveraging a vulnerability leaked by the Shadow Brokers [80]. More recently, Facebook, accused of privacy violations that could provide “material support” for terrorism potentially, was reported to face multibillion-dollar FTC fine [30]. Indeed, data brokers, as a boon to the cybercrime economy [81], have become an indispensable member of the illegally evolved supply chain called “cybercrime-as-a-service” [82].

While we do not have a clear picture of the information trading behaviors in the underground society, security researchers are taking more interest in exploring hacker communities. Initial studies of security experts have reached a consensus that one major motivation of hackers is profit-related (others include fame and skill improvement etc.) [83]. Our aim in this work is to study the profit-driven attacking behaviors in a hacker community, with a particular emphasis on the role of target information provided by a data broker in security using economic analysis. More precisely, we would like to understand the value of traded target information—both for the sellers of this information and for the attackers that buy it. Through an economic analysis of the attacking behaviors with information trading, we would be able to provide a simple glimpse of complex social structure and to better understand the phenomenon of hacking. This knowledge would provide insights for arriving at effective solutions to information-leakage-induced security problems.

We consider one or multiple attackers that have limited observation capabilities on the potential target. They can approach a data broker that holds the vulnerability of the target. The target vulnerability determines how much effort the attackers need to take in order to launch a successful attack. Without the information, the attacker may choose not to act, fail, or exert more effort than needed. The attackers could benefit from purchasing the information by launching a more targeted attack with less effort. Here we care about the value of the information for the attackers and how the data broker should price the information if they can obtain it, but how the data broker could obtain the information is beyond our focus. Besides, we talk about the scenario of multiple attackers when the target value can be shared among them if they all deliver successful attacks. The assumption of competition among attackers through dividing up the value of a single asset is appropriate when they share the benefits of private goods as illegal resource access (e.g., spectrum or other network resource utilization) and monopoly privileges (e.g., stealing electronically stored information about consumers' personal data for market exploration). Similar assumptions can be found in [34], which adopts a rent-seeking model of security games where the asset value is divided among the attackers and the defender. We are interested in whether there is a positive or negative network externality in the information market due to the competition among the attackers, that is, would the existence of more potential buyers increase the value of the data broker's information or decrease it. We also analyze an independent attacking scenario when the target is a type of "public good" and each successful attacker can obtain the whole target value regardless of other attackers' attacking behaviors. A comparison study is conducted between the competition and the independent scenario to better illustrate the impacts of the competition between attackers.

With the observation of the hierarchical and competitive structure in attacker behaviors, we present and study a multi-stage model of the information market. In Stage I, the data broker determines the information price to the attackers. In Stage II, the attackers decide whether to buy or not. In Stage III, after obtaining the target information, the attackers decide whether to attack the target or not. The composed game provides an integrated view of a security problem with competing attackers and target information trading. The research questions we aim to answer include: (a) How would the attacker change its attacking decision once it has bought some detailed information about the target's vulnerability from the data broker? (b) How does the competition between the attackers affect their information purchasing decision and attacking decisions? (c) For different attacker models (homogeneous/heterogeneous attackers), what are the conditions under which the attackers would benefit from the existence of the traded information? How does the value of information differ for different attackers? (d) Is it beneficial for the data broker to set a low price such that all attackers would buy the information? Or should the data broker enhance the price when there are more potential buyers rather than one? (e) How are the decisions affected when the data has lower quality (only partial information is available for trading)? Besides, we extend our model by taking the cooperative purchasing strategy into consideration, pertaining to two related questions: (f) when and how much do the attackers benefit from cooperative purchasing? (g) how does the cooperative purchasing strategy affect the risks of the target and the information price?

The problems are challenging due to the following two reasons. First, there is a lack of a systematic or quantitative framework to evaluate the information in a competitive crime community. Although it is intuitive that the more information, the better for the attackers, questions are still unexplored as what is the highest price that the attacker can accept? Does an attacker always benefit from buying the information if other attackers also buy it? Or is the information more valuable if other attackers do not buy it? To the best of our knowledge, this is the first work that tries to provide a unified quantitative framework of the security information market comprised of one data broker and multiple attackers. We will provide insights regarding the impacts of target information trading on various parties: the increased attacking probability of the target, the expected utility increase for the attackers, and the profit through selling the information for the data broker.

Second, the attacker behaviors are interdependent across multiple stages. On one hand, the attacking decisions, including whether or not to attack, and with how much effort, are affected by the attackers' knowledge of the target. On the other hand, whether or not to buy the information is determined by how much utility gain can be expected from attacking. The competition among multiple attackers makes these decisions even more complex. This is different from most competition analyses when a product can be sold to only one buyer and the game ends after the purchasing is done. Therefore, the structure of the game varies across the stages. We will model the game among the attackers as Bayesian games, to capture their limited observability, and model their purchasing-attacking decision process as a stochastic game. Besides, from the data broker's perspective, the purchasing probability of the buyer is not only determined by the competition game equilibrium among the attackers but also affected by the target value and the price. We will use a Stackelberg game to model the pricing and purchasing decisions of the players.

Our main contributions can be summarized as follows.

- While most traditional security game models assume that target information is obtained through attackers' self-observation and learning, we consider an information market in hacker communities and propose a game-theoretic framework, which captures the multi-stage correlated behaviors of attackers. This information market model better fits the practice of profit-seeking hacker communities with labor specialization. Our results show how the traded information would benefit or hurt the attackers when they are competing and heterogeneous. We also show that in this information channel, information accuracy is more valuable for a more attractive target.

- Much previous work focuses on interactions between a defender and a single or multiple independent attackers, without consideration of the competition/cooperation among the attackers or the role of other players that assist in attacking. We incorporate the strategic interactions between multiple attackers as in a Bayesian and stochastic game, and between the attackers and the data broker as in a Stackelberg game. Our analysis indicates that the value of information for the attackers could be weakened by their competition, and interestingly, the data broker might benefit from the competition between attackers. Besides, it is beneficial for the attackers to cooperate in purchasing only when the price is not high. We also show that, with the assistance of a data broker, the target suffers from higher risks even when the information price is too high to benefit the attackers. The risk is increased in a certain range confined by both the price and the target value.

- We provide the equilibrium solutions and characterize the conditions for the existence and uniqueness of the equilibrium under different target values. We show that if the target is not attractive enough, there may be multiple pure-strategy equilibria in the attackers' competition game. Whether there is a strictly dominant pure-strategy is determined by the target value, the target vulnerability, and the information price. Furthermore, in the subgame-perfect Nash equilibrium where the strategies are mutual best responses [84], it is not wise for the data broker to set a price low enough to attract all the buyers if the target is attractive enough to the attackers.

4.2 System Model and Problem Formulation

We consider two attackers trying to attack one potential target. The attackers have limited ability to obtain the vulnerabilities (or the protection level set by the defender) about the target. But they can purchase the information from a data supplier, who has full or partial knowledge about the target's vulnerabilities. The data supplier first sets a price for the information. Given the price, the attackers determine whether or not to make the purchase. Afterward (when the information has been revealed to the attackers), they need to decide whether to attack the target. All the players in the model are profit-motivated. The notations used throughout this chapter are given in Table 5.1.

If an attacker successfully attacks the target, its expected reward is $v > 0$; otherwise, it receives a payoff of zero. The value of v (also called the target value), reflecting the target attractiveness to the attackers, is common knowledge to all the players. We restrict the model to the target resource whose consumption by one agent would reduce consumption by others. That is, when multiple attackers successfully attack the target, they equally split the target value. In a two-attacker case, each gets a payoff of $\frac{1}{2}v$. This assumption relies on the fact that the target pool in reality is finite and attackers compete for a common

Table 4.1: List of Notation for Chapter 4.

Notation	Explanation
v	Target value
e_i	Attacker i's attacking effort $(i, j \in \{A, B\})$
$C(\cdot)$	Attacking cost ($C(e) = e$)
θ	Target protection level (minimum attacking effort)
f_i	Attacker i's expected utility
p	Information price
q_i^{attack}	Attacker i's attacking probability
q_i^{buy}	Attacker i's information purchasing probability
π	Data broker's expected profit.
u_i	Expected reward of a single successful attacker i
u_{ij}	Attacker i's expected reward with successful attackers i, j

asset pool [85]. Note that it holds in two realistic scenarios: Firstly, one circumstance is a case where the 'first-winner-takes-all" [86]. That is, the first attacker who mounts a successful attack receives the entire reward v, and the following attackers receive nothing. Considering the equal probability of being the first one to be successful, the expected reward of an attacker is $\frac{1}{2}v$ in a two-attacker case. Secondly, the target is assumed to be a quasi-public asset or display a negative network effect, whose value can diminish as more people use it. With the attackers being homogeneous, each attacker gets an equal share of the target value. This assumption of homogeneous attackers with equal share rule has been widely used in the security games literature ([87, 86, 85]). Apart from simplifying the analysis and exposition of the results, this model setup allows us to isolate the effect of the attacker heterogeneity and to highlight the effects of competition among attackers. We will relax these assumptions to analyze a more general case of heterogeneous attackers in Section 4.9.

We define the success of an attacker as follows: if the attacker's effort e in attacking is greater than or equal to the target's protection level by its defender (or owner), we say the attacker succeeds in the attack. The problem is, the attacker itself is not aware of the exact value of the target protection level, which determines the minimum level of effort for a successful attack. With a slight abuse of notation, let θ denotes the target protection level (from the other side: the target vulnerability) or the minimum attacking effort needed. A smaller value of θ indicates lower surveillance and thus less effort to launch a successful attack. Let us suppose the attackers only know the distribution of θ, which is uniformly distributed on $[0, 1]$ (normalized with respect to a sufficiently large upper bound that denotes the maximum possible defense capability of the defender). If an attacker's effort e is less than the actual value of θ, then it will fail. Note that the model focuses on the single-period attacking games where the attackers make one-shot decisions at the beginning with little information about the target vulnerability, and leaves out the multi-round attack problem where the information available to attackers could be updated from their observations in each round. As in [88], [89], and [90], for such a single-period attack game, the probability of a successful attack is assumed to be conditional on the attacker's strategic decision in terms of effort or investment allocation made before mounting the attack. This setup helps us to concentrate on the value of information acquired in black markets. The single-period framework can be justified by the fact that with the increasingly fierce confrontation in cyberspace, the process of cyber defense is typically characterized by rapid and frequent security patch upgrades, and as a result the effective lifespan of a fully disclosed vulnerability is short, shrinking the window of opportunity for the attacker [91, 92]. Therefore, in practice, there is often a small chance for multi-round attacks to take place, and the attack deployment decision can be made often only once. For example, a typical scenario that falls into this situation in practice is that after the first round of an attack, the vulnerability of the target being exploited by the attack is revealed to the defender, which will subsequently apply a security patch to fix that vulnerability, hence mitigating future rounds of the attack. The single-period setting also applies to the case where the duration of the target is short and hence it is impractical to mount multi-round attacks, such as eavesdropping on specific information that is transmitted once on a wireless network or jamming to disrupt a specific data transmission [92]. For those attacks that can be launched for multiple stages or rounds, this model applies to the first stage/round where little target information has been obtained, and the expected gain of a successful attack denotes the expected future gain for the next few rounds. While this model does not cover the cases of attackers' learning behaviors and advanced persistent threats, this model of profit-driven attackers purchasing information in black markets covers a significant fraction of realistic security concerns induced by information leakage.

Measured in both the success probability of an attack and the expected payoff, the attacker's total utility function with its attacking effort e is written as

$$f = \mathbf{1}_{e \geq \theta}(e) * v - C(e). \tag{4.1}$$

Here $\mathbf{1}_{e \geq \theta}(e)$ is an indicator function, defined as: $\mathbf{1}_{e \geq \theta}(e) = 1$ if $e \geq \theta$, else $\mathbf{1}_{e \geq \theta}(e) = 0$. $C(e)$ is the attacking cost that increases with the effort e. We will assume $C(e) = e$ for simplicity. Although this assumption represents a simple linear function between the effort and the cost, it is reasonable and would not affect the major insights obtained from the analysis.

The data supplier is a broker who collects and sells data about the target vulnerability or the target owner's protection level. This information tells how much effort is needed to launch a successful attack for the attackers, i.e., the actual value of θ. An attacker who buys the information could launch a targeted attack with exactly the minimum level of effort needed. We all also study the situation when the data broker only has partial information about the target, which means that the information could only tell a more accurate range of θ than the attacker has. We are interested in how the data broker chooses to sell the data and what is the information value for all the players, and ignore the details of how the broker acquires the data.

We provide a framework for analyzing how the attacker's optimal information purchasing and attacking decisions could be made in the face of the competition and uncertainty about the target vulnerabilities. The attacker's objective is to maximize the expected benefit from an attack (taking into account the attacker's target valuation, the success probability of an attack, and the cost involved in purchasing and attacking). The data broker sets the information price to maximize the expected profit (taking into account the purchasing probability of the attackers).

The model's timing proceeds as follows:

*Step*1. The data broker determines and broadcasts the information price p.

*Step*2. The attackers decide whether to buy the information or not. After the payments are made, the data broker delivers the target information to the buyer(s).

*Step*3. With the information available, the attackers decide how much effort will be taken in attacking (zero effort means not to attack).

*Step*4. After the attack, the corresponding utilities are gained by the attackers.

4.3 Single Attacker Model

As a benchmark, we introduce the model where a monopolist attacker will extract all surplus from successfully attacking the target. Considering the sequential-move nature of the bargaining process between the attacker and the data broker, a two-stage Stackelberg game [84] is employed to analyze the decisions of the players. In practice, the data broker (the Stackelberg game leader) considers what the best response of the attacker (the follower) is (note that the follower's best response to an action of the leader is a piece of known information to the leader), i.e. whether it will buy the information once it has been informed of the price. The data broker then picks a price that maximizes its expected profit, anticipating the predicted response of the follower. The attacker actually observes this and in equilibrium makes an optimal purchasing decision as a response. The subgame-perfect Nash equilibrium, normally deduced by "backward induction" [84], is obtained in the following analysis. First, for a given information price, we derive the attacker's optimal purchasing and attacking decisions. Second, with the prediction of the attacker's response, the data broker's optimal price is analyzed.

4.3.1 Optimal Decisions of the Attacker

The attacker needs to make the decision of whether to buy the target information from a data broker, by comparing the two expected utilities as follows.

Not Buy Information

If the attacker does not buy the information from the data broker, its expected utility function with effort level e is

$$f(e) = \int_0^e v d\theta - e = ve - e. \tag{4.2}$$

So the optimal solution is $e = 1$ with $f = v - 1$ if $v > 1$ and $e = 0$ with $f = 0$ if $v \le 1$.

Buy Information

If the attacker decides to buy the information θ at price p and to attack the target, it would attack with exactly the effort θ.

Case 1: $v > 1$

The attacker would always attack since $\theta \le v$ in this case, and its expected utility function is

$$f = \int_0^1 (v - \theta) d\theta - p = v - \frac{1}{2} - p. \tag{4.3}$$

Case 2: $v \le 1$

Only when $\theta < v$ would it attacks. Then its expected utility function is

$$f = \int_0^v (v - \theta) d\theta - p = \frac{1}{2} v^2 - p. \tag{4.4}$$

We can derive the attacker's optimal purchasing decision by comparing (4.3) and (4.4) with (4.2): When $v > 1$, if $v - \frac{1}{2} - p > v - 1$, or $p < \frac{1}{2}$, then the attacker would buy the information, otherwise it prefers not to buy the information. When $v \le 1$, if $\frac{1}{2}v^2 - p > 0$ or $p < \frac{1}{2}v^2$, then the attacker would buy the information, otherwise it prefers not to buy the information. Fig. 4.1 plots the regions of the attacker's optimal decisions with different values of information price and target value. The attacker buys the target information only in regions I and III. On the other hand, in region II, the attacker would attack with the greatest effort $e = 1$; while in region IV, the attacker would neither buy nor attack. Specifically, the value of information for the attacker lies in region I where it helps to deduce the effort taken, or region III where an attack is profitable when $\theta < v$. In other words, the value of the information for the attacker is an expected utility increase of $v - \frac{1}{2} - p - (v-1) = \frac{1}{2} - p$ if $v > 1$ and $p \leq \frac{1}{2}$ or $\frac{1}{2}v^2 - p$ if $v \leq 1$ and $p \leq \frac{1}{2}v^2$.

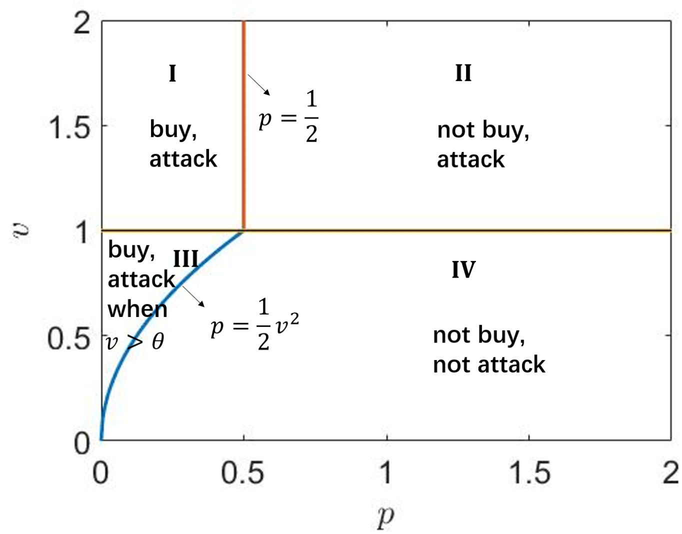


Figure 4.1: The optimal decisions of single attacker.

Besides, what the defender (or target owner) cares about is whether or not the attacker would choose to attack the target and with how much effort (i.e., successful or not). When no information is available to the attacker, it would not attack the target as long as $v \leq 1$. But when a data broker sells the information with a price low enough, the target would be successfully attacked even if $v \leq 1$. Therefore, the target is affected by the information trading in region III, which implies the importance of protecting the target information for the defender especially when the target value is not high for the attackers.

4.3.2 Optimal Pricing Decisions of the Data Broker

We assume that when the attackers are indifferent to whether to buy or not to buy, they always choose to buy in favor of less uncertainty. If $v \leq 1$, the information price cannot be set to be larger than $p = \frac{1}{2}v^2$, otherwise no profit can be gained by the data broker. Therefore, the Stackelberg game equilibrium strategy of the data broker is given as follows:

$$p^* = \begin{cases} \frac{1}{2}v^2 & \text{if } v \leq 1, \\ \frac{1}{2} & \text{if } v > 1. \end{cases} \tag{4.5}$$

Under this price, the attacker's equilibrium strategy is to buy the information and mount the attack when $\theta < v$. The corresponding expected profit of the data broker, denoted as π, is $\frac{1}{2}v^2$ when $v \leq 1$ and $\frac{1}{2}$ when $v > 1$. We can see that the information value for the data broker increases with the target value until the target becomes attractive enough to the attacker that it would attack anyway even without the information.

4.4 Competition Model

In this section, we consider the scenario when there are two attackers (A and B) that could buy the same data of a target from the data broker. The attackers make decisions independently. Following the work of [27], we restrict our attention to the case where the data set is sold only in one time-block at Step 2 and this trade information is common knowledge (i.e., the data broker is willing to publicize its total sales quantity). The interactions between the data broker and two attackers are formulated in a three-stage hierarchical order of decision making, as shown in Fig. 4.2. Stage I: the data broker, as the leader in the Stackelberg game, sets the information price. Stage II: the attackers, as the followers, play a simultaneous game of purchasing. Stage III: the attackers make their attacking decisions with the information available when they play a simultaneous game of attacking. We use backward induction to derive the equilibrium outcomes: we first derive the attackers' optimal attacking decisions and their expected utilities assuming they have or have not bought the information, and then analyze their optimal purchasing decisions. Finally, we obtain the optimal pricing decisions for the data broker.

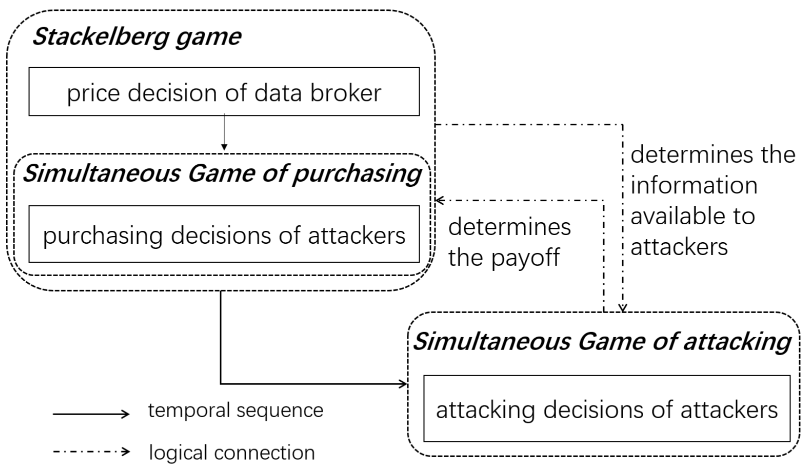


Figure 4.2: Hierarchical game structure.

4.4.1 Games of Attacking

For given purchasing decisions from two attackers, the attacking game is denoted by a tuple $\langle Q, S, U\rangle$. Set Q contains two players: $\{$attacker A, attacker B$\}$. Set S contains the strategies available to players: {attack, not attack} for each player. Set U contains the players' utility functions, which depend on the informational structure—that is, on which attackers purchase information. In the following analysis, we will characterize the solutions for the attacking game under different sets of purchasing decisions. We will use the solution concept of Nash equilibrium where no player can increase its own expected payoff by changing its strategy while the other player keeps its strategy unchanged.

Neither Buys Information

We first consider the situation when neither of the attackers decides to buy the information from the data broker. Whether or not the attackers would attack is determined by the value of the target. Therefore, we analyze the results of the attacking games with different values of v.

Case 1: $v > 2$

If both attackers decide to attack, with effort e_A and e_B respectively, we suppose $e_A \leq e_B$ without loss of generality. Then attacker A's expected utility is $f_A(e_A) = \frac{1}{2}v\int_0^{e_A} d\theta - e_A = (\frac{1}{2}v - 1)e_A$, and attacker B's expected utility is $f_B(e_B) = \frac{1}{2}v\int_0^{e_A} d\theta + v\int_{e_A}^{e_B} d\theta - e_B$. To maximize $f_A(e_A)$, we have $e_A = e_B = 1$. If only one attacker attacks, its optimal decision is $e = 1$ when $f = v - 1$, and the other attacker has zero utility. The payoffs matrix of the game when neither of them has bought the information is illustrated in Table 4.2 (with attacker A's strategies listed in rows and attacker B's strategies listed in columns). The only strictly dominant pure-strategy equilibrium can be analyzed as (attack, attack) with utility $f = \frac{1}{2}v - 1$ for both attackers.

Case 2: $1 < v \leq 2$

Table 4.2: Payoff table for game of attacking when neither of the attackers has the information.

	attack	not attack
attack	$\frac{1}{2}v-1, \frac{1}{2}v-1$	$v-1, 0$
not attack	$0, v-1$	$0, 0$

There are two pure-strategy Nash equilibria: (attack, not attack) and (not attack, attack). In such situations we will focus on the mixed-strategy Nash equilibrium solution [93], in which the player assigns a positive probability to every pure strategy. We suppose attacker A chooses to attack w.p. q_A^{attack} and attacker B attacks w.p. q_B^{attack}. Then in mixed-strategy Nash equilibrium, $f_B(attack) = q_A^{attack}(\frac{1}{2}v-1) + (1-q_A^{attack})(v-1) = f_B(not\ attack) = 0$, and a similar equation holds for attacker A. Therefore, $q_A^{attack} = q_B^{attack} = 2\frac{v-1}{v}$, and the expected utility for both attackers is $f = (2\frac{v-1}{v})(2\frac{v-1}{v})(\frac{1}{2}v-1) + 2\frac{v-1}{v}(1-2\frac{v-1}{v})(v-1) = 0$.

Case 3: $v \leq 1$

By using a similar analysis as above, we can derive that the only strictly dominant pure strategy is (not attack, not attack) with utility $f = 0$ for both attackers.

Both Buy Information

When both attackers buy information from the data broker, they will make the attacking decision after they obtain the information. Therefore, the attacking game is influenced by two factors: the target value and the minimum effort needed for a successful attack.

Table 4.3: Payoff table for game of attacking when both attackers buy the information.

	attack	not attack
attack	$\frac{1}{2}v-\theta-p, \frac{1}{2}v-\theta-p$	$v-\theta-p, -p$
not attack	$-p, v-\theta-p$	$-p, -p$

Case 1: $v > 2$

The attackers always benefit from attacking even if they split the value v since $\frac{1}{2}v > \theta$. Therefore, it is straightforward to derive that the only strictly dominant pure strategy is (attack, attack), and their expected utility is

$$f = \int_0^1 (\frac{1}{2}v - \theta)d\theta - p = \frac{1}{2}v - \frac{1}{2} - p. \tag{4.6}$$

Case 2: $1 < v \leq 2$

If both attackers decide to attack after they get the information θ, they both get a utility of $\frac{1}{2}v - \theta - p$. If only one attacker attack, then it gets a utility of $v - \theta - p$, while the other one gets $-p$. Their payoffs for this game are listed in Table 4.3. When $\frac{1}{2}v - \theta > 0$, the only pure-strategy Nash equilibrium is (attack, attack). When $\frac{1}{2}v - \theta \leq 0$, in mixed-strategy Nash equilibrium, we have $q_A^{attack} = q_B^{attack} = 2(1 - \frac{\theta}{v})$, and the expected utility for both attackers is $-p$. Therefore, the expected utility of each attacker is

$$f = \int_0^{\frac{1}{2}v} (\frac{1}{2}v - \theta)d\theta + \int_{\frac{1}{2}v}^1 0d\theta - p = \frac{1}{8}v^2 - p. \tag{4.7}$$

Case 3: $v \leq 1$

If $\theta \geq v$, neither of the attackers would attack. If $\theta < v$, the attacker gets a utility of $\frac{1}{2}v - \theta - p$ when both of them attack, and $v - \theta - p$ when only one attacker attacks. Therefore, the only pure-strategy Nash equilibrium is (attack, attack) when $\frac{1}{2}v - \theta > 0$, and in mixed-strategy Nash equilibrium when $\frac{1}{2}v - \theta \leq 0$, the attacker would attack w.p. $q_A^{attack} = q_B^{attack} = 2(1 - \frac{\theta}{v})$ and have an expected utility of $-p$. To sum up, the expected utility of each attacker is $f = \frac{1}{8}v^2 - p$.

Only One Attacker Buys Information

Without loss of generality, we consider the scenario where only attacker A buys the information. The payoffs for this game are listed in Table 4.4. It is important to note that attacker A's payoff function is its private information since the exact value of θ is not available to attacker B, and only its probability distribution is commonly known (with θ uniformly distributed on [0,1]). Therefore, the attacking game can be formulated as a Bayesian game with incomplete information [94]. We will analyze the Bayesian Nash equilibrium [94], where each attacker makes the decision to maximize its expected payoff. In this case, attacker A makes the decision after obtaining the value of θ from the data broker, while attacker B maximizes its expected payoff based on the distribution of θ.

Table 4.4: Payoff table for game of attacking when only attacker A buys the information.

	attack	not attack
attack	$\frac{1}{2}v - \theta - p$, $\frac{1}{2}v - 1$	$v - p - \theta, 0$
not attack	$-p, v - 1$	$-p, 0$

Case 1: $v > 2$

If both attackers decide to attack, then attacker B's utility function is $f_B(e_B) = \int_0^{e_B} \frac{1}{2}v d\theta - e_B = (\frac{1}{2}v - 1)e_B$, with $e_B = 1$ when $v > 2$. Attacker A's utility is therefore $\frac{1}{2}v - \theta - p$. If only attacker A attacks, then $f_A = v - p - \theta$, and $f_B = 0$. Else if only attacker B attacks, then $f_A = -p$, and $f_B = v - 1$. Therefore, the only pure-strategy Nash equilibrium is (attack, attack) with $f_A = \frac{1}{2}v - p - \frac{1}{2}$ and $f_B = \frac{1}{2}v - 1$.

Case 2: $1 < v \leq 2$

In this case, attacker B knows that if $\frac{1}{2}v - \theta \geq 0$, attacker A will certainly attack; that is, the probability of attacker A attacking is greater than or equal to the probability of

$\frac{1}{2}v - \theta \geq 0$: $q_A^{attack} \geq \int_0^{\frac{1}{2}v} d\theta = \frac{1}{2}v$. If we assume attacker B would attack w.p. q_B^{attack}, then its expected utility is $q_A^{attack} * q_B^{attack} * (\frac{1}{2}v - 1) + q_B^{attack} * (1 - q_A^{attack}) * (v - 1) = q_B^{attack} * (v - 1 - \frac{1}{2}vq_A^{attack})$. Since $q_A^{attack} \geq \frac{1}{2}v$, we have $v - 1 - \frac{1}{2}vq_A^{attack} \leq 0$. Therefore, to maximize attacker B's expected utility, $q_B^{attack} = 0$. Because attacker B always chooses not to attack, attacker A would attack. To sum up, the expected utilities are: $f_A = v - p - \frac{1}{2}$, and $f_B = 0$.

Case 3: $v \leq 1$

Attacker B would not choose to attack even when attacker A does not attack. In this case, attacker A chooses to attack only when $\theta > v$. Therefore, $f_A = \int_0^v (v - p - \theta) d\theta + \int_v^1 (-p) d\theta = \frac{1}{2}v^2 - p$, and $f_B = 0$.

4.4.2 Games of Purchasing

In the game of purchasing, the attackers decide whether to buy the information or not. Their joint strategies determine the specific payoff matrix of the attacking game to be played in the next stage. For example, if both attackers purchase the information, they will play the attacking game shown in Table 4.3; if only one attacker makes the purchase, they will play the game in Table 4.4. Note that only the distribution of θ is known to both attackers before making their purchasing decisions. Therefore, the whole decision process of the attackers can be modeled as a stochastic game, in which there are multiple players and the next state of the game depends on the joint action of the players [95]. In this subsection, we will derive the Nash equilibrium of the purchasing game under different values of v, as shown in Tables 4.5,4.6, and 4.7, where the attackers' payoffs are their expected utilities based on the equilibrium of the corresponding attacking games determined by their purchasing strategies.

Table 4.5: Payoff table for game of purchasing when $v \leq 1$.

	buy	not buy
buy	$\frac{1}{8}v^2 - p$, $\frac{1}{8}v^2 - p$	$\frac{1}{2}v^2 - p$, 0
not buy	0,$\frac{1}{2}v^2 - p$	0, 0

Case 1: $v \leq 1$

According to the payoffs in Table 4.5, if $p < \frac{1}{8}v^2$, the only pure-strategy Nash equilibrium is (buy, buy). If $\frac{1}{8}v^2 \leq p < \frac{1}{2}v^2$, there are two pure-strategy Nash equilibria: (buy, not buy) or (not buy, buy). In mixed strategy equilibrium, assume attacker A buys the information w.p. q_A^{buy} and attacker B buys w.p. q_B^{buy}. We have $f_B(buy) = q_A^{buy}(\frac{1}{8}v^2 - 1) + (1 - q_A^{buy})(\frac{1}{2}v^2 - 1) = f_B(not\ buy) = 0$, and a similar equation holds for attacker A. Therefore, $q_A^{buy} = q_B^{buy} = \frac{\frac{1}{2}v^2 - p}{\frac{3}{8}v^2}$, and the expected utility for both attackers is 0. If $p \geq \frac{1}{2}v^2$, the only pure-strategy Nash equilibrium is (not buy, not buy).

Table 4.6: Payoff table for game of purchasing when $1 < v \leq 2$.

	buy	not buy
buy	$\frac{1}{8}v^2 - p$, $\frac{1}{8}v^2 - p$	$v - \frac{1}{2} - p$, 0
not buy	0,$v - \frac{1}{2} - p$	0, 0

Case 2: $1 < v \leq 2$

According to the payoffs in Table 4.6, if $p < \frac{1}{8}v^2$, the only pure-strategy Nash equilibrium is (buy, buy). If $\frac{1}{8}v^2 \leq p < v - \frac{1}{2}$, there are two pure-strategy Nash equilibria (buy, not buy) or (not buy, buy). In mixed strategy equilibrium, we have $q_A^{buy} = q_B^{buy} = \frac{v - \frac{1}{2} - p}{v - \frac{1}{2} - \frac{1}{8}v^2}$. If $p \geq v - \frac{1}{2}$, the only pure-strategy Nash equilibrium is (not buy, not buy).

Case 3: $v > 2$

Table 4.7: Payoff table for game of purchasing when $v > 2$.

	buy	not buy
buy	$\frac{1}{2}v - \frac{1}{2} - p$, $\frac{1}{2}v - \frac{1}{2} - p$	$\frac{1}{2}v - \frac{1}{2} - p$, $\frac{1}{2}v - 1$
not buy	$\frac{1}{2}v - 1$, $\frac{1}{2}v - \frac{1}{2} - p$	$\frac{1}{2}v - 1$, $\frac{1}{2}v - 1$

According to the payoffs in Table 4.7, if $p \geq \frac{1}{2}$, we have $\frac{1}{2}v - \frac{1}{2} - p < \frac{1}{2}v - 1$, and the only pure-strategy Nash equilibrium is (not buy, not buy). If $p < \frac{1}{2}$, we have $\frac{1}{2}v - \frac{1}{2} - p \geq \frac{1}{2}v - 1$, and the only pure-strategy Nash equilibrium is (buy, buy).

Fig. 4.3 shows the equilibrium purchasing decisions under different values of price p and target value v.

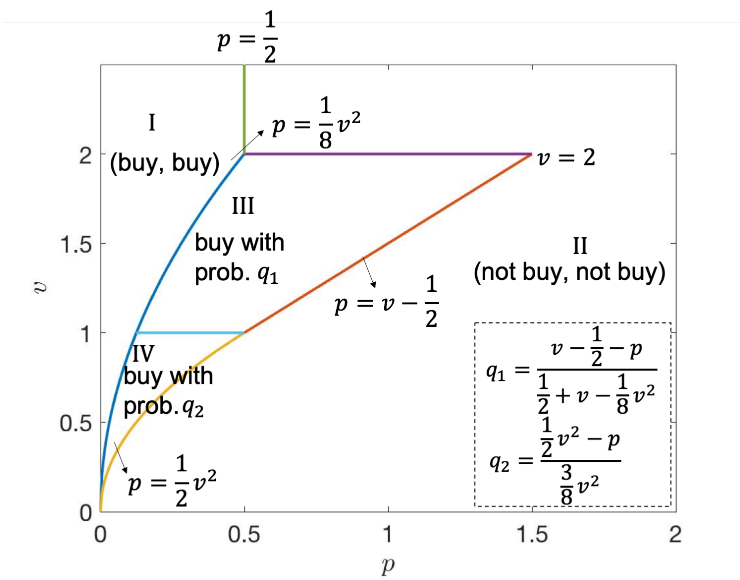


Figure 4.3: Optimal purchasing decisions of two attackers.

By integrating the results in the game of purchasing with those in the game of attacking, We can now analyze the impacts of information trading for the target. We already know that if no information is available, when $v > 2$, both attackers attack; when $1 < v \leq 2$, each attacker attacks w.p. $2\frac{v-1}{v}$, and when $v \leq 1$, no one attacks. While if

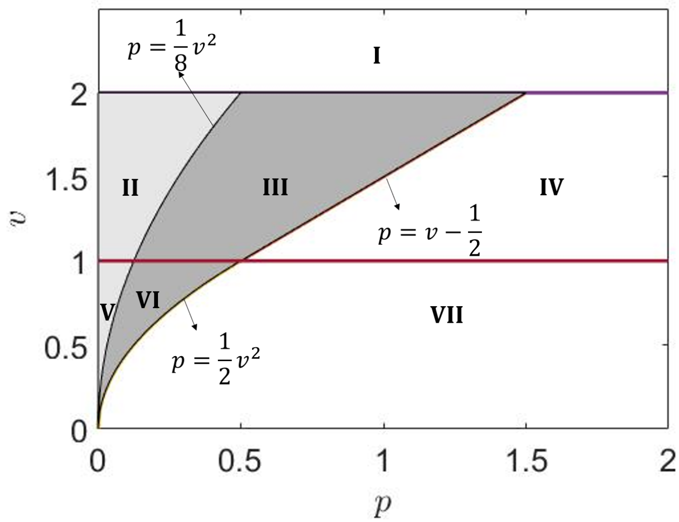


Figure 4.4: Regions where attacking probability of the target is increased.

the information is leaked and can be bought by the attackers at price p from a data broker, Fig. 4.4 summarizes the seven regions in parameter space with the shaded areas are where more possible attacks result from the information trading. The light grey region is where the attacking probability is definitely increased since the price is low enough that both attackers buy the information. The dark grey region is where an increase in attacking probability is possible, which is determined by the attackers' purchasing behaviors. The increase in risks due to the information leakage and trading can be shown in Table 4.8. The results imply that, when the value is highly attractive ($v \geq 2$), the attackers would launch the attack even without the information, and therefore the defender needs more defense effort; when $v < 2$, if the defender protects the information to the extent that the cost of acquiring and hence the price of information is high enough, the attack risk will not be increased, even if the information is leaked and traded.

Table 4.8: Attacking probability increase with information trading.

Region	Attacking probability increase
I, IV, VII	0
II	$\min[\frac{2}{v}-1, \frac{2}{v}(1-\theta)]$
III	$\frac{2}{v}(1-\theta)$ w.p. q_1^2, $\frac{2}{v}-1$ w.p. $q_1(1-q_1)$
V	$\min[1, 2(1-\frac{\theta}{v})]$
VI	1 when $v > \theta$ w.p. $q_2(1-q_2)$, $2(1-\frac{\theta}{v})$ w.p. q_2^2

One can also obtain the value of information for the attackers. If no information is available, when $v > 2$, both attackers obtain an expected utility of $\frac{1}{2}v - 1$; otherwise, both attackers get zero expected utility. If the target information can be bought, we could represent the value of information for the attackers as the amount of increase in the attacker's expected utility. If $v > 2$ and $p \leq \frac{1}{2}$, there is an expected utility increase of $\frac{1}{2} - p$; if $v \leq 2$ and $p \leq \frac{1}{8}v^2$, there is an increase of $\frac{1}{8}v^2 - p$. Fig. 4.5(a) shows the value of traded information for the attacker, which increases with the target value and decreases with the price. The results indicate that, in the mixed equilibrium of the competition game between the attackers, they are expected to benefit from the information only when $p \leq \frac{1}{8}v^2$ for $v \leq 2$ or $p \leq \frac{1}{2}$ for $v > 2$. However, even if the information does not benefit the attackers as the price increases, the target is expected to be attacked more likely with information leakage & trading (in regions III and VI of Fig. 4.4).

4.4.3 Optimal Pricing Decisions of the Data Broker

Now we further analyze the data broker's selling strategy in maximizing its profit. Intuitively, it could set either a low price such that both attackers buy or a high price that attackers buy with a certain probability. We show in Proposition 3.1 that its choice of pricing strategy depends on the attractiveness of the target.

The Stackelberg game equilibrium strategies are determined by the target value for the attackers. When the target is not attractive enough, it is not wise for the data broker to set a price low enough to attract two buyers. Specifically, at the equilibrium:

a) if $v \leq 1$, information is sold to the attackers at a price of $p^* = \frac{1}{4}v^2$, resulting each attacker making the purchase w.p. $2/3$;

b) if $1 < v \leq 2$, information is sold at $p^* = \frac{2v-1}{4}$, with a purchase probability of $\frac{v-\frac{1}{2}}{2v-1-\frac{1}{4}v^2}$ from each attacker;

c) else if $v > 2$, both attackers buy the information at $p^* = \frac{1}{2}$. ***Proof:*** (1) If $v > 2$, the data broker would not set a price larger than $\frac{1}{2}$, otherwise neither of the attackers is willing to make the purchase. In this case, the data broker's expected profit is $\pi^* = 2 * \frac{1}{2} = 1$ with optimal price $p^* = \frac{1}{2}$.

(2) If $1 < v \leq 2$, we consider two ranges of price: 1) $p \leq \frac{1}{8}v^2$, both attackers buy the information, which brings a profit of at most $\pi_1 = \frac{1}{4}v^2$ to the data broker with $p_1 = \frac{1}{8}v^2$. 2) $\frac{1}{8}v^2 < p \leq v - \frac{1}{2}$, the attackers would make the purchase w.p. $\frac{v-\frac{1}{2}-p}{v-\frac{1}{2}-\frac{1}{8}v^2}$. In this case, an expected profit of $\pi_2 = 2p\frac{v-\frac{1}{2}-p}{v-\frac{1}{2}-\frac{1}{8}v^2}$ will be gained. Because $\frac{\partial^2 \pi_2(p)}{\partial p^2} < 0$, the optimal price p_2 satisfies $\frac{\partial \pi_2(p)}{\partial p} = 0$, i.e., $p_2 = \frac{2v-1}{4}$. Since $\pi_1 < \pi_2$ for $1 < v \leq 2$, the data broker's expected profit is $\pi^* = \frac{(v-\frac{1}{2})^2}{2v-1-\frac{1}{4}v^2}$ with optimal price $p^* = \frac{2v-1}{4}$.

(3) If $v \leq 1$, similarly, we consider two ranges of price: 1) $p \leq \frac{1}{8}v^2$, for both attackers buying the information, the data broker would set a price equal to $\frac{1}{8}v^2$, which brings a profit of $\pi_1 = \frac{1}{4}v^2$. 2) $\frac{1}{8}v^2 < p \leq \frac{1}{2}v^2$, the attackers would make the purchase w.p. $\frac{\frac{1}{2}v^2-p}{\frac{3}{8}v^2}$. In this case, an expected profit of $\pi_2 = 2p\frac{\frac{1}{2}v^2-p}{\frac{3}{8}v^2}$ will be gained, which is maximized at $p = \frac{1}{4}v^2$.

Since $\pi_2 > \pi_1$, it is optimal for the data broker to set a price of $p^* = \frac{1}{4}v^2$, and $\pi^* = \frac{1}{3}v^2$.
■

Fig. 4.5(b) shows the data broker's optimal pricing strategy and corresponding expected profit. It indicates that the information value for the broker, represented as its expected profit, increases with the target value when $v \leq 2$, but when the target is attractive enough for the attackers ($v > 2$), the information value decreases to a certain value and remains unchanged.

If we compare the data broker's expected profit in the single-attacker scenario with that in the multi-attacker scenario, we find that, counter-intuitively, the data broker does not always benefit from having more potential buyers due to the competition between the attackers. Specifically, if the target value is small ($v \leq 1.24$), the data broker is expected to earn more when there is only one potential buyer; while if the target value is large ($v > 1.24$), the information value is larger for the data broker when there are more attackers.

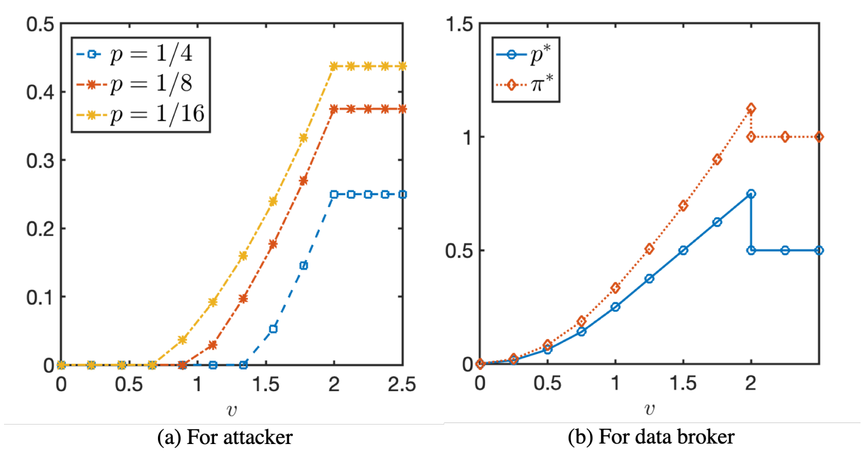


Figure 4.5: Value of traded information in competitive scenarios.

4.5 Extension: Partial information model

We consider now the possibility that the data supplier can only obtain partial information about the target, i.e., whether θ belongs to $[0, 0.5]$ or $[0.5, 1]$, but it cannot provide the exact value of θ. We analyze how this new informational structure affects the attackers' purchasing and attacking decisions and model the price of information with low data quality.

4.5.1 Games of Attacking

We also start with the competition game of attacking given different purchasing decisions.

Neither Buys Information

When neither of the attackers buys the information, the equilibrium results are the same as those in Section 4.5.1.

Both Buy Information

When both attackers buy the information, two cases are considered:

Case 1: $\theta \in [0, 0.5]$

If both of them attack, each obtains an expected utility of $f = \int_0^{0.5} \frac{1}{2}v * 2d\theta - \frac{1}{2} - p = \frac{1}{2}(v-1) - p$; else if only one attacks, then it obtains an expected utility of $f = \int_0^{0.5} v * 2d\theta - \frac{1}{2} - p = v - \frac{1}{2} - p$. Therefore, if $v > 1$, both of them would attack; if $\frac{1}{2} < v \leq 1$, the attacker would attack w.p. $q_A^{attack} = q_B^{attack} = \frac{2v-1}{v}$, and the expected utility for both attackers is $-p$; otherwise, neither would attack.

Case 2: $\theta \in [0.5, 1]$

If both of them attack, each obtains an expected utility of $f = \frac{1}{2}v - 1 - p$; else if only one attacks, then it obtains an expected utility of $f = v - 1 - p$. Therefore, if $v > 2$, both of them would attack; if $1 < v \leq 2$, $q_A^{attack} = q_B^{attack} = 2\frac{v-1}{v}$, and the expected utility for both attackers is $-p$; otherwise, neither would attack.

Considering the two cases $\theta \in [0, 0.5]$ and $\theta \in [0.5, 1]$ with equal probabilities for the attackers before they buy and obtain the information, the expected utility for the attacker when both buy the information is: $f = \frac{1}{2}(\frac{1}{2}(v-1) - p) + \frac{1}{2}(\frac{1}{2}v - 1 - p) = \frac{1}{2}v - \frac{3}{4} - p$ if $v > 2$, $f = \frac{1}{4}(v-1) - p$ if $1 < v \leq 2$, and $f = -p$ if $v \leq 1$.

Only One Attacker Buys Information

For attacker B who does not buy the information, if both attackers attack, it is expecting a utility of $\frac{1}{2}v - 1$; if it attacks but attacker A does not attack, its expected utility is $v - 1$.

Therefore, if $v > 2$, attacker B decides to attack. In this case, if attacker A gets that $\theta \in [0, 0.5]$, it would also attack, resulting an expected utility of $\frac{1}{2}v - \frac{1}{2} - p$, and if A gets that $\theta \in [0.5, 1]$, it would attack with an expected utility of $\frac{1}{2}v - 1 - p$. We thus have $f_A = \frac{1}{2}(\frac{1}{2}v - \frac{1}{2} - p) + \frac{1}{2}(\frac{1}{2}v - 1 - p) = \frac{1}{2}v - \frac{3}{4} - p$ and $f_B = \frac{1}{2}v - 1$. If $1 < v \leq 2$, attacker B knows that when $\theta \in [0, 0.5]$, attacker A would certainly attack, i.e., there is a higher probability that attacker A attacks. Therefore, attacker B would choose not to attack. Expecting this result, attacker A chooses to attack. We thus have $f_A = \frac{1}{2}(v - \frac{1}{2} - p) + \frac{1}{2}(v - 1 - p) = v - \frac{3}{4} - p$ and $f_B = 0$. If $v \leq 1$, attacker B decides not to attack. In this case, if attacker A gets that $\theta \in [0, 0.5]$, it would only attack when $v > \frac{1}{2}$, and if A gets that $\theta \in [0.5, 1]$, it would not attack. Therefore, we have $f_A = \frac{1}{4}v - \frac{1}{4} - p$ if $\frac{1}{2} < v \leq 1$, $f_A = 0$ if $v \leq \frac{1}{2}$ and $f_B = 0$.

4.5.2 Games of Purchasing

From the equilibrium analysis above, we know that if $v < \frac{1}{2}$, neither attacker would attack and therefore has no incentive to buy the information. Besides, the following three cases are considered according to different values of v. Tables 4.9, 4.10 and 4.11 list the payoffs of two attackers in the game of purchasing.

Case 1: $\frac{1}{2} < v \leq 1$

The only equilibrium is (not buy, not buy).

Table 4.9: Payoff table for game of purchasing when $\frac{1}{2} < v \leq 1$ under partial information.

	buy	not buy
buy	$-p, -p$	$\frac{1}{4}v - \frac{1}{4} - p$, 0
not buy	0, $\frac{1}{4}v - \frac{1}{4} - p$	0, 0

Case 2: $1 < v \leq 2$

If $\frac{1}{4}v - \frac{1}{4} - p \geq 0$, both attackers would buy; if $\frac{1}{4}v - \frac{1}{4} - p < 0$ and $v - \frac{3}{4} - p > 0$, in mixed strategy equilibrium, $q_A^{buy} = q_B^{buy} = \frac{v - \frac{3}{4} - p}{\frac{3}{4}v - \frac{1}{2}}$; else if $\frac{1}{4}v - \frac{1}{4} - p < 0$ and $v - \frac{3}{4} - p < 0$, neither attacker decides to buy the information.

Table 4.10: Payoff table for game of purchasing when $1 < v \leq 2$ under partial information.

	buy	not buy
buy	$\frac{1}{4}v - \frac{1}{4} - p$, $\frac{1}{4}v - \frac{1}{4} - p$	$v - \frac{3}{4} - p$, 0
not buy	0, $v - \frac{3}{4} - p$	0, 0

Case 3: $v > 2$

If $\frac{1}{2}v - \frac{3}{4} - p \geq \frac{1}{2}v - 1$, both attackers would buy; else, neither of the attackers buys the information.

Table 4.11: Payoff table for game of purchasing when $v > 2$ under partial information.

	buy	not buy
buy	$\frac{1}{2}v - \frac{3}{4} - p$, $\frac{1}{2}v - \frac{3}{4} - p$	$\frac{1}{2}v - \frac{3}{4} - p$, $\frac{1}{2}v - 1$
not buy	$\frac{1}{2}v - 1$, $\frac{1}{2}v - \frac{3}{4} - p$	$\frac{1}{2}v - 1$, $\frac{1}{2}v - 1$

Fig. 4.6 plots the attackers' optimal purchasing decisions of partial information in different ranges of v and p. The impact of partial information trading on the target is less than that of full information trading: the attacking probability increases only in the following two situations: (1) $1 < v \leq 2$, $p \leq \frac{1}{4}v - \frac{1}{4}$ and $\theta < 0.5$: both attackers would attack the target; and (2) $1 < v \leq 2$ and $\frac{1}{4}v - \frac{1}{4} < p \leq v - \frac{3}{4}$: both attackers would attack if they buy the information and find that $\theta < 0.5$, or the only one attacker who buys the information would attack.

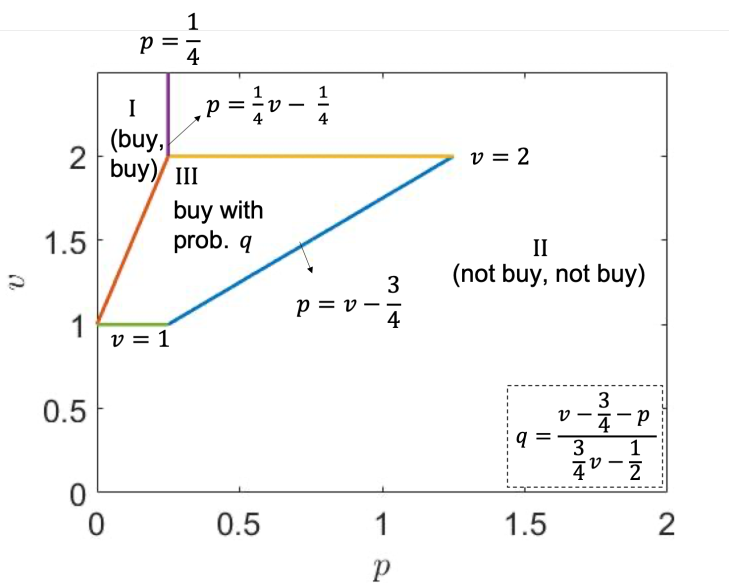


Figure 4.6: Optimal purchasing decisions of two attackers under partial information.

As for the value of partial information to the attackers, when $v > 2$ and $p < \frac{1}{4}$, there is an expected utility increase of $\frac{1}{4} - p$; when $1 < v \leq 2$ and $p \leq \frac{1}{4}v - \frac{1}{4}$, the attackers are expected to have a utility increased by $\frac{1}{4}v - \frac{1}{4} - p$, as illustrated in Fig. 4.7(a). Moreover, by comparing Fig. 4.3 and Fig. 4.6, we see that, for a lower-value target ($v < 1$), if the defender takes some effort to ensure that only partial information could be leaked, the attacking probability may decrease to zero.

4.5.3 Optimal Pricing Decisions of the Data Broker

Due to lower data quality, we are expecting a lower price compared to the scenario when full information is traded. Specifically, we have: In Stackelberg game equilibrium under partial information, the strategies of the players are:

a) if $v \leq 1$, the attackers would not buy the information at any price;

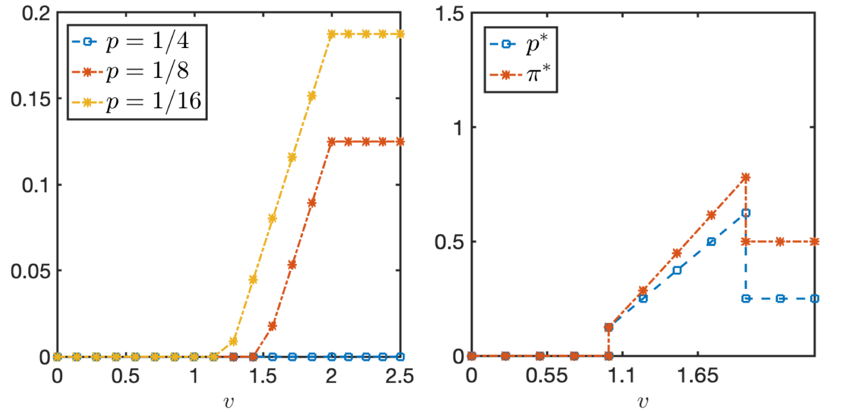
b) if $1 < v \leq 2$, information is sold at $p^* = \frac{1}{2}v - \frac{3}{8}$, with a purchase probability of $\frac{2v-\frac{3}{2}}{3v-2}$ for each attacker;

c) else if $v > 2$, both attackers buy the information at $p^* = \frac{1}{4}$. ***Proof:*** The proof is similar to that of Proposition 3.1, and thus omitted here. ■

One can now compare the results (shown in Fig. 4.7(b)) under partial information with those under full information. We could find that the price for partial information is $\frac{1}{8}$ lower when $1 < v \leq 2$ and $\frac{1}{4}$ lower when $v > 2$. That is, information accuracy is more valuable for the data broker or the attackers for a more attractive target.

4.6 Extension: Cooperative Purchasing

In this section, we are considering the cooperation between two attackers. Since the information can be reproduced with a negligible marginal cost, it is possible for two buyers to



(a) For attacker (b) For data broker

Figure 4.7: Value of traded information with partial information.

share the purchasing cost by sharing a copy of the data. We will call the process cooperative purchasing where two attackers only pay a total price p to the broker and share the information.

Naturally, there are several questions when cooperative purchasing behavior is of concern: (1) when would the attackers engage in cooperative purchasing? (2) how much will the attackers benefit from the cooperation? (3) will cooperative purchasing increase the risks of the target? and (4) if the data broker takes the possibility of cooperation between buyers into consideration, how would it set the price?

The new timeline of the process is: after the data broker announces the price, the attackers first decide whether to engage in cooperative purchasing. If not, each attacker will decide whether or not to buy the information by itself. Then with the information available, the attackers decide how much effort will be taken in attacking. The corresponding utilities are gained after the attack. We first analyze the attackers' attacking decisions after cooperative purchasing, and then compare the expected utilities with those in Section 4.5

to figure out when it is beneficial for them to cooperate; finally, the optimal price is derived for the data broker.

The payoffs for the game of attacking when they buy information cooperatively are listed in Table 4.12. It is straightforward to derive that when $v > 2$, the only strictly dominant pure strategy is (attack, attack) with an expected profit of $f = \frac{1}{2}(v - 1 - p)$ for each attacker; when $v \leq 2$, we have $q_A^{attack} = q_B^{attack} = \min[2(1-\frac{\theta}{v}), 1]$, with $f = \frac{1}{8}v^2 - \frac{1}{2}p$.

Table 4.12: Payoff table for game of attacking under cooperative purchasing.

	attack	not attack
attack	$\frac{1}{2}v - \theta - \frac{1}{2}p, \frac{1}{2}v - \theta - \frac{1}{2}p$	$v - \theta - \frac{1}{2}p, -\frac{1}{2}p$
not attack	$-\frac{1}{2}p, v - \theta - \frac{1}{2}p$	$-\frac{1}{2}p, -\frac{1}{2}p$

By comparing the expected profits with those obtained without cooperative purchasing, it is found that cooperative purchasing benefits the attackers only when the information price is lower than $\min[1, \frac{1}{4}v^2]$. We summarize the attackers optimal purchasing decisions in Fig. 4.8, which shows that the attackers engage in cooperative purchasing only in region I, and the attacker buys the information w.p. $q_A^{buy} = q_B^{buy} = \frac{v-\frac{1}{2}-p}{v-\frac{1}{2}-\frac{1}{8}v^2}$ in region II. The expected profit of an attacker is

$$f = \begin{cases} \frac{1}{2}(v - 1 - p) & \text{if } v > 2 \\ \frac{1}{8}v^2 - \frac{1}{2}p & \text{if } v \leq 2 \ \& \ p < \frac{1}{8}v^2 \\ 0 & \text{otherwise.} \end{cases} \tag{4.8}$$

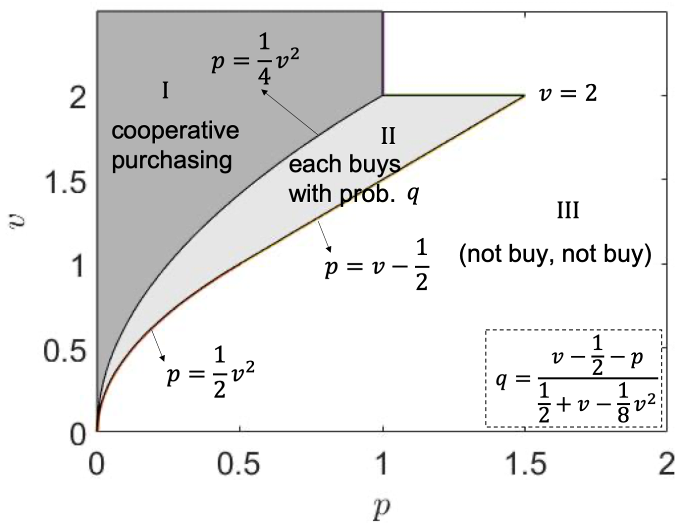


Figure 4.8: Optimal decisions of competing attackers under cooperative purchasing.

The benefit, defined as the expected utility improvement through cooperative purchasing, can be derived as

$$\Delta f = \begin{cases} \frac{1}{2}p & \text{if } v > 2 \;\&\; p < \frac{1}{2} \text{ or } v \leq 2 \;\&\; p < \frac{1}{8}v^2 \\ \frac{1}{2} - \frac{1}{2}p & \text{if } v > 2 \;\&\; \frac{1}{2} < p < 1 \\ \frac{1}{8}v^2 - \frac{1}{2}p & \text{if } v \leq 2 \;\&\; p < \frac{1}{8}v^2. \end{cases} \tag{4.9}$$

The results indicate that, only when the price is not high ($p < \frac{1}{4}v^2$ for $v \leq 2$, and $p < 1$ for $v > 2$), it is beneficial for the attackers to cooperate. We can also see a potential increase in the attacking probability resulted from cooperative purchasing when $1 < v \leq 2$ and $\frac{1}{8}v^2 < p < \frac{1}{4}v^2$: an increase of $\min[1, 2(1 - \frac{\theta}{v})] - 2(1 - \frac{1}{v})$ w.p. $(1 - q_A)^2$.

Taking the changes in the attackers' purchasing behaviors into consideration, the data broker's optimal price is:

In Stackelberg game equilibrium with the consideration of cooperative purchasing, the data broker's optimal price of the target information is given as:

$$p^* = \begin{cases} \frac{1}{4}v^2 & \text{if } v \leq 2, \\ 1 & \text{if } v > 2. \end{cases} \tag{4.10}$$

At this price, the attackers always engage in cooperative purchasing.

Proof: The proof is similar to that of Proposition 3.1 and thus omitted here. ■

Proposition 3.3 indicates that the data broker enhances its price when the target value is large ($v > 1$) if cooperative purchasing is considered. However, the data broker's expected profit gets decreased a little bit when the target value $v \leq 2$ due to the cooperative purchasing.

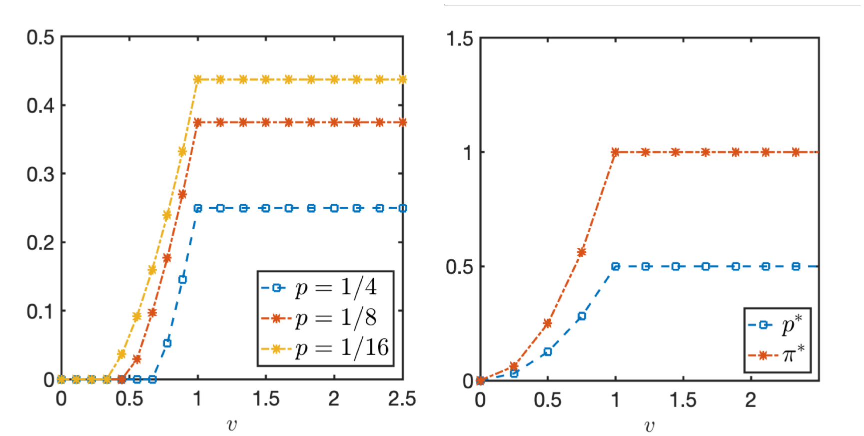
4.7 Alternative Setup: Independent Model

In this section, we consider the case where the attackers do not compete for the target value, i.e., each successful attacker receives a utility v in the case of multiple attackers instead of splitting the target value. This is referred to as the independent attack scenario in the following analysis. This assumption is suitable for the situation when the target is a type of "public good" that is non-rival (i.e., the consumption by one agent does not reduce consumption by others) [34]. We first analyze the optimal decisions of the attackers and the data broker; then a comparison between the results with those in Section 4.5 is made in order to investigate the impacts of competition.

Since there is no competition between the attackers, the model reduces to a standard Stackelberg game between the data broker and the attackers, with no simultaneous game between the attackers. In Section 4.4, we have derived that a single attacker would buy the information if the target value is high that $v > 1$ and the price satisfies $p \leq \frac{1}{2}$, or if $v < 1$ and $p \leq \frac{1}{2}v^2$. When there are two attackers, their purchasing and attacking decisions are the same as shown in Fig. 1. The value of information for the attacker is illustrated in Fig. 4.9(a). If we compare the results above with those of the competition model in Section 4.5, we conclude that, under the same target value and information price, it is less likely for the attackers to make the purchase in the competitive scenario. This result is consistent with our intuition and implies that the value of information for the attackers is weakened by their competition.

Similarly, the data broker's optimal price is $p^* = \frac{1}{2}v^2$ if $v \leq 1$, and $p^* = \frac{1}{2}$ if $v > 1$, as shown in Fig. 4.9(b). A comparison of the results with those in Fig. 4.5(b) shows us some interesting observations: first, when $v \leq 1.5$, the price is lower in a competitive scenario than in a independent scenario since the data broker needs to improve sales; while when $1.5 < v \leq 2$, the price is higher in competitive scenario for larger marginal profit; and when $v > 2$, the price is the same for both competitive and independent scenario since both attackers would buy the information as long as $p \leq \frac{1}{2}$. The data broker's corresponding expected profit for a two-attacker case without competition is v^2 if $v \leq 1$ and 1 if $v > 1$. Interestingly, the data broker benefits from the competition between attackers if $1.86 < v \leq 2$, when the optimal price in the independent scenario is much higher than that in the competitive scenario.

If we take cooperative purchasing into consideration, Fig. 4.10 shows the attackers' optimal decisions under different values of p and v. Compared with the results shown in Fig. 4.8 of the competitive scenario, cooperative purchasing benefits the attackers in more



(a) For attacker (b) For data broker

Figure 4.9: Value of information in independent scenarios.

cases related to different values of p and v in the independent scenario. The phenomenon can be explained as follows: intuitively, it is always beneficial to reduce purchasing cost when there is no competition in attacking; however, in the competitive scenario, the attackers face a risk of having to split the target value under cooperative purchasing. Therefore the final purchasing decision needs to be made by considering the trade-off between less purchasing cost and the increased competitive risk.

4.8 Extension: Heterogeneous Attackers

In this section, we discuss the robustness of our insights to an alternative modeling assumption, where the heterogeneous attackers do not split the target value but their utilities could be compromised by their competition. Specifically, attackers A and B are heterogeneous in that the benefits they derive from the same target could be different.

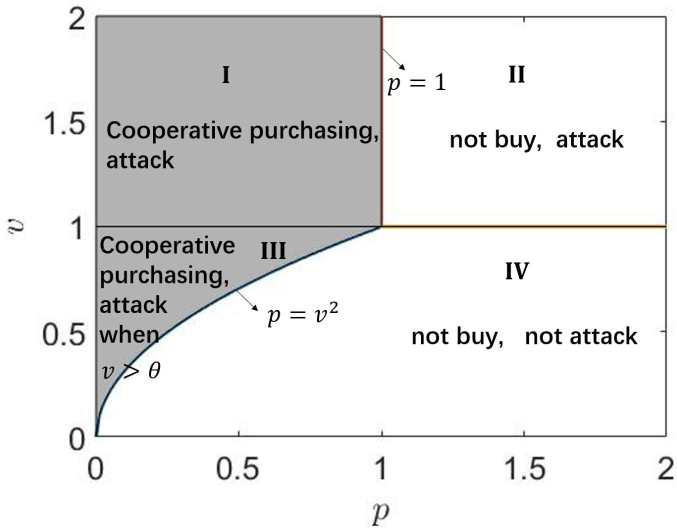


Figure 4.10: Optimal decisions of independent attackers under cooperative purchasing.

We denote the expected reward of attacker i as $u_i(v)$ if only i successfully attacks the target with a value of v, and $u_{ij}(v)$ if both attackers succeed ($i, j \in \{A, B\}$), where $u_{ij}(v) \leq u_i(v)$, $u_A(v) \neq u_B(v)$ and $u_{AB}(v) \neq u_{BA}(v)$. The equilibrium solutions of the game are determined by all the values of u_i and u_{ij} ($i, j \in \{A, B\}$). The game analysis process is similar to that for homogeneous attackers but the analytical solutions include a larger number of cases. Due to the page limit, we present one representative case to illustrate each game and briefly discuss the impacts of the attacker heterogeneity on the equilibrium results. In our numerical example, we investigate a situation involving a high-profit attacker and a low-profit attacker and examine how each player would benefit from the information trading.

4.8.1 Games of Attacking

In the games of attacking, considering whether each value of u_i and u_{ij} is larger than 1 (the largest value of θ), we will have 9 cases. Next we take a nontrivial case as an example where $u_A(v) > 1$, $u_B(v) > 1$, $u_{AB}(v) < 1$, $u_{BA}(v) < 1$ and present the following equilibrium results.

Neither Buys Information

When nobody buys the information, attacker i would attack w.p. $q_i^{attack} = \frac{u_i - 1}{u_i - u_{ij}}$ $(i, j \in A, B)$, and the expected utility is 0 for both attackers.

Both Buy Information

Attacker i will launch the attack when $u_{ij} > \theta$. If both u_{AB} and u_{BA} are smaller than θ, then attacker i would attack w.p. $q_i^{attack} = \frac{u_i - \theta}{u_i - u_{ij}}$.

Only One Attacker Buys Information

This is a game with asymmetric information. When only attacker A buys the information, the payoffs are listed in Table 4.13.

Table 4.13: Payoff table for game of attacking with heterogeneous attackers when only attacker A buys information.

	attack	not attack
attack	$u_{AB} - \theta - p$, $u_{BA} - 1$	$u_A - \theta - p$, 0
not attack	$-p$, $u_B - 1$	$-p$, 0

The expected utility of attacker A if it attacks is $E_A(attack) = q_B^{attack}(u_{AB} - \theta - p) + (1 - q_B^{attack})(u_A - \theta - p)$, and if it does not attack, $E_A(not\ attack) = -p$. Therefore, attacker A would attack if $E_A(attack) > -p$, or $q_B^{attack}(u_{AB} - u_A) + (u_A - \theta) > 0$. Similarly, attacker B would attack if $E_B(attack) = q_A^{attack}(u_{BA} - u_B) + (u_B - 1) > 0$. Denote $\Delta_1 = u_{AB}(u_{BA} - u_B) + (u_B - 1)$, and suppose attacker B chooses to attack, then $q_A^{attack} = Prob(E_A(attack) > -p) = u_{AB}$, and $q_B^{attack} = Prob(E_B(attack) > 0) = Prob(\Delta_1 > 0)$. If $\Delta_1 > 0$, then $q_B^{attack} = 1$, and $f_A = \frac{1}{2}u_{AB}^2 - p$, $f_B = \Delta_1$. Else if $\Delta_1 \leq 0$, then $q_B^{attack} = 0$ and $q_A^{attack} = Prob(u_A > \theta) = 1$, $f_A = u_A - \frac{1}{2} - p$, $f_B = 0$.

Similarly, when only attacker B buys the information, If $\Delta_2 = u_{BA}(u_{AB} - u_A) + (u_A - 1) > 0$, then $q_A^{attack} = 1$, and $f_B = \frac{1}{2}u_{BA}^2 - p$, $f_A = \Delta_2$. Else if $\Delta_2 \leq 0$, then $q_A^{attack} = 0$ and $q_B^{attack} = 1$, $f_B = u_B - \frac{1}{2} - p$, $f_A = 0$.

4.8.2 Games of Purchasing

Based on the equilibrium results above, we have four cases in games of purchasing depending on the values of Δ_1, Δ_2: ($\Delta_1 \leq 0$, $\Delta_2 > 0$), ($\Delta_1 > 0$, $\Delta_2 \leq 0$), ($\Delta_1 \leq 0$, $\Delta_2 \leq 0$), and ($\Delta_1 > 0$, $\Delta_2 > 0$). Suppose $u_{AB} \leq u_{BA}$ without loss of generality, Table 4.14 lists the payoffs for the case of ($\Delta_1 \leq 0$, $\Delta_2 \leq 0$) as an example.

Table 4.14: Payoff table for game of attacking with heterogeneous attackers when $\Delta_1 \leq 0$, $\Delta_2 \leq 0$ and $u_{AB} \leq u_{BA}$ ($T = u_{BA}u_{AB} + u_B(u_{BA} - u_{AB}) - \frac{1}{2}u_{BA}^2$).

	buy	not buy
buy	$\frac{1}{2}u_{AB}^2 - p, T - p$	$u_A - \frac{1}{2} - p, 0$
not buy	$0, u_B - \frac{1}{2} - p$	$0, 0$

According to Table 4.14, if $(\frac{1}{2}u_{AB}^2 - p)(u_A - \frac{1}{2} - p) < 0$ and $(T - p)(u_B - \frac{1}{2} - p) < 0$, attacker A chooses to buy w.p. $q_A^{buy} = \frac{u_A - \frac{1}{2} - p}{u_A - \frac{1}{2} - \frac{1}{2}u_{AB}^2}$, and attacker B buys w.p. $q_B^{buy} = \frac{u_B - \frac{1}{2} - p}{u_B - \frac{1}{2} - T}$. Otherwise, there is only one pure-strategy Nash equilibrium.

As an illustration purpose, we consider attacker B to be able to benefit more from attacking the target than attacker A. Fig. 4.11 shows the equilibrium purchasing decisions when $u_A = 0.7v$, $u_B = v$, $u_{AB} = 0.25v$, and $u_{BA} = 0.85v$. Our results show that if the price is low enough (in region I, $p < \frac{1}{2}u_{AB}^2$), both attackers purchase the information. The risk of the target, in terms of the attacking volume, is increased by the information trading in both regions I and III under some conditions ($u_B > \theta$ if $v < 1$, $u_{AB} > \theta$ otherwise). These observations are consistent with those in our base model.

We compare the results with those shown in Fig. 4.3 and find that attacker B, the high-profit attacker, has more tendency to buy the information than attacker A. In region III, only attacker B buys the information. An intuitive explanation is: the target is less attractive and therefore the information is of less value to attacker A who lacks the incentive to engage in information trading. Besides, Fig. 4.12(a) plots the value of traded information for the attackers under different target value when $p = \frac{1}{16}$. It is interesting to find that: If the target value is small ($v < 1.414$), only attacker B benefits from the traded information since the marginal gain for attacker A is too low to make the purchase. As the target value increases, attacker A benefits more from the information, while attacker B benefits less in the range of $1.414 \leq v < 4$. If $2.697 \leq v < 4$, the existence of the traded information even hurts attacker B. This is because attacker A is more likely to mount the attack with the traded information, resulting in the competition between the attackers and therefore a decrease in attacker B's expected utility. If the target value is large enough ($v \geq 4$), both attackers benefit from the traded information since they would launch attacks even without the information.

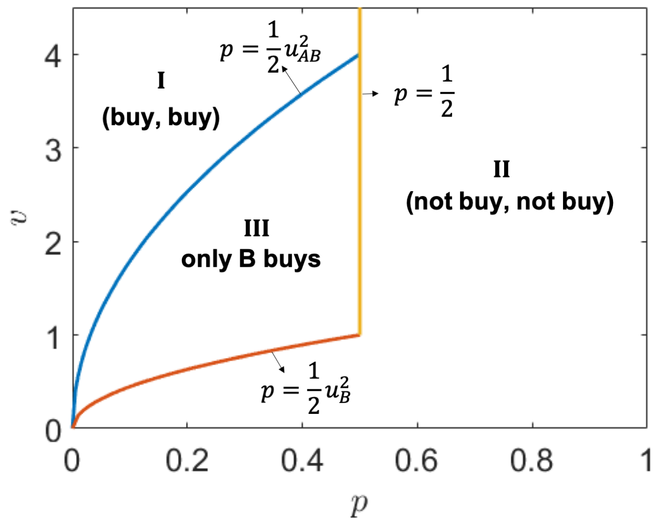


Figure 4.11: Optimal purchasing decisions of heterogeneous attackers.

4.8.3 Optimal Pricing Decisions of the Data Broker

The changes in the attackers' purchasing behaviors lead to a different pricing policy of the data broker, as illustrated in Fig. 4.12(b). In the example, if $v \leq 1$, information is sold at $p^* = \frac{1}{2}v^2$; if $1 < v \leq 2.828$ (when $u_{AB}^2 < 0.5$), $p^* = 0.5$; else, $p^* = \frac{1}{2}u_{AB}^2$. It is seen that, when v is large enough that $u_{AB}^2 > 0.5$, the data broker lowers the price in order to attract both attackers; otherwise, the data broker tends to win only the higher-profit attacker. Similar to the case of homogeneous attackers, the data broker benefits from having more potential buyers if the target value is large ($v > 2.828$).

As for the impact of competition on the optimal price, one can notice that, if the target value is not small ($2.828 < v < 4$), the attackers benefit from their competition in terms of having a lower information price set by the data broker to attract both attackers; while if $v < 1.428$, attacker A would face a higher price due to the competition because

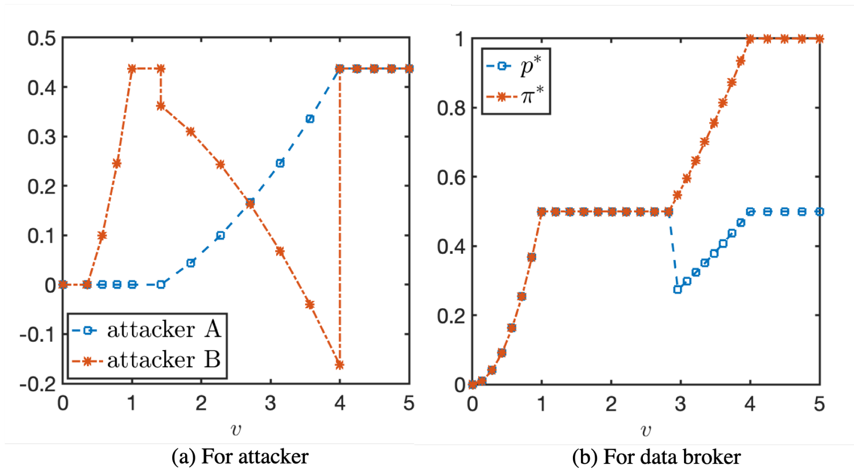


Figure 4.12: Value of traded information with heterogeneous attackers.

attacker B could accept a higher price than attacker A. This result is different from our base model, where the competition decreases the price for the low-value target ($v < 1.5$) and increases it for the high-value target ($1.5 < v < 2$). That being said, the impact of attackers' competition on the information price is complicated by several factors including both the attackers' heterogeneity and the target value.

Chapter 5

Incentive Mechanism Design for Crowd Sourcing to Service Users

5.1 Introduction

With the rapid advances in information and communication technologies, we are witnessing a paradigm shift away from traditional business processes to crowdsourcing. By outsourcing works to the crowd[96], companies access more easily the diverse labor pool, new ideas and solutions, enhanced efficiency, and reduced cost. Among the recent successful practices, numerous businesses outsource tasks such as data sensing, content creation, and product design to *their own service users*, instead of to a less-specific, more public group. As shown in Fig. 5.1(a), the service provider serves its users and at the same time uses the crowdsourcing data contributed by some of its users (i.e., participants) to improve the service quality and efficiency. For example, traffic monitoring applications aggregate the traffic data directly from the drivers (i.e., the users) during their traveling via a network connection and provide the information back to the drivers [46, 97]. Other examples include Universal Studios Wait Times [98] for waiting time information, China Software Developer Network for technical support, Dietsensor for diet nutrition [99], and Brilliance for product design ideas [100]. The benefits of leveraging the existing user base for crowdsourcing are multifold: Firstly, it eliminates the need for enrolling workers from third-party platforms. Information can be easily and cost-effectively collected through already established communication channels with the users. Secondly, users' data reflects the true market needs and thus makes the business strategy more targeted and efficient. Besides, companies could enjoy the benefit of enhanced consumer loyalty from more user engagement.

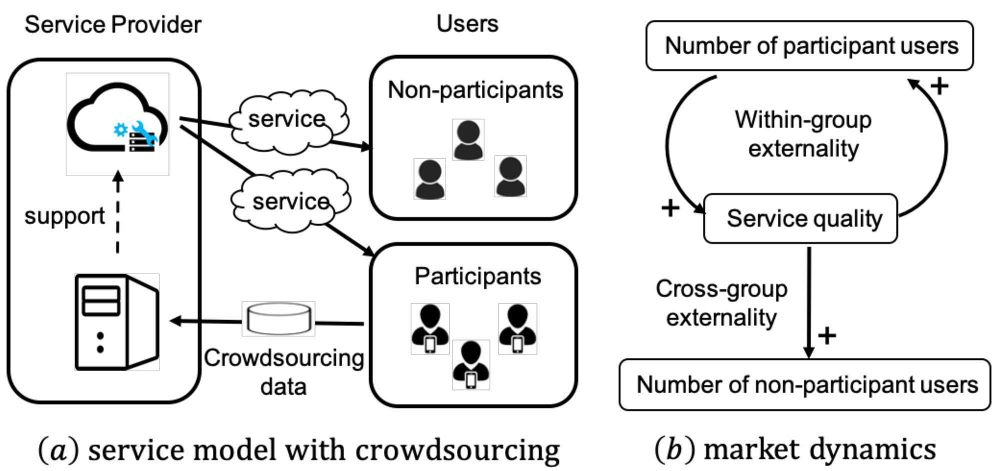


Figure 5.1: Illustration of (a) a service model with crowdsourcing and (b) network externalities. The service provider uses the crowdsourcing data contributed by its own users to support a better quality of service.

Nevertheless, a rational user will free-ride on the contributions of others. It may also have privacy concerns about providing its data or incurs some cost arising from contributing. Therefore, incentive mechanisms are needed in the recruitment of contributors out of the end-users. In this regard, two types of markets exist in the business process: service markets and crowdsourcing markets. In the service market, the end-users subscribe to the service delivered by the service provider with a specific service level and price. On the other hand, in the crowdsourcing market, the service provider "buys" the contributed data from some of its end users by rewarding them for contribution. It decides the optimal reward in the crowdsourcing market to attract more contributors and finally maximize the expected profits in the service market. So far, most crowdsourcing incentive mechanism studies focus on using financial reward, in the form of money or virtual cash [43], to encourage more contributions[37, 101, 52, 102]. The contributors, who are usually recruited through a crowdsourcing platform, are assumed to be decoupled from the service users. The only motivation for them in crowdsourcing is to get a financial reward.

However, the users' intrinsic demand/desire for better services, which could have been exploited as a very powerful tool to foster the engagement of users, is often neglected. Such an intrinsic incentive comes from the fact that the contributed data from the crowdsourcing participants, after appropriate processing and aggregation, can be used to add value to the service which is provided to the users themselves [40]. For example, the traffic data collected from the drivers are processed at the traffic monitoring applications to provide more timely traffic information service with a much broader geographical coverage [46]. That is to say, the service quality is endogenous and there exists an intrinsic motivation for the service users to engage in crowdsourcing. Failure to consider such an intrinsic incentive may lead to underestimation of crowdsourcing, giving too little of an extrinsic incentive, and substantial loss of contributors.

Moreover, network effects are inherent in the market considering the positive impacts of crowdsourcing on the service quality (as shown in Fig. 5.1(b)). Note that the users can be divided into two groups according to whether they participate in crowdsourcing: participant users (or contributors) and non-participant users (pure consumers). On one hand, a typical within-group network externality exists among the participant users: a higher number of participants contributes more data to enhance the quality of service, which attracts a greater number of potential participant users. On the other hand, a positive cross-group network externality exists as a better service also brings more consumers (non-participant users). As a motivating example, consider the case of community websites that provide technical support for users. The more users contribute to the service, by providing data, or answering others' questions, the more help or benefit a user can get from the community, and further the more users would subscribe to and contribute to the website. How the market evolves under these two network externalities and what is the market equilibrium are the key questions for a thorough understanding of the incentives of crowdsourcing.

One related work that considers the intrinsic reward is [103], which derives the optimal extrinsic reward based on the contributor's effort level. They assume that the network effects are the same for all users or the influences between users represented by the social relationship matrix are public information, which could be rather difficult to obtain when the number of users is huge. However, the positive impact of crowdsourcing on service quality and thus the market share, which is a major initiative of implementing crowdsourcing in practice, has not been explicitly explored. Furthermore, the questions are not studied yet about how the dynamics of the subscription and the participation levels interact with each other, how the service price and competition among service providers impact the incentive mechanism and the market equilibrium, as well as how the neglect of intrinsic incentive would affect the decision on the extrinsic reward. The answers to these questions are crucial for service providers to take the best advantage of crowdsourcing.

Different from most existing work that decouples the decisions of service subscription and contribution, we explicitly explored the users' dual role and examined how the intrinsic incentive and extrinsic reward together reshape the market shares. The problems are challenging due to the following reasons.

First, there lacks a systematic or quantitative framework to investigate the market behaviors under crowdsourcing. Traditional one-sided service markets are usually determined by a predetermined fixed service level and related studies are conducted with the assumption of a certain form of the demand function. The market we considered is more conceptually of a two-sided market, with one group of pure consumers and the other group being crowdsourcing participant users or contributors (as shown in Fig. 5.1(a)). The change of a single user's participating behavior in crowdsourcing affects both the purchasing and the participating behavior of others. In another word, the market dynamics are characterized by both the within-group externality (i.e. more contributors encourage more contributors) and the cross-group network externality (i.e. more contributors attract more pure consumers). Besides, the market behavior analysis is further complicated by the users' heterogeneity in both service usage level and privacy concerns.

Second, the heterogeneity in the service providers' marketing strategies and the competition make the analysis even more complex, especially when both extrinsic and intrinsic incentives are incorporated. We consider two types of service providers: a high-quality service provider and a basic service provider, both providing financial rewards for crowdsourcing participation. The high-quality service provider benefits from the user-contributed data in providing a service with a predetermined high-quality level. The basic service provider relies mainly on the user-contributed data to enhance the service quality.

When users' choice of services is of concern, there are two key questions to pose: 1) what is the market equilibrium when the two service providers compete for the same population of users? 2) how does their competition affect the rewards? These questions are not easy to answer due to both the difference in the service providers' marketing strategies and the critical roles played by the rewards in reshaping the market shares, which are often neglected in the literature.

Our research differs from the literature on the following fundamental/significant novelties:

- To the best of our knowledge, this is the first model to address endogenous user segmentation in crowdsourcing incentive mechanism design with the consideration of endogenous service quality. The modeling of the two-way interdependence between the service and the users, which better fits the practice, is a unique contribution of our work and has not been considered in the literature.

  Our model considers a two-sided market with both within-group and cross-group network externalities in the field of crowdsourcing incentive mechanism design, whereas other studies of the wireless market mainly focus on the traditional one-sided market model or only take one type of network externality into consideration. By considering the two-sided externality market model, our work is able to give unique insights regarding the evolution of communities that serve themselves via crowdsourcing. Such insights are not possible to obtain by the traditional one-sided market models.

- Several interesting findings in our work are not intuitive in the first place. Firstly, we show that, counter-intuitively, failure to take into account the users' intrinsic incentive leads to too little extrinsic incentive, rather than a higher financial reward as a makeup. This is different from the finding in [104] in which the authors state that more intrinsic rewards would help reduce the amount of extrinsic rewards that a crowdsourcer will need to provide. Secondly, we delineate how the users' behaviors dynamically evolve and prove the convergence of the market dynamics under some mild conditions related to both the service price and the reward. Lastly, we showed how the competition makes a difference in reshaping the markets, which cannot be intuitively or trivially predicted without a thorough analysis.

All of the detailed proofs are provided in the Appendix.

5.2 System Model

In this section, we describe our model of crowdsourcing incentive mechanism. This model applies to profit-driven service providers that rely on collecting and processing user-related data for final services and benefit from user-contributed information in improving the service quality level or reducing the investment cost. To make our description more concrete, the following presentation is based on the case of crowdsensing. However, our model applies more broadly to the general context of crowdsourcing.

We consider a service area where a population of users has the choice between the services of the same type offered by two service providers. Motivated by the current commercial applications, we consider two service providers with different market strategies: one provider with predetermined high service quality and one provider with service quality dependent on the user-contributed data. Both providers make money by charging a subscription fee (or service price) from their users. Note that the concept of service quality is general. It indicates the expected utility a user can achieve from using the service. For instance, it measures the accuracy or timeliness of the information provided to the users in the case of traffic monitoring applications.

5.2.1 High-quality Service Provider (HSP)

The high-quality service provider harnesses both user-contributed data and data from its own static wireless sensor devices to offer high-quality service with a predetermined level. There are many examples of services that use both crowdsourcing and traditional modes of data selection. For instance, Universal Studios Wait Times and Disneyland Wait time collect both user-contributed waiting time information for various attractions and the official waiting time information disseminated by the theme park operators [98]. In Haze Watch which measures the air quality, the data collected by mobile phones complements static high-fidelity data captured by traditional meteorological stations [105].

For the HSP, a traditional dedicated sensor network guarantees a high quality of service at the expense of substantial deployment and maintenance costs. The user-contributed data is used as a supplement to the traditional sensing data, which helps to reduce the data collection burden of the traditional sensor network and thus reduces its cost of installing and maintaining sensors. Intuitively, the more users participating in the crowdsensing, the less investment on the traditional sensing network. Take Disneyland Wait time for example, as more mobile users update and share their waiting time information, the theme park could hire fewer operators to collect the information. Therefore, the user-contributed data could benefit the service provider in terms of reducing labor costs for recruitment and training employees, as well as dealing with turnover. Note that we consider the scenario when the HSP's target service level is predetermined which could be sufficiently high. This situation occurs when the service provider's target market typically consists of users with high usage levels or high sensitivity to the service quality. That means the service quality level does not depend on the actual number of contributors. The main reason for the HSP to implement crowdsensing is to find the lowest-cost ways to hit the high service level target, but not to enhance the service quality.

5.2.2 Basic Service Provider (BSP)

The other service provider, referred to as the basic service provider, offers a basic service and relies mainly on user-contributed data to enhance the service quality. Therefore, the service quality of BSP depends on the number of contributors. Examples of such community-based applications that rely on crowd-sourced information are abundant and diverse. For example, to facilitate price comparison of grocery items, LiveCompare [106] provides basic pricing information from online grocery stores and relies on photographic data collected by users to allow for grocery bargain hunting through participatory sensing. Another example is iStockphoto [107] that provides online microstock photography.

To encourage more contributors, the BSP informs all the users that the service quality can be improved if users contribute their data. That means the users could benefit themselves by providing their data. Meanwhile, both HSP and BSP provide a discount on the service price to each contributor. The discount can be viewed as a reward for the sensing work. Index service price p and reward w terms by a provider such that p_h and p_b describe service prices of the HSP and the BSP, respectively. We have $w_h \leq p_h$ and $w_b \leq p_b$. To focus on the effects of incentive for crowdsensing, we assume that both the basic or target service level and the corresponding service prices and have already been strategically determined, and defer a discussion of the marketing strategy to establish the optimal price and service level to future work. The case when a basic service provider requires users' participation in crowdsourcing but offers no financial reward is also analyzed and provided in the Appendix due to the page limit. In our model, we consider the situation when each crowdsourcing participant receives the same reward. This assumption is justified in the following contexts:

- In some cases, there is no need to differentiate the contributors in their contribution because the sensing tasks are similar and simple in that it only requires periodic data reporting. Typical examples include physical sensing tasks such as spectrum sensing and traffic monitoring [108]. Besides, a real-world complex crowdsourcing task is often split into a large number of atomic tasks in the form of binary choices that requires trivial cognitive load, as has been shown in a study on 27 million crowdsourcing tasks that are dominated by boolean questions [109] for applications in basic data-driven operations such as filtering [110]. In these cases, the contributions from different contributors are modeled as identical regardless of their participation behaviors and results.

- There are situations where it is impractical to differentiate the participants' contributions because the number of contributors is huge or the quality of the data contributed is hard to measure [49]. For example, on online platforms drawing on the experiences of their communities members to assist their users in choosing between alternative service providers (e.g. TripAdvisor), the quality of the information provided by individuals is hard to evaluate, and homogeneous contributions from users are often assumed[111].

Moreover, we assume the reward remains unchanged in a certain period[1] to reduce the complexity in designing and implementing the incentive mechanism and leave the dynamic rewards for future work. The assumption is reasonable since the static reward is simple, scalable, and has been widely used in literature (e.g. [49, 112, 103]). On the contrary, dynamic incentives such as the bidding-based incentive schemes are time-consuming and may incur high overhead in the process of information exchange and data quality evaluation, especially for large-scale crowdsourcing tasks. Note that in this work, we focus on the more fundamental goal of using an incentive to encourage more users to participate and contribute data, while leaving the issue of truthfulness to other orthogonal techniques that have wider applicability and maybe in a better position than incentive mechanisms to provide truthfulness.

5.2.3 User Model

Each user decides whether to subscribe to a service provider and whether to contribute its data. A user can choose one of the five options (denoted by s):

(i) $s = 0$: Do not subscribe to any service;

(ii) $s = 1$: Subscribe to HSP but do not participate in crowdsensing;

(iii) $s = 2$: Subscribe to HSP and participate in crowdsensing;

(iv) $s = 3$: Subscribe to BSP but do not participate in crowdsensing;

[1] The duration of one period is decided by the service providers' market plan. Our model allows the possibility of changing the rewards across periods. In this research, we only focus on one period.

(v) $s = 4$: Subscribe to BSP and participate in crowdsensing.

Each user is rational and will make a choice that maximizes its payoff. The type of users who are enthusiastic about crowdsourcing and not interest-driven is beyond the scope of this work. The *payoff* function of a potential user is defined to be the difference between the achieved utility and the service price, plus the difference between the reward and the private loss if participating in crowdsourcing. We denote v_h and v_b as the expected *utility* that a user can obtain from subscribing to the services provided by HSP and BSP, respectively. In line with [113, 114, 115], v_h and v_b can be also seen as the equivalent revenues a user gets by using the services. We assume that $v_b < v_h$, even when all the users participate in crowdsensing for BSP. This assumption is reasonable as the user-contributed data is not always reliable and when the HSP's target service quality is high enough. Since the values of v_h and v_b are determined by and reflect the providers' service quality levels, we will use the terms service quality and the expected utility provided by the service providers interchangeably in our models.

Let α denote the user's usage level of the service or its evaluation for the achieved utility, and l being the user's private loss in participating crowdsourcing. Then, the payoff of user i with parameters (α_i, l_i) is

$$\Pi_i^{\mathrm{E}} = \begin{cases} 0 & \text{if } s = 0, \\ \alpha_i v_h - p_h & \text{if } s = 1, \\ \alpha_i v_h - p_h + w_h - l_i & \text{if } s = 2, \\ \alpha_i v_b - p_b & \text{if } s = 3, \\ \alpha_i v_b - p_b + w_b - l_i & \text{if } s = 4. \end{cases} \tag{5.1}$$

Note that we consider that the users are heterogeneous in their service usage and privacy concern, with their types characterized by the following two parameters:

(1) Usage level α. α characterizes the proportion of period a user spent in using the service or its desire for the service. We assume that for each user, its usage level is independent of the service it subscribes to. Without loss of generality, we normalize α to [0, 1] and assume that it is exogenously determined as uniformly distributed in the user population, where $\alpha = 0$ corresponds to no usage at all and $\alpha = 1$ to full usage [116]. Using the uniform distribution to approximate users' type regarding their evaluation for the service is commonly used for analytical tractability (e.g. [117, 115, 113, 118, 119, 120]). The consideration of more general distributions often does not change the main insights. α can also be explained as a user's sensitivity to the service quality [113] or evaluation for the achieved utility [115]. The achieved utility or usage value of the service, denoted as u, is a function of both the service quality and the usage level α: $u = \alpha v_h$ or $u = \alpha v_b$. The assumption on the linear relationship between the achieved utility, u, and the service quality level, v_h or v_b, has been widely made in literature [113, 114, 115].

(2) Evaluation of private loss l. Generally, a user has its own evaluation of personal loss in reporting private data (e.g., location), denoted as l. The loss may come from nuisance costs either from privacy concerns such as data breach accidents or privacy costs originating from the user's per se taste for privacy [121]. The parameter l can also represent the user's cost arising from sensing and contributing (e.g., telecommunication charges and battery consumption) [98]. The distribution of the users' evaluations is given exogenously in our model. Without loss of generality, we assume l follows a uniform distribution over the range $[0, L]$. The uniform distribution is assumed for analytical tractability. Using

the uniform distribution to approximate the contributors' cost in crowdsourcing is widely used (e.g. [112]). To capture the fact that some users may be never willing to participate in crowdsensing tasks due to high privacy concerns, we further assume that $L > v_h$.

The HSP's service users are the users with choice $s \in \{1, 2\}$, and the BSP's service users are the users with choice $s \in \{3, 4\}$. In the following analysis, we will call users with choice $s = 1$ or $s = 3$ as (pure) consumers, and the ones with choice $s = 2$ or $s = 4$ as service contributors. The population can be divided into five segments according to their choices. Let m_s denotes the fraction of users with choice s. Obviously, $\sum_s m_s = 1$. A service provider's market share is referred to as the fraction of users subscribing to its service. Without loss of generality, we normalize the total number of potential users to one, as has been widely adopted in the marketing literature [122]. Table 1 summarizes the key notation used in our model analysis.

5.3 Monopoly Market Analysis of an HSP

In this section, we study the scenario where there is only one service provider, HSP. We consider only the nontrivial case of $p_h - w_h < v_h$, since if $p_h - w_h \geq v_h$, there will be no user subscribing to the service. We first derive the market shares under a given reward. Then the optimal value of the reward is derived to maximize the HSP's profit.

5.3.1 Analysis of the Market Share

Under the assumption that α follows a standard uniform distribution, and u is uniformly distributed: $u \sim U[0, v_h]$, users are uniformly distributed on the $v_h \times L$ square, as illustrated in Fig. 5.2. A user with parameters (α, l) subscribes to the HSP only if $\Pi^{\mathrm{E}}_{s=1} > 0$ and

Table 5.1: List of Notation for Chapter 5.

Notation	**Explanation**
v_h	HSP's service quality level
v_b	BSP's service quality level ($v_b = \underline{v} + g(m_4)$)
$\underline{v}$	BSP's basic service quality level
α	User's usage level
u	Utility derived by a user from using a service
l	User's evaluation of private loss ($l \sim U[0, L]$)
Π^{E}	User's payoff
Π^{H}	HSP's profit
Π^{B}	BSP's profit
p_h	HSP's service price
p_b	BSP's service price
w_h	HSP's reward given to contributors
w_b	BSP's reward given to contributors
s	Users' choice with five values:
	0: not subscribing to any service;
	1: subscribing to HSP, not participating in crowdsensing;
	2: subscribing to HSP, participating in crowdsensing;
	3: subscribing to BSP, not participating in crowdsensing;
	4: subscribing to BSP, participating in crowdsensing
m_s	Fraction of users with choice s, with $\widetilde{m}_s$ denoting its Equilibrium value
$g(\cdot)$	Contribution to BSP's service quality by crowdsourcing
$C(\cdot)$	HSP's investment cost
$C_r(\cdot)$	Cost reduction for HSP by crowdsourcing
β_j	Marginal value of users' contribution to HSP in cost reduction
	($j = 0$ in linear network externalities; $j = 1$ in quadratic network externalities)
γ	Marginal value of users' contribution to BSP in service improvement

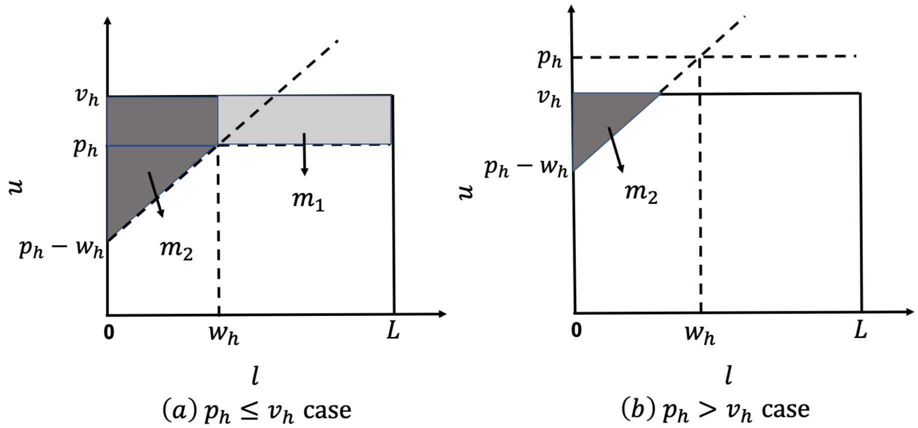


Figure 5.2: Market share of an HSP in monopoly market.

contribute to the service when $\Pi^{\mathrm{E}}_{s=2} > \Pi^{\mathrm{E}}_{s=1}$. Therefore, conditional on p_h and v_h, m_1 and m_2 can be computed below:

$$m_1 = \begin{cases} \frac{(L-w_h)(v_h-p_h)}{Lv_h} & \text{if } p_h \leq v_h, \\ 0 & \text{if } p_h > v_h. \end{cases} \tag{5.2}$$

$$m_2 = \begin{cases} \frac{w_h(v_h-p_h)+0.5w_h^2}{Lv_h} & \text{if } p_h \leq v_h, \\ \frac{(v_h-p_h+w_h)^2}{2Lv_h} & \text{if } p_h > v_h. \end{cases} \tag{5.3}$$

One can show that m_1 decreases with w_h but both m_2 and m_1+m_2 increases with w_h. That means a higher reward for crowdsensing participation would bring more consumers to the HSP but also lower down the unit revenue per user. The HSP needs to consider this trade-off when maximizing its expected profit and the optimal reward is addressed in the subsequent section.

5.3.2 Service Provider's Optimal Reward Decision

The HSP's profit function, which is the amount of service revenue less than the investment costs in a traditional sensor network, has the following expression:

$$\Pi^{\mathrm{H}} = p_h m_1 + (p_h - w_h)m_2 - C(m_2). \tag{5.4}$$

The first two terms correspond to the revenues from the users with choice $s = 1$ and $s = 2$, respectively. $C(m_2)$ is the HSP's investment and maintenance cost in traditional sensor network for achieving the service quality v_h. Due to the randomness and unreliability of users' data, we assume that, even if all the potential users participate in crowdsensing, HSP still needs to invest to ensure high service quality. That is, $C(1) > 0$. For illustration purpose, we write $C(m_2)$ as $C(m_2) = C_0 - C_r(m_2)$. Note that function $C_r(\cdot)$ reflects the cost reduction induced by the user-contributed data. As more users participating in crowdsensing helps to reduce the data collection burden of the traditional sensor network, which indicates that the provider can invest less in traditional sensing, we assume that $C_r'(m_2) \geq 0$ (at least when m_2 is not large. Furthermore, $C_r(m_2)$ is non-negative, concave, and continuously differentiable. These attributes reflect the diminishing marginal cost reduction and can be satisfied by a wide range of functions[115]. Note that the values of both m_1 and m_2 are dependent on the reward w_h given by HSP, i.e., they are functions of w_h as shown in (5.2) and (5.3). By setting $\frac{d\Pi^{\mathrm{H}}}{dw_h} = 0$, we will have:

Proposition 5.1. In monopoly market with an HSP, the HSP's optimal reward for users participating in crowdsensing is: min$\{$max$\{0, w_h^*\}, p_h\}$, where w_h^* satisfies:

$$\begin{cases} w_h(1.5w_h - 3p_h + 2v_h) = Lv_hC_r' & \text{if } p_h \leq v_h, \\ (w_h + v_h - p_h) = \frac{2Lv_hC_r'}{(3w_h - 3p_h + v_h)} & \text{if } p_h > v_h. \end{cases} \tag{5.5}$$

For ease of exposition, we investigate two specific forms of network externalities, where the cost is characterized as a linear and a quadratic function of m_2, respectively.

Linear network externalities

In this subsection we consider the case of linear network externalities, given by $C_r(m_2) = \beta_0 m_2$, which has been widely used in literature (e.g. [113], [123], [124]). Here β_0 ($0 < \beta_0 < C_0$) indicates the marginal value of the users' contribution. The value of β_0 is evaluated by the HSP regarding what extent the users' data could help save its investment cost. The larger β_0, the more crowd-sourced information can contribute. Its value is affected by multiple factors such as the data type, data quality, and the data mining model used by the HSP. Its value can be obtained through an extensive market investigation on public crowdsourcing practices or data mining on its historical investment cost. Then we have the following results regarding the HSP's optimal decision on reward:

Proposition 5.2. Given the linear network externally function, the HSP's optimal reward for users participating in crowdsensing is:

i) if $p_h \leq v_h$ and $\beta_0 > \frac{p_h}{2}(4 - \frac{3p_h}{v_h})$, or if $p_h > v_h$ and $\beta_0 > \frac{v_h}{2}$, then $w_h^* = p_h$;

ii) if $p_h \leq v_h$, and $\beta_0 \leq \frac{p_h}{2}(4 - \frac{3p_h}{v_h})$,
then $w_h^* = w_1^* = \frac{\sqrt{(2v_h - 3p_h - \beta_0)^2 + 6\beta_0(v_h - p_h)} - (2v_h - 3p_h - \beta_0)}{3}$;

iii) else, $w_h^* = w_2^* = p_h + \frac{2\beta_0 - v_h}{3}$.

It is easy to derive that the value of w_h^* increases with β_0, which results in a larger value of m_2 or $m_2 + m_1$ and a smaller value of m_1. This result suggests that higher marginal contribution from users' data leads to a higher reward to the contributors, and thus more consumers and more contributors. Fig. 5.3 plots the regions of the HSP's optimal reward decision with different values of the service price and marginal value of users' contribution. As the figure shows, the following interesting insights can be revealed: (1) If the marginal contribution from users' data is high enough, the HSP would not charge the users participating in crowdsensing tasks for using the service. In this way, it encourages the highest amount of contributors. The threshold of the marginal contribution above which the HSP gives the highest reward ($w_h^* = p_h$) is nonlinear with the service price. (2) If the marginal users' contribution β_0 is higher than $\frac{v_h}{2}$, the HSP never sets a service price higher than v_h. Otherwise, the profit of HSP would be negative since no user chooses $s = 1$ and no revenue can be earned from the users choosing $s = 2$. (3) One can derive that $w_h^* = 0$ only when $p_h \leq \frac{2v_h}{3}$ and $\beta_0 = 0$. That is, when the service price p_h is high ($p_h > \frac{2v_h}{3}$), even if the user-contributed data does not help to reduce the HSP's investment cost ($\beta_0 = 0$), the reward to contributors can still be used to attract more consumers. This finding also suggests that the HSP may not have the incentive to set a service price higher than $\frac{2v_h}{3}$ when crowdsensing doesn't contribute to investment cost reduction.

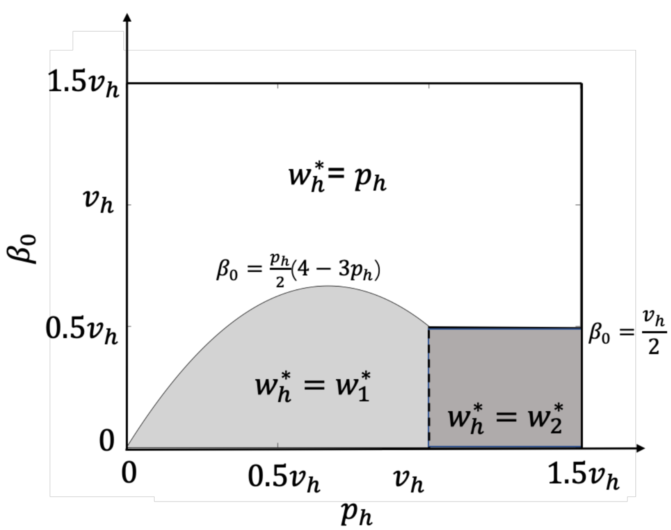


Figure 5.3: HSP's optimal reward w_h^* vs. (p_h, β_0) with linear network externalities.

Quadratic network externalities

We now consider the case of quadratic network externalities that are given by $C_r(m_2) = -m_2^2 + 2\beta_1 m_2$. The network externalities are positive before the peak point $\beta_1 \in (0.5, 1]$ and negative beyond. This attribute reflects the fact that, when m_2 exceeds a certain threshold, the contribution increment of users becomes negligible; in contrast, the operational cost keeps increasing as m_2 increases. Therefore, $C_r(m_2)$ may first increase then decrease as m_2 increases. Similar to β_0 in the case of linear network externalities, the value of β_1 indicates the marginal value of the users' contribution and can be evaluated through empirical studies. The quadratic network externalities have been used in literature (e.g., [123, 125]) to describe the non-linear characteristic of market effect. To illustrate the impacts of the marginal contribution from users' data and the service price, respectively, we provide the computational results by varying the value of β_1 from 0.6 to 1.0 at an increment of 0.05 and the value of p_h from 20 to 27 at an increment of 1.0. The values of input parameters are $L = 28, v_h = 25$. Since we are only interested in the equilibrium outcomes, we ignore the mark tilde ($\sim$) of m_s for the equilibrium results in the rest of the numerical analysis. As shown in Fig. 5.4, a larger value of β_1 would lead to larger w_h and therefore more contributors and more users. Besides, due to the enhanced reward for contribution, more consumers choose to participate in crowdsourcing, which results in a smaller value of m_1. However, there is a negligible increase in the HSP's expected profit as the marginal user contribution increases. This is due to the increased investment cost in crowdsourcing as can be seen from both the higher reward and the larger number of contributors. Fig. 5.5 reveals that a higher service price induces HSP to offer a higher reward to the crowdsourcing contributors so as to attract the users. However, with the service level remains unchanged, there will be fewer pure consumers if the service price is enhanced, while the number of contributors increases due to the increase in the reward w_h. Therefore, to maintain the market share, the HSP who sets a higher service price needs to enhance its reward simultaneously, which results in a larger proportion of users participating in crowdsourcing.

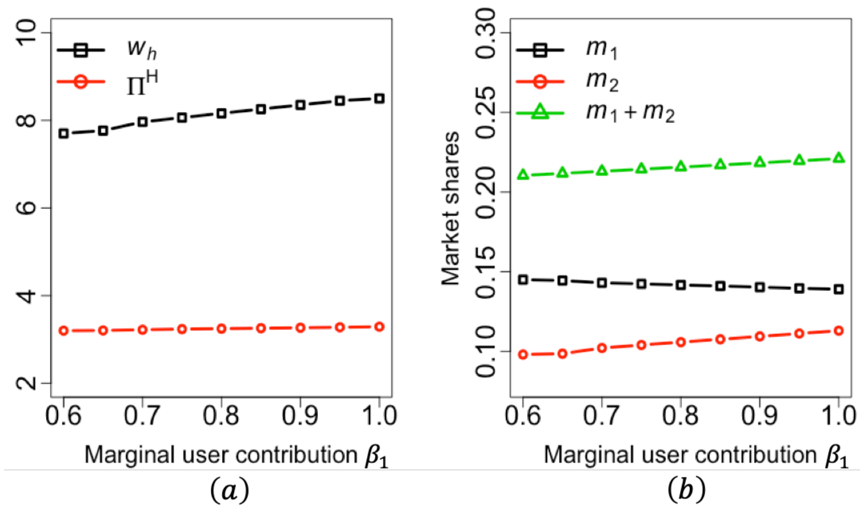


Figure 5.4: Optimal results of monopoly market of HSP with quadratic network externalities vs. β_1 ($p_h = 20$).

5.4 Monopoly Market Analysis of a BSP

In this section, we consider a monopoly market with one BSP whose service quality increases with the number of contributors. In this model, there exists a sequence of time periods indexed $t = 1, 2, ...$ At each period t, every user decides whether to subscribe and contribute to the service of BSP. We restrict our model to the case when $p_b - w_b < \underline{v}$ to avoid the trivial situation when no user would subscribe.

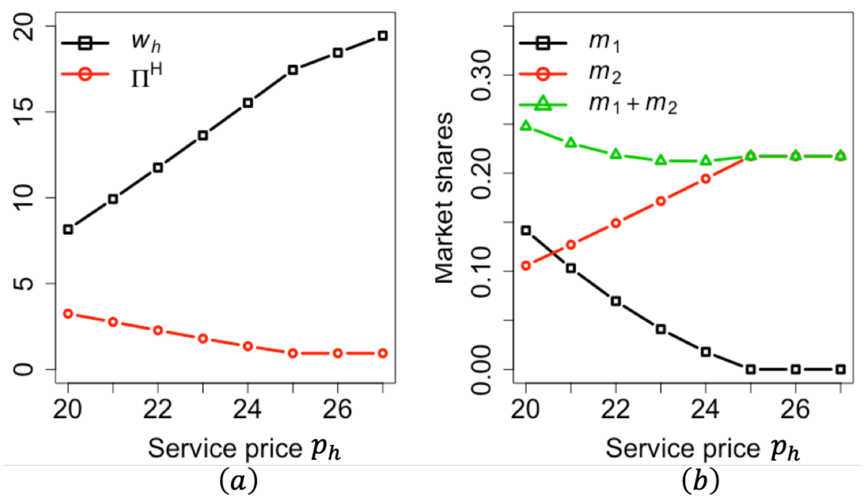


Figure 5.5: Optimal results of monopoly market of HSP with quadratic network externalities vs. p_h ($\beta_1 = 0.8$).

5.4.1 Analysis of the Market Share

Let $m_s[t]$ ($s = 0, 3, 4$) denotes the fraction of population with choice s in period t. Then $m_0[t] + m_3[t] + m_4[t] = 1$. The achieved service quality in period t is

$$v_b[t] = \underline{v} + g(m_4[t]) \tag{5.6}$$

where $\underline{v}$ is the basic quality of the BSP's service. The function $g(\cdot)$ represents the increased service quality contributed by crowd-sourced information, and actually reflects the positive network externality. The expression of $g(\cdot)$ is application dependent. For the sake of generality, we only assume $g(\cdot)$ is continuous and differentiable, and $g(0) = 0$, $g'(m_4) > 0$, $g(1) \leq v_h - \underline{v}$. A user' achieved utility u of the service depends on the basic service quality $\underline{v}$, the number of contributors in crowdsensing $m_4[t]$, and its usage level α: $u = \alpha * v_b[t] = \alpha * (\underline{v} + g(m_4[t]))$.

Similar to the case illustrated in Fig. 5.2, users are uniformly distributed on the $v_b[t] \times L$ square in period t. The fractions of users with choice $s = 3$ and 4 in period $t + 1$ are functions of the service quality in period t:

$$m_3[t+1] = \begin{cases} \frac{(L-w_b)(v_b[t]-p_b)}{Lv_b[t]} & \text{if } p_b \leq v_b[t], \\ 0 & \text{if } p_b > v_b[t]. \end{cases} \tag{5.7}$$

$$m_4[t+1] = \begin{cases} \frac{w_b(v_b[t]-p_b)+0.5w_b^2}{Lv_b[t]} & \text{if } p_b \leq v_b[t], \\ \frac{(v_b[t]-p_b+w_b)^2}{2Lv_b[t]} & \text{if } p_b > v_b[t]. \end{cases} \tag{5.8}$$

Inherent in the above formulations is the naive or static expectation [98], which states that the users assume the service quality of BSP will remain the same as in the previous period. Based on this assumption, the users make the decisions of subscription and crowdsensing participation.

Different from (5.3) in the monopoly market with an HSP, the value of $v_b[t]$ and thus users' utilities change over time since they are determined by $m_4[t]$. On the other hand, it is straightforward to derive that both $m_3[t+1]$ and $m_4[t+1]$ increase with $v_b[t]$. This interdependence between m_4 and v_b characterizes the within-group network externality in the group of contributors, and the impact of m_4 on m_3 describes the cross-group network externality between the contributors and the pure consumers. We denote the difference in term of the fraction of population with choice s between two time steps $t+1$ and t, by $\Delta m_s = m_s[t+1] - m_s[t]$ $(s = 0, 3, 4)$. Positive and negative values of Δm_s express the increase and decrease in the number of users with choice s. The equilibrium value of the market shares is realized when $\Delta m_s = 0$. In line with [115, 113], we define the market equilibrium under a given reward w_b as follows:

Definition 5.1. (Monopoly Market Equilibrium with BSP) In monopoly market with a BSP, $(\widetilde{m}_3, \widetilde{m}_4)$ is an equilibrium point of the market share if it satisfies $\widetilde{m}_s = m_s[t+1] = m_s[t]$ for all $s \in \{3, 4\}$.

Once an equilibrium point is reached, the fractions of consumers and contributors remain unchanged. In the following analysis, we show that the convergence of the market share depends on the values of p_b, $\underline{v}$ and $g(\cdot)$. Since only the value of m_4 is affected by the network externality, and both m_3 and m_0 are fixed when m_4 does not change anymore, we will focus on the convergence of m_4 in the subsequent analysis according to different ranges of the service price p_b: low price ($p_b \leq \underline{v}$), medium price ($\underline{v} < p_b \leq \underline{v} + g(1)$), and high price ($p_b > \underline{v} + g(1)$).

Case 1: $p_b \leq \underline{v}$. In this case, we always have $p_b \leq v_b[t]$. By applying the contraction mapping method [126], we show in Theorem 5.1 the existence of a market equilibrium, and provide a sufficient condition under which there exists a unique equilibrium.

Theorem 5.1 (Market Equilibrium under Low Price). *In monopoly market with a BSP, if $p_b \leq \underline{v}$, the market dynamics converges to the unique equilibrium point starting from any initial point $m_4[0] \in [0, 1]$ if*

$$max_{m_4 \in [0,1]} \frac{g^{'}(m_4)}{(\underline{v} + g(m_4))^2} < \frac{2L}{p_b^2}, \tag{5.9}$$

and the market equilibrium satisfies

$$\widetilde{m}_4 = \frac{w_b(\underline{v} + g(\widetilde{m}_4) - p_b) + 0.5w_b^2}{L(\underline{v} + g(\widetilde{m}_4))}. \tag{5.10}$$

The insight is that if the service quality is not very sensitive to the number of contributors, the market dynamics have only one possible equilibrium point. Note that the condition (5.9) is sufficient but not necessary for the equilibrium uniqueness and the convergence of market dynamics. Suppose the condition (5.9) is satisfied, Corollary 5.1 further characterizes the equilibrium by finding its upper bound, i.e., the largest possible proportion of users that participate in crowdsensing.

Corollary 5.1 (Maximum Equilibrium under Low Price). In monopoly market with a BSP, if $p_b \leq \underline{v}$ and the condition (5.9) holds, the market equilibrium point reaches its maximum M_1 when $w_b = p_b$, which satisfies

$$M_1 = \frac{p_b}{L} - \frac{p_b^2}{2L(\underline{v} + g(M_1))}. \tag{5.11}$$

Case 2: $p_b > \underline{v}+g(1)$. In this case, we always have $p_b > v_b[t]$. Theorem 5.2 discusses the existence and uniqueness of a market equilibrium point when the service price is high.

Theorem 5.2 (Market Equilibrium under High Price). *In monopoly market with a BSP, if $p_b > \underline{v} + g(1)$, the market dynamics converges to the unique equilibrium point starting from any initial point $m_4[0] \in [0, 1]$ if*

$$max_{m_4 \in [0,1]} g^{'}(m_4) < 2L. \tag{5.12}$$

and the market equilibrium satisfies

$$\widetilde{m}_4 = \frac{(\underline{v} + g(\widetilde{m}_4) - p_b + w_b)^2}{2L(\underline{v} + g(\widetilde{m}_4))}. \tag{5.13}$$

Corollary 5.2 (Maximum Equilibrium under High Price). In monopoly market with a BSP, if $p_b > \underline{v} + g(1)$ and the condition (5.12) holds, the market equilibrium point reaches its maximum M_2 when $w_b = p_b$, which satisfies

$$M_2 = \frac{\underline{v} + g(M_2)}{2L}. \tag{5.14}$$

Case 3: $\underline{v} < p_b \leq \underline{v} + g(1)$. Now we turn to the case of medium service price. Since $g(\cdot)$ is a continuous function, there exists a specific value of m_4, denoted as m_{p_b}, that satisfies $p_b = \underline{v} + g(m_{p_b})$. Our analysis shows that, if the conditions for equilibrium uniqueness hold for both low and high price cases, then there is also a unique market equilibrium in the medium price case.

Theorem 5.3 (Market Equilibrium under Medium Price). *In monopoly market with a BSP, if $\underline{v} < p_b \leq \underline{v} + g(1)$, then*

(1) if $w_b \leq \sqrt{2Lp_b m_{p_b}}$, then there exists at least one market equilibrium $\widetilde{m}_4 \in [0, m_{p_b}]$, which satisfies (5.13)*; and if the condition* (5.12) *holds, the equilibrium within $[0, m_{p_b}]$ is unique.*

(2) else if $w_b > \sqrt{2Lp_b m_{p_b}}$, then there exists at least one market equilibrium $\widetilde{m}_4 \in (m_{p_b}, 1]$, which satisfies (5.10)*; and if the condition* (5.9) *holds, the equilibrium within $[m_{p_b}, 1]$ is unique.*

The market converges to the unique equilibrium point starting from any initial point $m_4[0] \in [0,1]$ *if both conditions* (5.9) *and* (5.12) *hold.*

Theorem 5.3 indicates that, if the reward for crowdsensing participation is low enough, the market dynamics may converge to a point less than m_{p_b}, otherwise, there exists a market equilibrium larger than m_{p_b}.

Corollary 5.3 (Maximum Equilibrium under Medium Price). In monopoly market with a BSP, given p_b, if $\underline{v} < p_b \leq \underline{v} + g(1)$ and the conditions (5.9) and (5.12) hold, the market equilibrium point $\widetilde{m}_4$ reaches its maximum value when $w_b = p_b$, specifically: (1) if $p_b \leq 2Lm_{p_b}$, then its maximum value is M_2; and (2) if $p_b > 2Lm_{p_b}$, then its maximum value is M_1.

In summary, from (5.10) and (5.13), we are able to see how the market shares are determined by both the financial reward w_b and the intrinsic incentive through the function $g(m_4)$. This result differs from the one shown in (5.3) when only financial incentive works. Besides, Corollaries 5.1, 5.2, and 5.3 state that the reward that brings the largest number of contributors is always equal to the service price. In another word, the BSP attracts the largest number of contributors when it does not charge any service fee to contributors for subscribing to the service. While this result is intuitive, we are more interested in the question of what is the optimal reward of the BSP to maximize its profit.

5.4.2 BSP's Optimal Reward Decision

The BSP's steady-state profit function (profit obtained when the market equilibrium is reached), comprised of the revenues from the users with choice $s = 3$ and $s = 4$ respectively, is given by

$$\Pi^{\mathrm{B}} = p_b \widetilde{m}_3 + (p_b - w_b)\widetilde{m}_4. \tag{5.15}$$

Note that the BSP's investment cost of providing the basic service is fixed and does not affect its optimal reward decisions, hence we omit it in our model. Under the unique conditions (5.9) and (5.12), the value of $\widetilde{m}_4$ is the market equilibrium obtained from either (5.10) or (5.13), dependent on the reward w_b. Since it is difficult to solve the optimal reward that maximizes (5.15), we transform the original profit maximization problem with w_b as the decision variable to an equivalent problem using $\widetilde{m}_4$ as the decision variable. First we need to prove that there is a one-to-one correspondence between the reward w_b and the market share $\widetilde{m}_4$. It suffices to show that only one value of w_b satisfies (5.10) or (5.13) when $\widetilde{m}_4$ is given. From (5.10), we derive that there is only one positive solution of w_b:

$$\begin{aligned} w_b = w_{b1}(\widetilde{m}_4) &= p_b - \underline{v} - g(\widetilde{m}_4) \\ &+ \sqrt{(p_b - \underline{v} - g(\widetilde{m}_4))^2 + 2\widetilde{m}_4 L(\underline{v} + g(\widetilde{m}_4))}. \end{aligned} \tag{5.16}$$

From (5.13), there is only one positive solution of w_b:

$$\begin{aligned} w_b &= w_{b2}(\widetilde{m}_4) \\ &= p_b - \underline{v} - g(\widetilde{m}_4) + \sqrt{2\widetilde{m}_4 L(\underline{v} + g(\widetilde{m}_4)}. \end{aligned} \tag{5.17}$$

Note that if $w_b = p_b - (\underline{v} + g(\widetilde{m}_4)) - \sqrt{2\widetilde{m}_4 L(\underline{v} + g(\widetilde{m}_4))} < 0$, no user will contribute to the service. Therefore, once the BSP chooses the market share $\widetilde{m}_4$, it has equivalently chosen the reward w_b. We denote $p_{b,0}$ as the service price that satisfies $p_{b,0} = \underline{v} + g(m_{b,0})$, where $m_{b,0}$ satisfies $\underline{v} + g(m_{b,0}) = 2Lm_{b,0}$. Then, the equivalent profit maximization problem is to maximize the profit function described below based on different ranges of p_b:

Case 1: $p_b \leq \underline{v}$, where $\widetilde{m}_4 \in [0, M_1]$,

$$\begin{aligned} \Pi^{\mathrm{B}}(\widetilde{m}_4) = &\frac{(L - w_{b1}(\widetilde{m}_4))(\underline{v} + g(\widetilde{m}_4) - p_b)}{L(\underline{v} + g(\widetilde{m}_4))} p_b \\ &+ \widetilde{m}_4(p_b - w_{b1}(\widetilde{m}_4)) \end{aligned} \tag{5.18}$$

Case 2: $p_b > p_{b,0}$, where $\widetilde{m}_4 \in [0, M_2]$,

$$\Pi^{\mathrm{B}}(\widetilde{m}_4) = \widetilde{m}_4(p_b - w_{b2}(\widetilde{m}_4)) \tag{5.19}$$

Case 3: $\underline{v} < p_b \leq p_{b,0}$, where $\widetilde{m}_4 \in [0, M_1]$, if $\widetilde{m}_4 \leq m_{p_b}$, then $\Pi^{\mathrm{B}}(\widetilde{m}_4)$ is written in (5.19); else if $\widetilde{m}_4 > m_{p_b}$, then $\Pi^{\mathrm{B}}(\widetilde{m}_4)$ is written in (5.18).

The profit maximization problem above is an easy-to-solve nonlinear programming problem. Once the optimal value of $\widetilde{m}_4$ is obtained, one can calculate the optimal reward

w_b^* through (5.16) or (5.17), the service quality through (5.6), and finally the value of $m_3^{\{eq\}}$ through (5.7).

5.4.3 Numerical Examples

We now numerically illustrate the BSP's optimal decision and the corresponding market equilibrium. As the values of w_b^* and $\widetilde{m}_4$ are dependent on the function of $g(\cdot)$, for illustration purpose, we investigate a linear network externality $g(m_4) = \gamma m_4$ [123] in the following examples. For $\gamma > 0$ the externality is positive. The value of γ represents the marginal value of the users' contribution, which is affected by how well the BSP performs by taking advantage of the users' data. It can be obtained through empirical studies on the benefits of crowdourcing using historical or public data. The condition (5.9) becomes $\gamma < \frac{2L\underline{v}^2}{p_b^2}$, and the condition (5.12) becomes $\gamma < 2L$. Both conditions are always satisfied under the assumption of $L > v_h \geq \underline{v} + g(1)$.

One may intuit that when the marginal contribution from users' data γ increases, BSP should raise the reward to encourage more contributors. We use a concrete example shown in Fig. 5.6 (with $L = 28, \underline{v} = 15$) to delineate the intuition. As Fig. 5.6 shows, when γ increases, the BSP raises its reward to encourage more contributors, leading to a higher service quality level and a larger number of users. It is interesting to see that, the number of pure consumers does not change apparently with γ. This phenomenon is caused by the impacts of two factors: enhanced reward which encourages more consumers to contribute (to change user choices from $s = 3$ to $s = 4$), and enhanced service quality which attracts more consumers (to change user choices from $s = 0$ to $s = 3$ or $s = 4$). In this example, m_3 first slightly decreases with γ because the impact of enhanced reward is higher and then increases with γ when the impact of enhanced service quality is higher. This trend is different from the one in the case of the monopoly market with HSP, which shows that m_1 decreases with the marginal contribution from users' data. Such a difference is caused by the network effect. Note that the optimal reward is always lower than the reward that brings the maximum number of participating users, which equals p_b. To see how the service price affects the outcome performance, we plot the results under different values of p_b in Fig. 5.7. Similar to the case of a monopoly market with an HSP, a higher service price at the BSP leads to fewer consumers but a higher reward w_b and therefore a larger proportion of contributors. Furthermore, when the service price is high enough ($p_b \geq 16$), the percentage of users that subscribe to BSP but don't participate in crowdsensing tasks remains at $m_3 = 0$ since the service price is so high that $p_b > v_b$.

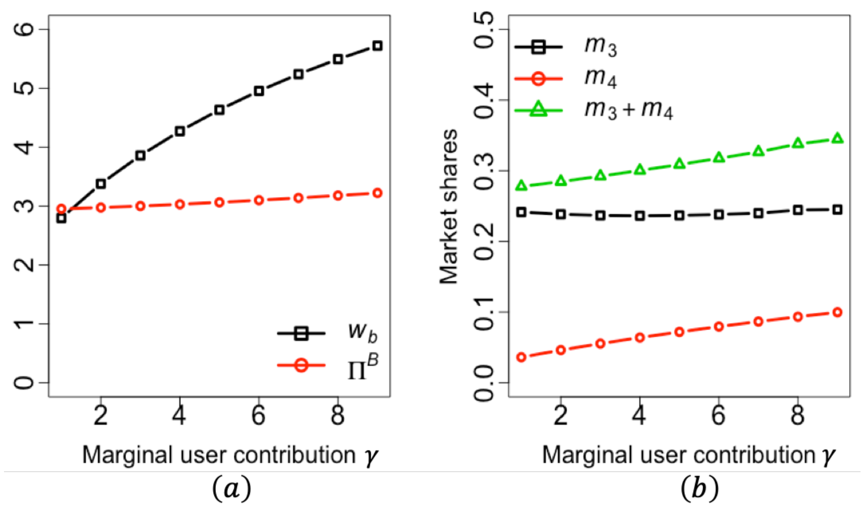


Figure 5.6: Optimal results of monopoly market of BSP vs. γ ($p_b = 11$).

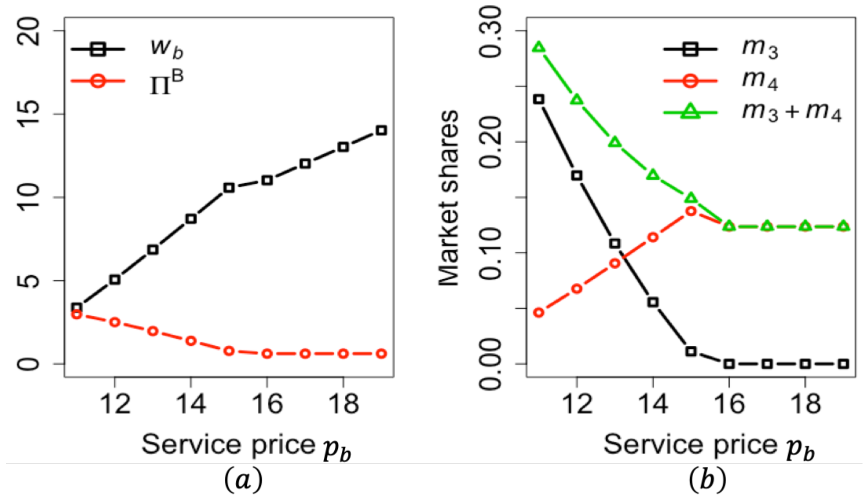


Figure 5.7: Optimal results of monopoly market of BSP vs. p_b ($\gamma = 2$).

5.4.4 Impact of Ignoring the Network Effect

To emphasize the importance of taking the users' intrinsic demand into consideration in incentive designing, we examine the scenario where the BSP ignores the intrinsic incentive of consumers; i.e., it assumes $g(\cdot) = 0$. The following Corollary gives the BSP's optimal reward decision:

Corollary 5.4. In a monopoly market with a BSP, if the existing intrinsic incentive of consumers is ignored by the BSP, given p_b, the BSP's optimal reward w_b^* satisfies:

$$w_b^* = \begin{cases} 0 & \text{if } p_b \leq \frac{2v}{3}, \\ 2p_b - \frac{4v}{3} & \text{if } \frac{2v}{3} < p_b \leq \underline{v}, \\ p_b - \frac{v}{3} & otherwise. \end{cases} \tag{5.20}$$

Using the same input parameters values as in Fig. 5.7, we obtain the numerical results when p_b increases from 11 to 19 as shown in Fig. 5.8. By comparing the results with those considering the network effect, we could find that failure to take into account the users' intrinsic incentive leads to too little extrinsic incentive from the BSP, which could entice too little participation from the consumers, especially at low service price. The reason is that, when the BSP ignores the users' intrinsic incentive for crowdsensing, it would underestimate the number of contributors and the contributions from users. Its underestimation causes the BSP to invest less in crowdsensing; that is, the BSP reduces the reward to the contributors. Therefore, to take the best advantage of crowdsourcing, it is important to incorporate consumers' intrinsic incentive with network effect into the incentive mechanism if such effects indeed exist.

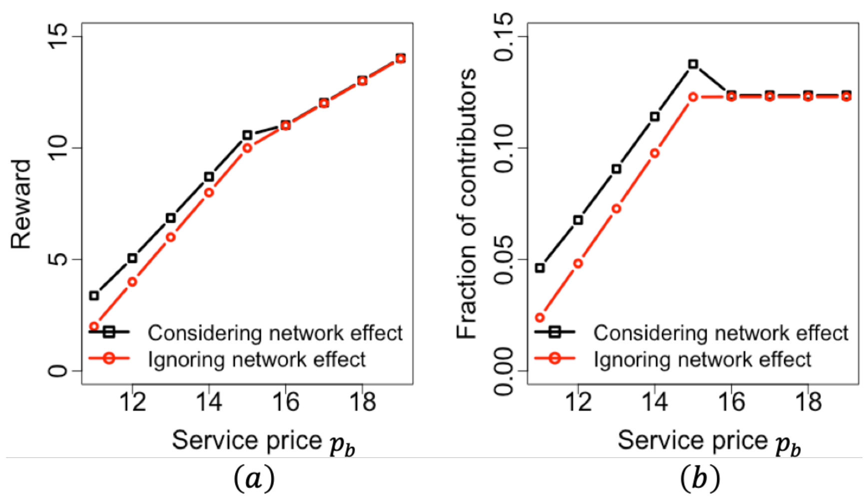


Figure 5.8: Optimal results of monopoly market of BSP when ignoring the network effect ($\gamma = 2$).

5.5 Competitive Market Analysis

In this section, we extend the monopoly model to the duopoly case of two service providers, one HSP, and one BSP, competing for a population of users. Each user chooses to subscribe to at most one service and decides whether to contribute to the service it subscribes to. We first analyze the market dynamics and give an algorithm to derive the possible equilibrium. Then we discuss the reward strategies of the HSP and the BSP using a game-theoretical model. Finally, a set of numerical examples are given to illustrate the impacts of competition.

5.5.1 Analysis of the Market Share

With five choices available ($s = 0, 1, 2, 3, 4$), a user's optimal choice is the one that brings the largest payoff. For example, in period $t+1$, a user with parameters (α, l) subscribes and participates in crowdsensing at BSP if and only if $\Pi^{\mathrm{E}}_{s=4} \geq \max\{\Pi^{\mathrm{E}}_{s=0}, \Pi^{\mathrm{E}}_{s=1}, \Pi^{\mathrm{E}}_{s=2}, \Pi^{\mathrm{E}}_{s=3}\}$, that is:

$$\begin{aligned} & w_b \geq l \text{ and} \\ & \frac{p_b - w_b + l}{v_b[t]} \leq \alpha \leq \min\{\frac{p_h - p_b + w_b - l}{v_h - v_b[t]}, \frac{p_h - p_b + w_b - w_h}{v_h - v_b[t]}\}. \end{aligned} \tag{5.21}$$

We can divide the user population into five segments according to their choices. If we illustrate the whole population as a $1 \times L$ square (as shown in Fig. 5.9), the five segments are bounded by ten lines, including two vertical lines $l = w_h$, $l = w_b$, four horizontal lines: $\alpha_1 = \frac{p_h}{v_h}$, $\alpha_2[t] = \frac{p_b}{v_b[t]}$, $\alpha_3[t] = \frac{p_h - p_b}{v_h - v_b[t]}$, $\alpha_4[t] = \frac{p_h - p_b + w_b - w_h}{v_h - v_b[t]}$, and four straight lines: $\alpha_5[t] = \frac{p_h - p_b + w_b - l}{(v_h - v_b[t])}$, $\alpha_6[t] = \frac{p_h - p_b - w_h + l}{(v_h - v_b[t])}$, $\alpha_7[t] = \frac{p_h - w_h + l}{v_h}$, $\alpha_8[t] = \frac{p_b - w_b + l}{v_b[t]}$. Fig. 5.9 gives an illustrative example of the market shares (for convenience, we omit the time indexes $[t]$ of all the variables in the figure). One can observe that only the users with both low usage levels (α) and low private loss evaluation (l) would subscribe to BSP and participate in crowdsensing tasks. The users with both high usage level and high loss evaluation would subscribe to HSP without crowdsensing participation. The values of the market shares at period t can be calculated as

$$m_1[t+1] = \int_{w_h}^{L} \int_{\max\{\alpha_5[t], \alpha_1, \alpha_3[t]\}}^{1} \frac{1}{L} d\alpha dl \tag{5.22}$$

$$m_2[t+1] = \int_{0}^{w_h} \int_{\max\{\alpha_6[t], \alpha_4[t], \alpha_7[t]\}}^{1} \frac{1}{L} d\alpha dl \tag{5.23}$$

$$m_3[t+1] = \int_{w_b}^{L} \int_{\alpha_2[t]}^{\min\{\alpha_3[t], \alpha_6[t]\}} \frac{1}{L} d\alpha dl \tag{5.24}$$

$$m_4[t+1] = \int_{0}^{w_b} \int_{\alpha_8[t]}^{\min\{\alpha_4[t], \alpha_5[t]\}} \frac{1}{L} d\alpha dl. \tag{5.25}$$

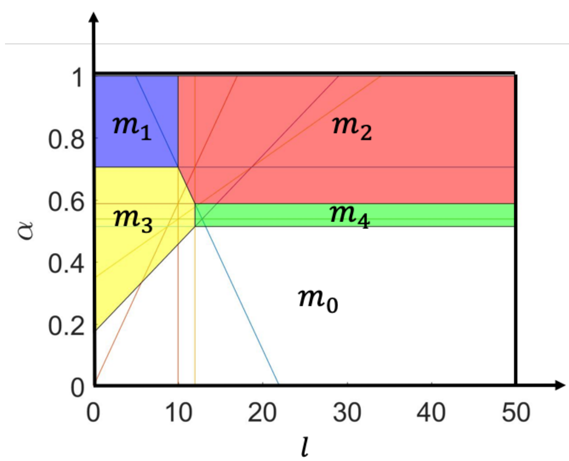


Figure 5.9: (color online) Market shares in a competitive market.

The competitive market dynamics reach an equilibrium when m_s remain unchanged. The definition of the competitive market equilibrium is given below:

Definition 5.2 (Competitive Market Equilibrium). In competitive market, $(\widetilde{m}_1, \widetilde{m}_2, \widetilde{m}_3, \widetilde{m}_4)$ is an equilibrium point of the market share if it satisfies $\widetilde{m}_s = m_s[t+1] = m_s[t]$ for all $s \in \{0, 1, 2, 3, 4\}$.

Since it is challenging to prove the existence and uniqueness of the competitive market equilibrium, we will use Algorithm 1 for finding the equilibrium point (where ϵ is a threshold with a small value, e.g., $\epsilon = 10^{-4}$). The basic idea is to first find the equilibrium of $\widetilde{m}_4$, and then calculate the boundaries of the segments, based on which we can derive the equilibrium point $\widetilde{m}_s$ $(s = 1, 2, 3)$. The reason for our focus on the convergence of m_4 in the first step is that, when m_4 is fixed, the value of v_b and thus the boundaries between the five user segments do not change anymore, and therefore we have $\Delta m_s = 0$ $(s \in \{1, 2, 3, 4\})$ as long as $\Delta m_4 = 0$. As the service quality, v_b increases with the number of contributors, which tends to attract more users subscribing to BSP and participating in crowdsensing, the changing of m_4 is monotonic. Therefore, the market share is expected to converge to a unique equilibrium under the conditions (5.9) and (5.12). Indeed, we observe through numerical simulations that, the market always converges to a unique equilibrium under fixed rewards. To conserve space, more analyses of $m_4[t]$ are available in the Appendix.

5.5.2 Service Providers' optimal Reward Decisions

With the consideration of the market response, both BSP and HSP make their reward decisions to maximize their own profits, (5.4) and (5.15), respectively. We restrict our

Algorithm 1: Competitive Market Equilibrium

Input: Service prices p_h, p_b, rewards w_b, w_h, service qualities v_h, $\underline{v}$, maximum user loss evaluation L, HSP's investment cost function $C(\cdot)$, users' contribution function $g(\cdot)$ at BSP

Output: $\widetilde{m}_3$, $\widetilde{m}_4$, $\widetilde{m}_1$ and $\widetilde{m}_2$

Initialize $t = 0, m_4[0] = 0$;
repeat
  $t \leftarrow t + 1$;
  update $v_b[t]$ according to (5.6);
  update $m_4[t]$ according to (5.25);
until $m_4[t] - m_4[t-1] \leq \epsilon$;
$\widetilde{m}_4 = m_4[t]$;
Calculate $\widetilde{m}_1$, $\widetilde{m}_2$ and $\widetilde{m}_3$ according to (5.22), (5.23), and (5.24) respectively.

analysis to the situation of static reward strategy and simultaneous market entry of the two service providers, to remove the impacts of the first-mover advantage when evaluating the impacts of competition. The expected profit of each service provider is determined by both its reward decision and the market shares $\widetilde{m}_i$, which are also dependent on the reward decision of the other service provider. We denote $w = \{w_h, w_b\}$ as the decision profiles of the two service providers and define the notion of Nash equilibrium (NE) as one solution concept of the reward game:

Definition 5.3. The strategy profile $w^* = \{w_h^*, w_b^*\}$ constitutes a Nash Equilibrium of the reward game if $\Pi^{\mathrm{H}}(w_h^*, w_b^*) \geq \Pi^{\mathrm{H}}(w_h, w_b^*), \forall w_h \in [0, p_h]$, and $\Pi^{\mathrm{B}}(w_h^*, w_b^*) \geq \Pi^{\mathrm{B}}(w_h^*, w_b)$, $\forall w_b \in [0, p_b]$.

The NE of the reward game yields strategic reward policies of both HSP and BSP, in which none of the service providers would unilaterally change its decision to increase its profit. Due to the difficulty in characterizing the close-form expression of market equilibrium, it is difficult to prove the existence of the NE solution, even though there is expected to be a one-to-one relationship between the market equilibrium and the strategy profile. Therefore, we will focus on using a distributed algorithm with a sequential updating mechanism to obtain the NE. The key idea is that the service providers take turns updating their reward decisions until convergence. Firstly, we randomly choose w_h and w_b as our starting point. Then, in each iteration, BSP or HSP updates its profit-maximizing reward in turn given the reward decision of the other service provider. This iterative procedure continues until none of them could enhance its profit by changing the decision anymore, at which point the algorithm returns the Nash equilibrium. The distributed algorithm is summarized in Algorithm 2. We note that in arriving at the NE of the reward game, the information shared between HSP and BSP includes the service prices and qualities, the cost, and users' marginal contributions. The service price and quality is public information representing the service providers' marketing strategies. The cost, composed largely of the investment cost on traditional appliances, can be estimated from the supply market. It is also likely to evaluate the value of user-contributed information through data mining techniques.

5.5.3 Numerical Examples

In this section, we numerically illustrate the Nash Equilibrium of the reward game and the impacts of competition. Unless specified otherwise, we assume that $C(m_2) = 1 + m_2^2 - 2\beta_1 m_2, g(m_4) = \gamma m_4, m_4[0] = 0$, $L = 28, \underline{v} = 15, p_b = 11, p_h = 20, v_h = 25, \gamma = 7$, $\beta_1 = 0.7$. Firstly, we show in Fig. 5.10 the equilibrium outcomes of the market shares for the two operators with different rewards given to the contributors. The equilibrium outcomes of the monopoly markets with only HSP or BSP under the same input parameters are also plotted in Fig. 5.11. Several observations can be made here: First of all, a greater

Algorithm 2: Distributed Algorithm for Reward Game of HSP and BSP

Input: Service prices p_h,p_b, service qualities v_h,$\underline{v}$, maximum user loss evaluation L, HSP's investment cost function $C(\cdot)$, users' contribution function $g(\cdot)$ at BSP

Output: w_b^*, w_h^*, Π^{B*} and Π^{H*}

Generate sets V_{w_h} and V_{w_b} by discretizing $[0, p_h]$ and $[0, p_b]$, respectively, with $N_h = |V_{w_h}|, N_b = |V_{w_b}|$;

Initialize $w_b \in [0, p_h], w_h \in [0, p_b]$, $n = 0$, $\Pi^{\mathrm{H}}[0] = 0$, $\Pi^{\mathrm{B}}[0] = 0, m_4[0] = 0$;

repeat

  $n \leftarrow n + 1$;

  for $w_{b,k} \in V_{w_b}, k \in \{1, ..., N_b\}$ **do**

    Calculate $\widetilde{m}_3$ and $\widetilde{m}_4$ by substituting $w_{b,k}$ and w_h into Algorithm 1;

    $\Pi_k^{\mathrm{B}} = \Pi^{\mathrm{B}}(\widetilde{m}_3, \widetilde{m}_4, w_{b,k})$ according to (5.15);

  $k^* = \arg\max_{k \in \{1,...,N_b\}} \Pi_k^{\mathrm{B}}$; $\Pi^{\mathrm{B}}[n] = \pi_{b,k^*}, w_b = w_{b,k^*}$;

  for $w_{h,k} \in V_{w_h}, k \in \{1, ..., N_h\}$ **do**

    Calculate $\widetilde{m}_1$ and $\widetilde{m}_2$ by substituting $w_{h,k}$ and w_b into Algorithm 1;

    $\Pi_k^{\mathrm{H}} = \Pi^{\mathrm{H}}(\widetilde{m}_1, \widetilde{m}_2, w_{h,k})$ according to (5.4);

  $k^* = \arg\max_{k \in \{1,...,N_h\}} \Pi_k^{\mathrm{H}}$; $\Pi^{\mathrm{H}}[n] = \pi_{h,k^*}, w_h = w_{h,k^*}$;

until $\Pi^{\mathrm{H}}[n] - \Pi^{\mathrm{H}}[n-1] \leq \epsilon$ *and* $\Pi^{\mathrm{B}}[n] - \Pi^{\mathrm{B}}[n-1] \leq \epsilon$;

reward at a service provider brings more contributors for it while fewer contributors at its competitor (the other service provider). Secondly, a comparison between Fig. 5.10 and Fig. 5.11 shows how the competition makes a difference in reshaping the market. Specifically, the competition has a large influence on the market shares of the BSP as the HSP attracts many users from it. Finally, the impact of the BSP's reward w_b is largely weakened by the competition as illustrated in Fig. 5.10(b).

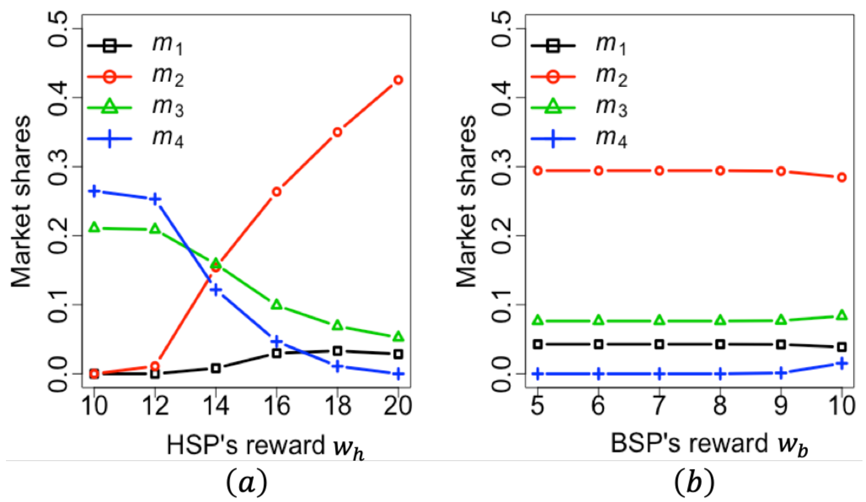


Figure 5.10: Market shares in a competitive market vs. (a) w_h and (b) w_b.

Next, we study the impacts of the marginal user contribution β_1 and γ on the equilibrium results, respectively. As shown in Fig. 5.12 and Fig. 5.13, the marginal contribution from users' data β_1 at the HSP's side has a negligible impact on the results due to its stable service level. A larger value of γ, the marginal user contribution at the BSP's side, apparently increases the reward given by the BSP and its number of contributors, while decreasing the number of contributors to the HSP. Besides, a comparison between Fig. 5.12 and Fig. 5.4 or between Fig. 5.13 and Fig. 5.6 indicates that, more contributors

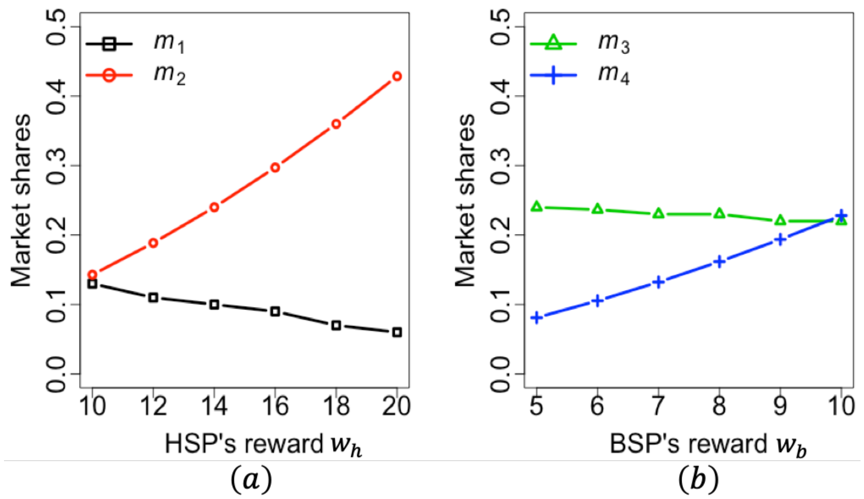


Figure 5.11: Market shares in a monopoly market with an HSP or a BSP vs. (a) w_h and (b) w_b.

(represented by the value of $m_2 + m_4$) are attracted in competitive market than in the case of monopoly market of either HSP or BSP. The reason is that a service provider, facing a competitor, needs to enhance the reward for a larger market share. Moreover, due to larger rewards and corresponding more contributors, the HSP invests less on the traditional sensor network and users experience better service from the BSP.

5.6 Appendices

This section provides the proofs for Chapter 5.

1. Proof for Proposition 5.2.

Proof: By substituting m_1 and m_2 in (5.4) with (5.2) and (5.3), respectively, the HSP's profit function can be analyzed in two cases:

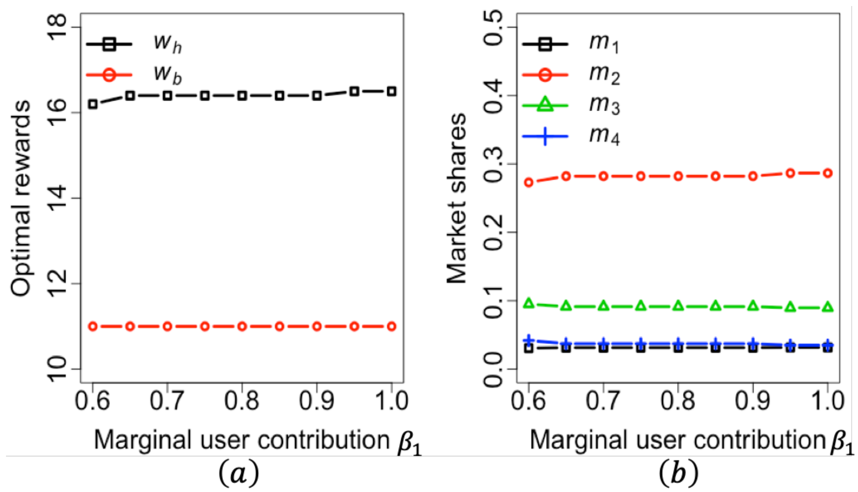


Figure 5.12: Optimal results of a competitive market with different β_1.

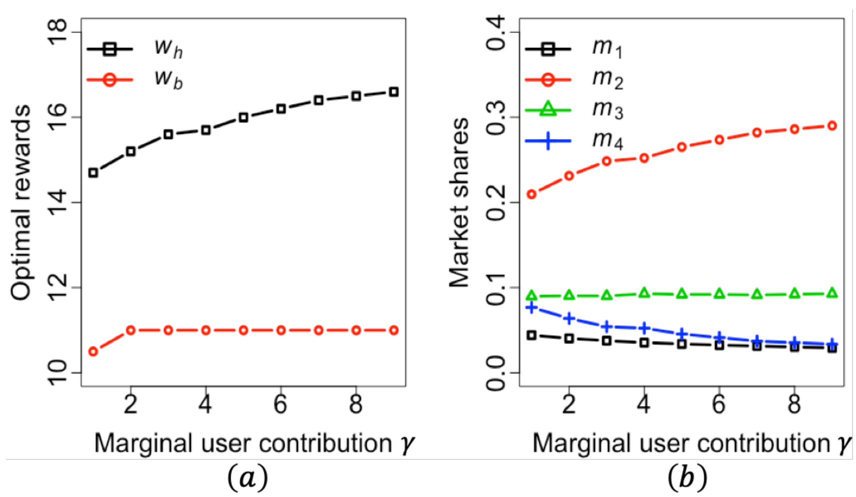


Figure 5.13: Optimal results of a competitive market with different γ.

(1) *Case 1*: if $p_h \leq v_h$,

$$\Pi^{\mathrm{H}} = \frac{(v_h - p_h)(p_h L - w_h^2 + \beta_0 w_h) + 0.5 w_h^2 (p_h + \beta_0 - w_h)}{L v_h} - C_0 \tag{5.26}$$

Let's assume that $E_1 = \frac{d\Pi^{\mathrm{H}}}{dw_h} = \frac{(v_h - p_h)\beta_0 + w_h(3p_h + \beta_0 - 2v_h) - 1.5w_h^2}{Lv_h}$. Note that E_1 is a quadratic form equation that has at maximum two roots (i.e., $E_1 = 0$). We call these roots $w_{h,1}$ and $w_{h,2}$, and they can be written by the following expression:

$$w_{h,1,2} = \frac{(2v_h - 3p_h - \beta_0) \pm \sqrt{(2v_h - 3p_h - \beta_0)^2 + 6\beta_0(v_h - p_h)}}{-3}. \tag{5.27}$$

We are interested in determining these roots (i.e., if they occur in $[0, p_h]$) for values of v_h, p_h, and a, and it is easy to derive that $w_{h,1} \leq 0$ and $w_{h,2} \geq 0$. Let us study the behavior of E_1 by taking its derivation and calculating its global maximum point and the corresponding reward, i.e.,

$$\frac{dE_1}{dw_h} = \frac{3p_h + \beta_0 - 2v_h - 3\overline{w}_h}{Lv_h} = 0. \tag{5.28}$$

So E_1 has a global maximum point at

$$\overline{w}_h = \frac{3p_h + \beta_0 - 2v_h}{3}. \tag{5.29}$$

Note that $\overline{w}_h$ is less than 0 for $\beta_0 < 2v_h - 3p_h$ and it is greater or equal to p_h for $\beta_0 > 2v_h$. Therefore we analyze the HSP's optimal reward in the following scenarios related to different ranges of β_0.

(a) If $\beta_0 \leq 2v_h - 3p_h$, then $\frac{dE_1}{dw_h} = \frac{d^2\Pi^{\mathrm{H}}}{dw_h^2} < 0$, that is, E_1 decreases with w_h. Since $w_{h,1} < 0$, when $w_h \in [0, p]$, we have: if $w_h = w_{h,2}$, then $E_1 = 0$; if $w_h < w_{h,2}$, then $E_1 > 0$; and if $w_h > w_{h,2}$, then $E_1 < 0$. Therefore, Π^{H} is maximized at $w_h^* = \min\{p_h, w_{h,2}\}$.

(b) If $2v_h - 3p_h < \beta_0 \leq 2v_h$, then E_1 inceases with w_h for $w_h \leq \overline{w}_h$ and E_1 decreases with w_h for $w_h > \overline{w}_h$. As $E_1(w_h = w_{h,1}) = 0$ and $E_1(w_h = w_{h,2}) = 0$, we have: if $w_h \in [0, \overline{w}_h]$, then $E_1 > 0$; if $w_h \in (\overline{w}_h, w_{h,2}]$, then $E_1 > 0$; else if $w_h > w_{h,2}$, $E_1 < 0$. Therefore, Π^{H} is maximized at $w_h^* = \min\{p_h, w_{h,2}\}$.

(c) If $\beta_0 > 2v_h$, then $\frac{dE_1}{dw_h} = \frac{d^2\Pi^{\mathrm{H}}}{dw_h^2} > 0$, that is, E_1 increases with w_h. Besides, $w_{h,2} > \overline{w}_h$. Now we have $E_1 > 0$ for all $w_h \in [0, p_h]$. Therefore, Π^{H} is maximized at $w_h^* = p_h$. In this case, the HSP only profits from serving the users with high privacy concern and do not participate in crowdsensing tasks.

Because $w_{h,2} > p_h$ when $\beta_0 > \frac{p_h}{2}(4 - \frac{3p_h}{v_h})$, and $\frac{p_h}{2}(4 - \frac{3p_h}{v_h}) < 2v_h$, we can conclude that: under high marginal contribution from users' data ($\beta_0 > \frac{p_h}{2}(4 - \frac{3p_h}{v_h})$), $w_h^* = p_h$; and under low marginal contribution from users' data ($\beta_0 \leq \frac{p_h}{2}(4 - \frac{3p_h}{v_h})$), $w_h^* = \frac{(2v_h - 3p_h - \beta_0) - \sqrt{(2v_h - 3p_h - \beta_0)^2 + 6\beta_0(v_h - p_h)}}{-3}$.

(2) *Case 2*: if $p_h > v_h$

$$\Pi^{\mathrm{H}} = (p_h - w_h + \beta_0)\frac{(v_h - p_h + w_h)^2}{2Lv_h} - C_0. \tag{5.30}$$

Let's assume that $E_2 = \frac{d\Pi^{\mathrm{H}}}{dw_h} = \frac{(v_h - p_h + w_h)(3p_h + 2\beta_0 - v_h - 3w_h)}{2Lv_h}$ in the case of $p_h > v_h$. Then E_2 has at maximum two roots written by:

$$w_{h,1} = p_h - v_h \tag{5.31}$$

$$w_{h,2} = p_h + \frac{2\beta_0 - v_h}{3} \tag{5.32}$$

We can see that $w_{h,2} > p_h$ when $\beta_0 > 0.5v_h$, and $w_{h,2} < 0$ when $\beta_0 < 0.5(v_h - 3p_h)$.

By setting

$$\frac{dE_2}{dw_h} = \frac{3p_h + \beta_0 - 2v_h - 3\overline{w}_h}{Lv_h} = 0 \tag{5.33}$$

we have

$$\overline{w}_h = \frac{3p_h + \beta_0 - 2v_h}{3} > 0. \tag{5.34}$$

The HSP's optimal reward can also be analyzed related to different ranges of β_0:

(a) if $\beta_0 > 2v_h$, then $w_{h,2} > p_h$, $\overline{w}_h > p_h$ and $\frac{dE_2}{dw_h} = \frac{d^2\Pi^{\mathrm{H}}}{dw_h^2} > 0$, that is, E_2 increases with w_h. Now we have: if $w_h = w_{h,1}$, then $E_2 = 0$; if $w_h < w_{h,1}$, then $E_2 < 0$; and if $w_h > w_{h,1}$, then $E_2 > 0$. Therefore, Π^{H} is maximized at $w_h = 0$ or $w_h = p_h$. Because $w_h > p_h - v_h$, we have $w_h^* = p_h$.

(b) If $\beta_0 \leq 2v_h$, then E_2 increases with w_h for $w_h \leq \overline{w}_h$ and E_2 decreases with w_h for $w_h > \overline{w}_h$. As $E_2(w_h = w_{h,1}) = 0$ and $E_2(w_h = w_{h,2}) = 0$, we have: if $w_h \in (w_{h,1}, \overline{w}_h]$, then $E_2 > 0$; if $w_h \in (\overline{w}_h, w_{h,2}]$, then $E_2 > 0$; else if $w_h > w_{h,2}$, $E_2 < 0$. Therefore, Π^{H} is maximized at $w_h = \min\{p_h, w_{h,2}\}$.

Therefore, we can conclude that, for $p_h > v_h$: if $\beta_0 > 0.5v_h$, then $w_h^* = p_h$; else $w_h^* = p_h + \frac{2\beta_0 - v_h}{3}$. □

2. Proof for Theorem 5.1.

Proof: (1) Proof for the existence and uniqueness of the equilibrium:

We first define $H_1(m_4) = \frac{w_b(\underline{v} + g(m_4) - p_b) + 0.5w_b^2}{L(\underline{v} + g(m_4))} - m_4$ for $m_4 \in [0, 1]$. By definition, $\widetilde{m}_4$ is an equilibrium point if and only if it is a root of $H_1(m_4)$.

It is easy to check that $H_1(0) = \frac{w_b(\underline{v}-p_b)+0.5w_b^2}{L\underline{v}} > 0$ and $H_1(1) = \frac{w_b(\underline{v}+g(1)-p_b)+0.5w_b^2}{L(\underline{v}+g(1))} - 1 = \frac{(w_b-L)(\underline{v}+g(1))-(p_b-0.5w_b)}{L(\underline{v}+g(1))} < 0$. As $H_1(m_4)$ is continuous on $[0,1]$, $H_1(m_4)$ has at least one root on $[0,1]$.

If $\frac{g^{'}(m_4)w_b(p_b-0.5w_b)}{L(\underline{v}+g(m_4))^2} < 1$ for all $m_4 \in [0,1]$, then $H_1{'}(m_4) < 0$, and $H_1(m_4)$ is strictly decreasing on $[0,1]$. Therefore, $H_1(m_4)$ has only one root if $\max_{m_4 \in [0,1]} \frac{g^{'}(m_4)}{(\underline{v}+g(m_4))^2} < \frac{L}{w_b(p_b-0.5w_b)}$.

Since $w_b(p_b-0.5w_b)$ takes its maximum value $0.5p_b^2$ when $w_b = p_b$, $\frac{g^{'}(m_4)w_b(p_b-0.5w_b)}{L(\underline{v}+g(m_4))^2} < 1$ always holds when $\max_{m_4 \in [0,1]} \frac{g^{'}(m_4)}{(\underline{v}+g(m_4))^2} < \frac{2L}{p_b^2}$.

(2) Proof for the convergence of the market:

We define $F_1(m_4) = \frac{w_b(\underline{v}+g(m_4)-p_b)+0.5w_b^2}{L(\underline{v}+g(m_4))}$. Using the contraction mapping method [126], $\forall m_{4,a}, m_{4,b} \in [0,1]$, if there is a real number $\kappa \in [0,1)$ such that $|F_1(m_{4,a}) - F_1(m_{4,b})| \leq \kappa |m_{4,a} - m_{4,b}|$, then m_4 converges to a fixed point $\widetilde{m}_4$ satisfying (5.10).

Let $m_{4,a}$ and $m_{4,b}$ be two different real numbers arbitrarily chosen from the interval $[0,1]$, and suppose without loss of generality that $m_{4,a} > m_{4,b}$.

Since $F_1(\cdot)$ is continuous and differentiable on $[0,1]$, according to the mean value theorem, there exists $m_{4,c} \in (m_{4,a}, m_{4,b})$ such that

$$F_1^{'}(m_{4,c}) = \frac{F_1(m_{4,a}) - F_1(m_{4,b})}{m_{4,a} - m_{4,b}}. \tag{5.35}$$

Then, if the condition (5.9) holds, we have

$$\begin{aligned} &|F_1(m_{4,a}) - F_1(m_{4,b})| = |F_1^{'}(m_{4,c})||m_{4,a} - m_{4,b}| \\ &\leq \kappa_1 |m_{4,a} - m_{4,b}| \end{aligned} \tag{5.36}$$

where $\kappa_1 = \max_{m_4 \in [0,1]} \frac{g^{'}(m_4) w_b (p_b - 0.5 w_b)}{L(\underline{v} + g(m_4))} < 1$. $\square$

3. Proof for Corollary 5.1.

Proof: From (5.10), it is easy to prove that $\frac{dM_1}{dw_b} > 0$ when the condition (5.9) holds. Therefore, $\widetilde{m}_4$ reaches its maximum value when $w_b = p_b$. $\square$

4. Proof for Theorem 5.2.

Proof: (1) Proof for the existence and uniqueness of the equilibrium:

We define $H_2(m_4) = \frac{(\underline{v} + g(m_4) - p_b + w_b)^2}{2L(\underline{v} + g(m_4))} - m_4$ for $m_4 \in [0, 1]$. Because $H_2(0) = \frac{(\underline{v} - p_b + w_b)^2}{2L\underline{v}} > 0$ and

$H_2(1) = \frac{(\underline{v} + g(1) - p_b + w_b)^2 - 2L(\underline{v} + g(1)))}{2L(\underline{v} + g(1))} < 0$, we have: $H_2(m_4)$ has at least one root on $[0, 1]$.

If $g^{'}(m_4)[1 - (\frac{p_b - w_b}{\underline{v} + g(m_4)})^2] < 2L$, then ${H_2}'(m_4) < 0$, and $H_2(m_4)$ is strictly decreasing on $[0, 1]$. Therefore, $H_2(m_4)$ has only one root if $\max_{m_4 \in [0,1]} g^{'}(m_4)[1 - (\frac{p_b - w_b}{\underline{v} + g(m_4)})^2] < 2L$.

We only consider the case when $\frac{p_b - w_b}{\underline{v} + g(m_4)} \leq 1$, otherwise $\pi_{\alpha,l} \leq 0$ and no user would subscribe to the service. Because $[1 - (\frac{p_b - w_b}{\underline{v} + g(m_4)})^2]$ takes the largest value 1 when $p_b = w_b$, so as long as $\max_{m_4 \in [0,1]} g^{'}(m_4) < 2L$, we have ${H_2}'(m_4) < 0$.

(2) Proof for the convergence of the market:

We define $F_2(m_4) = \frac{(\underline{v} + g(m_4) - p_b + w_b)^2}{2L(\underline{v} + g(m_4))}$. Using similar procedure to that of Theorem 1, if the condition (5.12) holds, we can prove that $\forall m_{4,a} \in [0, 1]$, $\forall m_{4,b} \in [0, 1]$,

$$|F_2(m_{4,a}) - F_2(m_{4,b})| \leq \kappa_2 |m_{4,a} - m_{4,b}| \tag{5.37}$$

where $\kappa_2 = \max_{m_4 \in [0,1]} \frac{g^{'}(m_4)[1 - (\frac{p_b - w_b}{\underline{v} + g(m_4)})^2]}{2L} < 1$. $\square$

5. Proof for Corollary 5.2.

Proof: The proof is similar to that of Corollary 5.1 and we omit the detail here. $\square$

6. Proof for Theorem 5.3.

Proof: (1) Proof for the existence and uniqueness of the equilibrium:

If $m_4[t] \leq m_{p_b}$, then $p \geq v_b[t]$, we have $\Delta m_4 = m_4[t+1] - m_4[t] = H_2(m_4[t])$.

Since $H_2(0) > 0$ and $H_2(m_{p_b}) = \frac{w_b^2}{2L(\underline{v}+g(m_{p_b}))} - m_{p_b}$, we can derive that, if $w_b \leq \sqrt{2Lp_b m_{p_b}}$, then $H_2(m_{p_b}) < 0$ and there exists at least one market equilibrium $\widetilde{m}_4 \in [0, m_{p_b}]$. Similarly, we can prove that if $w_b > \sqrt{2Lp_b m_{p_b}}$, then there exists at least one market equilibrium $\widetilde{m}_4 \in (m_{p_b}, 1]$.

The proof for the uniqueness of the equilibrium is similar to those for Theorem 5.1 and Theorem 5.2, and is omitted here.

(2) Proof for the convergence of the market:

We define

$$F(m_4) = \begin{cases} F_1(m_4) = \frac{w_b(\underline{v}+g(m_4)-p_b)+0.5w_b^2}{L(\underline{v}+g(m_4))} & \text{if } m_4 > m_{p_b}, \\ F_2(m_4) = \frac{(\underline{v}+g(m_4)-p_b+w_b)^2}{2L(\underline{v}+g(m_4))} & \text{if } m_4 \leq m_{p_b}. \end{cases} \tag{5.38}$$

Let $m_{4,a}$ and $m_{4,b}$ be two different real numbers arbitrarily chosen from the interval $[0, 1]$, and suppose without loss of generality that $m_{4,a} > m_{4,b}$.

If $m_{4,a} > m_{4,b} > m_{p_b}$ or $m_{4,b} < m_{4,a} < m_{p_b}$, we have already proved that there is a real number $\kappa \in [0, 1]$ such that $|F(m_{4,a}) - F(m_{4,b})| \leq \kappa |m_{4,a} - m_{4,b}|$ in the proofs of Theorems 1 and 2, where $\kappa = \max\{\kappa_1, \kappa_2\} < 1$.

Now we consider the case when $m_{4,a} > m_{p_b} > m_{4,b}$. Note that $F_1(m_{p_b}) = F_2(m_{p_b})$, we have

$$
\begin{aligned}
&|F(m_{4,a}) - F(m_{4,b})| = |F_1(m_{4,a}) - F_2(m_{4,b})| \\
&= |F_1(m_{4,a}) - F_1(m_{p_b}) + F_2(m_{p_b}) - F_2(m_{4,b})| \\
&\leq |F_1(m_{4,a}) - F_1(m_{p_b})| + |F_2(m_{p_b}) - F_2(m_{4,b})| \\
&\leq \kappa_1 |m_{4,a} - m_{p_b}| + \kappa_2 |m_{p_b} - m_{4,b}| \\
&\leq \max\{\kappa_1, \kappa_2\}(|m_{4,a} - m_{p_b}| + |m_{p_b} - m_{4,b}|) \\
&\leq \max\{\kappa_1, \kappa_2\}(|m_{4,a} - m_{4,b}|)
\end{aligned}
\tag{5.39}
$$

Since $\kappa_1 < 1$ and $\kappa_2 < 1$, we have $\max\{\kappa_1, \kappa_2\} < 1$.

Based on the contraction mapping theorem, we conclude that m_4 converges to a fixed point. □

7. Proof for Corollary 5.3.

Proof: The proof is similar to that of Corollary 5.4 and we omit the detail here. □

8. Proof for Corollary 5.4.

Proof: The network externality is neglected by setting $g(\cdot) = 0$. The BSP's profit maximization problem then is equivalent to that of the HSP by setting $C(\cdot) = 0$ and using p_b instead of p_h, $v_b = \underline{v}$ instead of v_h. Therefore, we have: if $p_b \leq \underline{v}$, then $w_b^* = \frac{\sqrt{(2\underline{v}-3p_b)^2} - (2\underline{v}-3p_b)}{3}$; else, $w_b^* = p_b - \frac{\underline{v}}{3}$. □

9. Analysis of the market share for BSP in competitive market

In this section, we provider further analysis of the market shares in competitive markets.

Since when m_4 is fixed, the boundaries between the five user segments are fixed, we focus on the convergence of m_4. The value of m_4 is dependent on the values of α_4, α_8 and α_5. Fig. 5.14 shows the possible ranges of contributors. The part of users with choice $s = 4$ is represented by the part of the shaded area that is to the left of $l = w_b$. Note that as the reward decreases from $w_b = p_b$ to $w_b = 0$, that is, as we move the line $l = w_b$ from right to left, $m_4[t+1]$ decreases from $\hat{m}[t+1]$ to 0, where $\hat{m}[t+1]$ is the percentage of the shaded area and can be computed as

$$\hat{m}[t+1] = \begin{cases} S_1[t]/L & \text{if in cases (a),(c)} \\ (S_1[t] - S_2[t])/L & \text{if in cases (b),(d)} \\ (S_3[t] - S_4)/L & \text{if in case (e)} \\ 0 & \text{if } \alpha_8 \geq \alpha_4. \end{cases} \tag{5.40}$$

where $S_1[t] = \frac{1}{2}v_b[t](\alpha_4[t] - \frac{p_b - w_b}{v_b[t]})^2$, $S_2[t] = \frac{1}{2}(\alpha_4[t] - \frac{p_h}{v_h})(\alpha_4[t]v_b[t] - p_b + w_b - w_h)$, $S_3[t] = \frac{(v_b[t] - p_b + w_b)^2}{2v_b[t]}$, and $S_4 = \frac{(v_h - p_h)^2}{2v_h}$. $\square$

10. Special Case: Requiring consumers' data contribution without any financial reward

We now consider the case when a basic service provider requires users' participation in crowdsourcing but offers no financial reward (SBSP), and each user decides whether to subscribe to the service or not. It is a commonly used strategy by many service providers in practice to save the operation cost. For instance, Waze [51] provides traffic information and requires users to contribute their data in consuming the service, and every user, before using the service, has to accept the agreement which specifies that Waze receives and uses its detailed location and route information. We analyze here the market equilibrium under such a non-reward strategy and compare the results with those when a reward is provided, in order to shed more lights on optimizing the marketing strategies when crowdsouring is implemented.

Mathematically, this case means $w_b = 0$ and a user with parameters (α, l) would subscribe to the service only if its payoff is positive: $\Pi^{\mathrm{E}}_{s=4} = \alpha v_b - p_b - l \geq 0$. Therefore, to attract subscribers, the service price needs to be lower than the basic service level $\underline{v}$.

The market share, as shown in Fig. 5.15, can be updated as:

$$m_4[t+1] = \frac{[\underline{v} + g(m_4[t]) - p_b]^2}{2L(\underline{v} + g(m_4[t]))} \tag{5.41}$$

Then we can obtain the following results which characterize the equilibrium point of the market share.

Proposition 5.3. In monopoly market with a SBSP which requires user data contribution to consume the service and offers no reward, there exists at least one market equilibrium. If the following condition is satisfied,

$$\max_{m_4 \in [0,1]} g^{'}(m_4) < 2L. \tag{5.42}$$

then starting from any initial point $m_4[0] \in [0, 1]$, the market always converges to a unique equilibrium point which satisfies

$$\widetilde{m}_4 = \frac{(\underline{v} + g(\widetilde{m}_4) - p_b)^2}{2L(\underline{v} + g(\widetilde{m}_4))}. \tag{5.43}$$

We now compare the results to those in the case of a BSP which offers reward in the form of discount, to explore the benefits of crowdsourcing under different marketing strategies. The essential questions we aim to answer are: (1) which strategy (BSP's vs. SBSP's) is more favorable than the other in terms of attracting contributors or service consumers; and (2) the results of which case are more sensitive to the network effect or the service price. We show the numerical results in Fig. 5.16 under different values of γ or p_b.

It is seen that the BSP's strategy can easily beat the SBSP's strategy in attracting more consumers under the same service price since data contribution is not mandatory by the BSP. However, the number of contributors to the SBSP decreases with p_b and could be larger than that to the SBSP when the service price is low. The reason is that, the BSP's reward in the form of a discount is less compelling to the users when the service price is not high, while the SBSP could attract more consumers by setting a low service price. This result may explain in part why some applications such as Waze offer free service. Although the SBSP's profit is much lower than the BSP's in this case, we are not comparing the service providers' expected profits here, since they may have multiple income sources such as advertising revenue except from revenues from users.

Another interesting observation is that the number of contributors for the SBSP is less sensitive to the network effect than it is in the case of a BSP. This is due to the BSP's more proactive strategy on setting a reward when leveraging the network effects. In the following analysis we will restrict our discussion on the case of BSP to better focus on the network effects, and examine how its influence can change when there is a market competition.

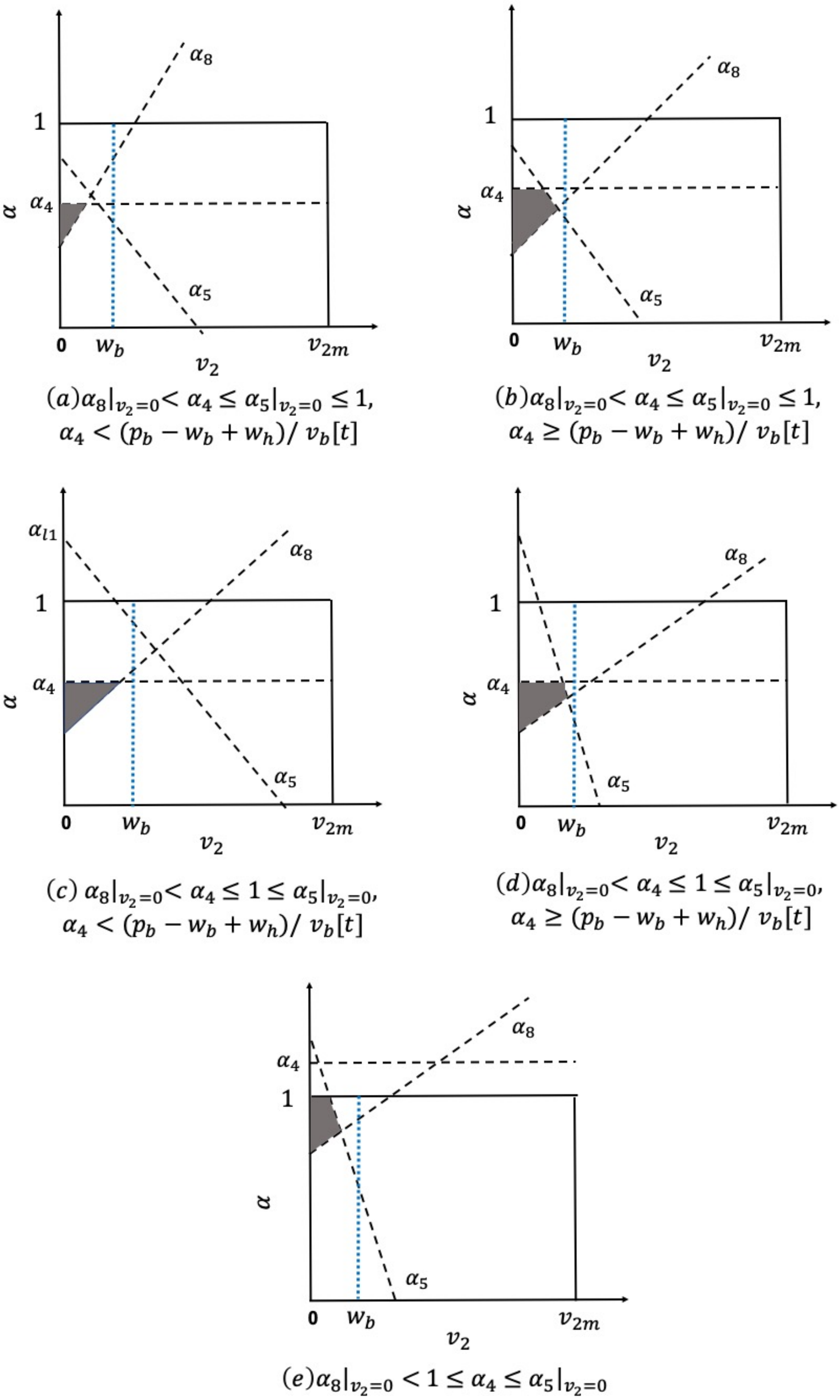


Figure 5.14: The fraction of users subscribing to BSP with crowdsensing participation in competitive market (shown as the shaded area to the left of $l = w_b$)

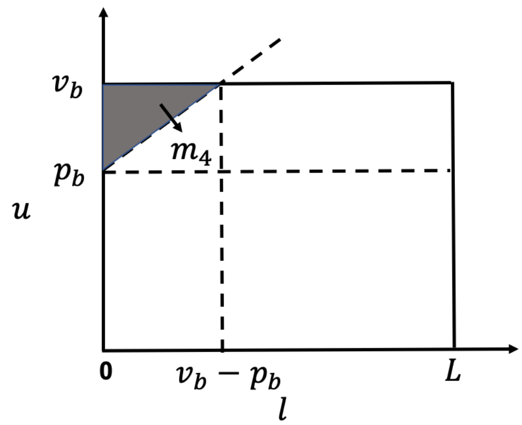


Figure 5.15: Market share of a SBSP.

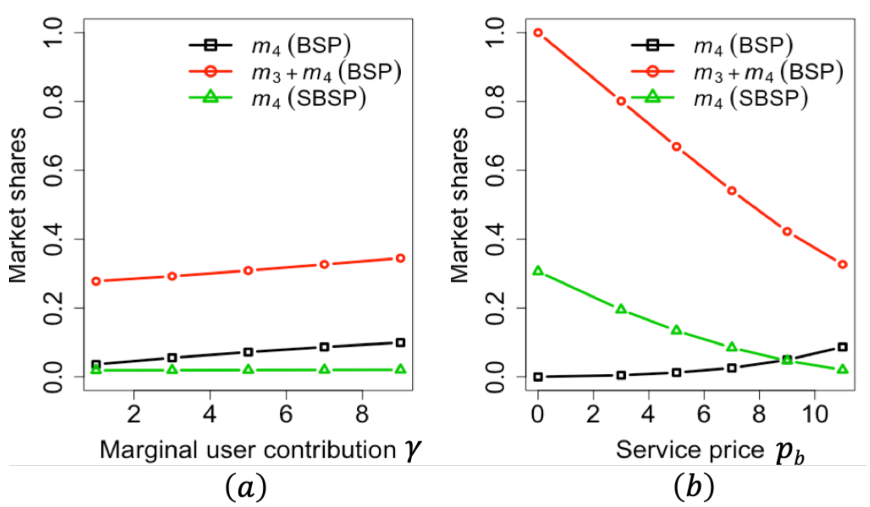


Figure 5.16: Market shares with different (a) p_b and (b) γ.

Chapter 6

Conclusion and future work

In this work, we have studied how the cooperation and competition among different players can have an impact on the efficiency and security of the cyber ecosystems. Specifically, the following issues are addressed in this dissertation and some future work are also pointed out here along these directions:

We proposed an optimal NIS schemes based on the equilibrium of the backup reservation game between two MNOs. The decision-making conditions are identified for MNOs to adopt and implement the backup reservation contract under both independent and competitive scenarios. The interactive decisions of the two operators, namely the reservation price and the reservation quantity, are derived under uncertain demand risks. The strategic backup reservation based infrastructure sharing framework is shown to have the potential to improve both the capacity utilization of the resource-owning operator and the virtual operator's service level to the users. The benefits of such a scheme are determined by various factors such as the demand uncertainty, the potential redundant capacity, and the competitive intensity. These findings lay the foundation for a clear guideline for the operators to choose their partners and determine their action plans when NIS is desired in volatile and competitive markets. For future research, it would be worthwhile to relax the assumption of the availability of complete information between MNOs. Furthermore, extending this study to three or more operators will be of interest to assess the generality of the conjectures. In the case of multiple VNOs competing for the resources of a single ROO, two possible strategies may be considered: (1) Iteration of partner selection and reservation: The ROO chooses the most beneficial VNO j to cooperate through backup reservation, according to its own redundant resources S, the VNOs' demand patterns, and service prices; and then VNO determines the reservation quantity R_j, after which the ROO updates the redundant resource quantity to $(S - R_j)^+$, and chooses the next VNO to cooperate. The above process is repeated until no benefit improvement can be earned from the NIS for the ROO. This method is easier to implement, but still leaves space for further resource utilization improvement. (2) Capacity allocation and flexible contract: The ROO would design a contract with each VNO that specifies the reservation price, the bounds of reservation quantity, as well as penalties for deviating from these agreed-upon quantities. This flexible contractual setting is supposed to further enhance the resource utilization but more sophisticated. In the case of multiple ROOs, the optimal reservation decisions would change according to the equilibrium in the ROOs' price competition game. We leave these as potential topics for future research.

We have also studied a security problem with target information trading from an economic perspective. The interaction between a data broker and two attackers is formulated as a Stackelberg game where the data broker acts as the leader setting the price with the consideration of possible responses from the attackers. The competition between two attackers is modeled as a type of stochastic game. We have evaluated the value of the information from the perspectives of different players respectively, which is related to the acceptable price and the expected utility increase for the attackers, the changes in the attacking probabilities for the target, as well as the data broker's optimal selling strategy. We discover several interesting insights into the information market in the hacker community. For example, if the target is not so attractive, the information value for the attackers will be weakened by their competition, but the data broker might benefit from their competition under some conditions. However, the data broker does not always benefit from having more potential buyers considering the competition between the attackers, especially when cooperative purchasing is expected under a low target value. In the case of heterogeneous attackers, the data broker prefers to set a high price to attract only the high-profit attacker when the target value is not high. Besides, information accuracy is more valuable of a more attractive target for the attackers and the data broker. Our results also provide some insights to the defense strategy: to protect the information from leakage would avoid attacks if the target value is low enough, but when the target is highly attractive, more effort should be taken into the protection of the target itself than the protection of the information. Several directions for future research can stem from our work. First, it will be worthwhile to investigate a specific type of attack. Second, the situation where the data broker does not reveal its total sales quantity is a problem that the attackers may encounter. Finally, the consideration of price discrimination with heterogeneous attackers and collective attacks instead of individual attacks would be important future research directions.

In crowdsourcing to service users, network externality arises when both the service quality and the consumers' utility depend on the number of consumers who participate in crowdsourcing. In this work, we first study the market equilibrium with the consideration of such network externality. Our model gives new dimensions to characterize consumer behavior related to service subscription and crowdsourcing participation. Then we analyze the optimal reward decision to maximize the service provider's expected profit. This study represents the first effort in understanding the intrinsic incentives of consumers for better service in crowdsourcing, and the impacts of competition between service providers on the incentive mechanisms.

Future research directions include considering cooperation between the service providers to share their user-contributed data and a penalty cost for users switching between service providers. Moreover, the current research assumes that the users are interest-seeking and rational in making decisions on crowdsourcing. This assumption can be relaxed to include user behavior patterns by considering the following modeling procedures in line with [127]: (1) Survey of user behaviors: To understand users' different behaviors toward participating in crowdsourcing, a survey can be conducted on a particular crowdsourcing program. (2) User segmentation: Based on the results of the survey, the entire user base can be divided according to their behaviors into different segments. For instance, one segment contains reward-driven users, and the other consists of enthusiastic contributors. We could also allow for the consideration of users who are driven by both reward and enthusiasm represented by an overlapping area between different segments. (3) Market dynamic analysis and decision-making on the optimal rewards. Using similar analysis presented in this work, models can be built to find out the market equilibrium and the more targeting rewards. We leave these as potential topics for future research.

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