

**Experimental Characterization of Rotational Stiffness and Moment Capacity in
Plain-Embedded Steel H-Pile to Bent Cap Connections**

by

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Abstract

Steel H-pile to concrete bent cap connections relying on plain pile embedment are standard in short-span highway bridges across the United States. Current practice treats these connections as pinned or fixed, yet actual behavior falls between the two. Rotational stiffness, and therefore the moment demand distribution in the bent, depends on embedment depth, but no systematic experimental basis existed for HP pile sections to quantify this relationship.

This dissertation presents a full-scale experimental program to characterize the moment capacity and rotational stiffness of plain HP pile-to-concrete bent cap connections over a range of embedment depths. Eight specimens were subjected to lateral load tests to generate connection moment demand. The test matrix included HP10×42 piles at embedment-to-pile-depth ratios (L_e/d) of 0.5 to 1.5, HP12×53 piles at $L_e/d = 0.5$ and 1.0, and one strong-axis HP10×42 test at $L_e/d = 1.0$.

For HP10 and HP12 piles, a transition in governing failure mode was identified between $L_e/d = 0.75$ and $L_e/d = 1.0$. Connections at $L_e/d \geq 1.0$ failed by ductile steel flange local buckling, and those at $L_e/d \leq 0.75$ failed by brittle concrete bearing and crushing. No connection behaved as pinned or fixed. The measured degrees of fixity ranged from 50% at $L_e/d = 0.5$ to 93% at $L_e/d = 1.5$, although the measured rotational stiffness values are still very far from the theoretical fixity. The AASHTO LRFD minimum embedment of 12 inches coincides with the empirical failure mode transition boundary for HP10 and HP12 sections but is insufficient for larger pile sizes, or other pile types.

A power-law regression, $K_\theta/EI = 0.78 \text{ (ft}^3\text{rad)}^{-1} (L_e/d)^2$, was developed from the undamaged weak-axis specimens. The near-quadratic exponent is physically consistent with the concrete bearing couple mechanism, supporting the expression as a hand-calculation formula rather than a purely empirical fit. Prior lateral loading reduced rotational stiffness by 19 to 65% per loading event while moment capacity dropped by only approximately 13%, demonstrating that stiffness and capacity are governed by independent mechanisms.

The experimental data were implemented in FB-MultiPier through a four-point nonlinear connection spring guideline, validated against all eight specimens and two full-scale pile bents from prior studies. This research provides the experimental, analytical and numerical basis for treating connection stiffness and moment capacity as two separate design checks, each with its own limit state, rather than the prevailing pinned-or-fixed assumption.

Artificial Intelligence (AI) Use Disclosure Statement

In the preparation of this work, the following Artificial Intelligence (AI) tools were used: Grammarly. This tool was used primarily as a writing assistance tool designed to enhance the quality and clarity of written communication. The author acknowledges full responsibility for the intellectual content of this work and has ensured that all AI-assisted sections have been reviewed and revised for accuracy and appropriate academic style. All AI-generated content was reviewed and validated for relevance, appropriateness, and accuracy before incorporation into the final document to maintain scholarly integrity of this research.

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List of Symbols and Abbreviations

Symbol	Definition	Units
a	Shear span (distance from load application to cap face)	in.
A_p	Cross-sectional area of the HP pile	in ²
b_f	Pile flange width	in.
D or d	Pile section depth (used interchangeably with d for HP sections)	in.
E	Pile modulus of elasticity	ksi
EI	Pile flexural rigidity	kip·ft ²
f'_c	Specified compressive strength of concrete	psi or ksi
f_y	Steel yield strength	ksi
I	Pile moment of inertia	in ⁴ or ft ⁴
k_{fh}	Fixed-head lateral spring stiffness	kip/in
k_{partial}	Partial (semi-rigid) lateral spring stiffness	kip/in
L_e	Pile embedment length into the concrete bent cap	in.
L_e/d	Embedment-to-pile-depth ratio	—
M	Applied moment	kip-ft
M_{conn}	Moment capacity at the connection interface	kip-ft
M_j	Experimental connection moment at failure	kip-ft
M_n	Nominal moment capacity	kip-ft
M_p	Plastic moment capacity of the pile section	kip-ft
M_u	Ultimate (failure) moment capacity of the connection	kip-ft
M_y	Yield moment of the pile section	kip-ft

Symbol	Definition	Units
P	Applied lateral load	kip
P/M	Lateral force-to-moment ratio at the connection	in ⁻¹
T	Resisting concrete bearing strength	kip
t _f	Pile flange thickness	in.
t _w	Pile web thickness	in.
V _u	Nominal shear capacity of the connection	kip
Z	Plastic section modulus of the pile	in ³
A	Concrete compression zone height parameter (Kappes et al., 2021)	—
B	Compression zone depth factor (Kappes et al., 2021)	—
β ₁	Concrete stress block factor per ACI 318	—
η _{fh}	Dimensionless fixed-head stiffness coefficient (Shishlov et al., 2022)	—
K _θ	Pile head rotational stiffness at the connection	kip-ft/rad
Λ	Degree of fixity (0 = pinned; 1 = fixed)	—
n _h	Coefficient of horizontal subgrade reaction	pci
ρ	Connection efficiency (M _j / M _p)	—
AASHTO	American Association of State Highway and Transportation Officials	
ACI	American Concrete Institute	
ALDOT	Alabama Department of Transportation	
ASTM	American Society for Testing and Materials	
BEF	Beam on Elastic Foundation	

Symbol	Definition	Units
CFT	Concrete-Filled Tube	
DAQ	Data Acquisition System	
DOT	Department of Transportation	
FEM	Finite Element Method	
HP	Wide-flange H-pile section (e.g., HP10×42)	
LRFD	Load and Resistance Factor Design	
MEMS	Micro-Electro-Mechanical Systems	
NBI	National Bridge Inventory	
PHRS	Pile Head Rotational Stiffness	
PTPC	Pile-to-Pile Cap connection	

Chapter 1:

Introduction

According to Rakestraw et al. (2024), in many bridges, the foundation system consists of driven piles embedded in pile caps (or footings), where axial loads, lateral loads and moments are transferred from the superstructure to the soil and/or bedrock. Additionally, the piles and pile caps can be subjected to severe lateral deflections, due to seismic loads, water or wind loads or vessel impact. These loads can result in high local curvature and moment demands along the pile length. Pile bents are a common foundation system used to support highway bridges.

When considering these pile-to-pile cap connections of these structures, design tools are capable of two boundary conditions - pinned or fixed. A pinned connection allows for transfer of axial and lateral forces, but no moment, allowing large rotations to eliminate excessive moment build-up. A fixed connection, on the other hand, besides axial and lateral forces, also allows the transfer of moment to the top of the pile. Additionally, Poulos and Davis (1974) state that, considering elastic solutions, horizontally loaded structures may deflect approximately 75% to 50% less when piles are fully fixed to the structure, instead of simply pinned. Figure 1 exemplifies the two conditions.

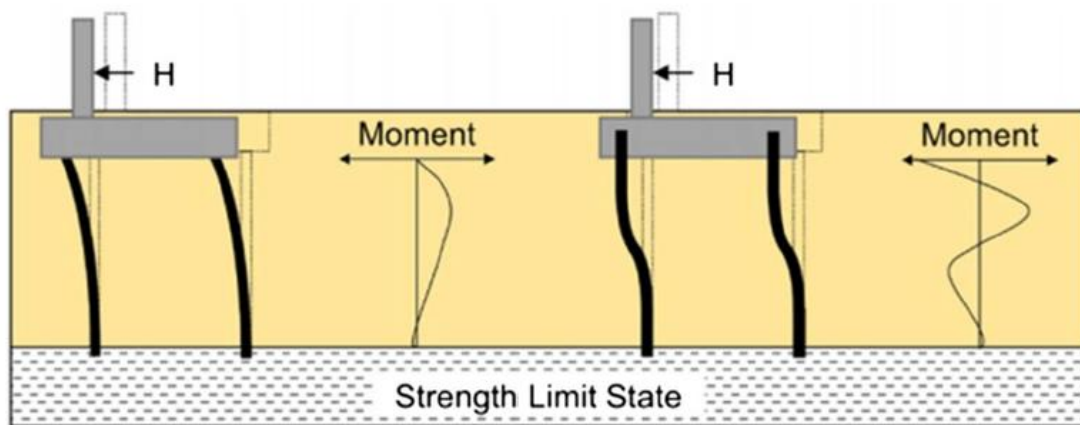


Figure 1. Bending moments of pinned (left) and fixed (right) pile to pile cap connections (Wilson et al, 2006, and Arockiasamy and Arvan, 2022).

However, it is known that actual conditions of pile-to-cap connections in the field are almost never purely fixed or purely pinned. It is somewhere between the two, a partial fixity.

There are many approaches to address partially fixed connections, and they range from moment capacity, rotational stiffness, or all the way to a full non-linear finite element approach modeling the entire cap cross-section. These approaches are either overly simplified, lack experimental data or are too complex for routine application.

There are many ways to increase the capacity of a pile to cap connection. In bridge design, selecting an adequate length of embedment of the connection between steel H-pile and the bent cap can greatly influence the level of fixity of the connection (Arvan and Arockiasamy, 2023). Piles embedded to an adequate length will resist moments, shears, and axial loads (fully fixed connection), whilst piles with limited embedment depth will not resist moments (simply pinned connection). Figure 2 shows how varying the pile embedment into the cap changes the behavior of the connection.

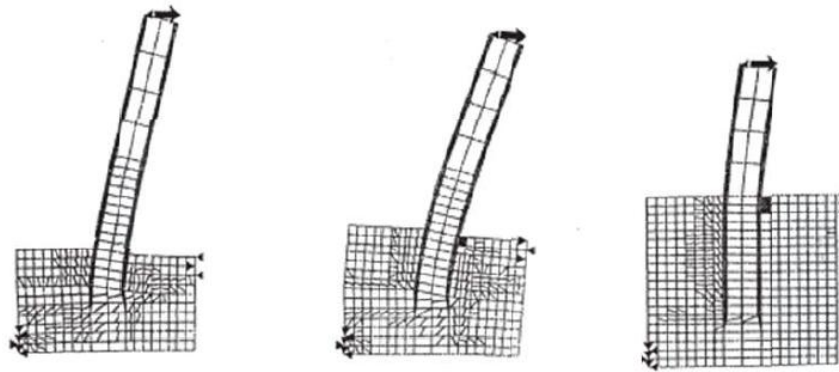


Figure 2. FEM model of embedded column base subjected to moment and shear force (Pertold et al, 2000a).

In two previous experimental efforts, the response of ALDOT standard pile bents subjected to lateral loads was studied with static load tests (Marshall et al, 2017, and Anderson et al, 2018) and dynamic load tests (braking loads, Marshall et al, 2020). While the response of these bents subjected to overload was demonstrated, the embedment of the piles was not a focus in these studies.

For the design of new bridges, most structural and geotechnical engineers can follow simple guidelines to ensure that their connection design remains fixed. However, most of the existing pile bents in the United States were designed without regard for this topic, following only the designer's experience. This means that many existing pile bents do not have, in fact, fixed condition, nor a pinned condition. The actual condition is somewhere in between.

Therefore, the purpose of this research is to characterize the behavior of plain-embedded steel H-pile to concrete bent cap connections and to determine a preliminary relationship between rotational stiffness and embedment length, through a program consisting of literature synthesis, full-scale laboratory experiments, and analytical and numerical modeling. To accomplish this, four specific objectives are defined:

- (1) Characterize the governing failure modes and moment capacities of plain HP pile-to-bent cap connections across a range of embedment depths, for two pile sections and two bending orientations, using full-scale lateral load testing.

- (2) Evaluate the predictive accuracy of existing analytical methods for connection moment capacity and rotational stiffness using the experimental results.
- (3) Develop an empirical relationship between connection rotational stiffness and embedment depth, and place the measured behavior within the pinned-to-fixed fixity spectrum.
- (4) Implement the experimentally derived connection behavior in a practice-oriented numerical modeling framework suitable for routine bridge bent analysis, and validate it against both the laboratory specimens and full-scale pile bent load tests from prior studies.

Chapter 2:

Pile-to-Cap Connections

1. Pile-to-Pile Cap Connections and Embedment

The connection between piles and pile caps is one of the main factors that control the transfer of forces in bridge pile bents or any other foundation type. This connection can range from a pinned condition, allowing only the transfer of axial and lateral forces, to a fixed condition, which enables the transfer of these forces along with the development of moment capacity at the pile head. The fixity at the pile-to-pile cap (PTPC) connection significantly influences the response of the foundation system to loads, affecting both moment and deflection at the pile top. According to Davisson (1970), a pinned connection typically results in zero moment at the pile top with larger deflections, whereas a fixed connection enables maximum moment transfer, reducing deflections under lateral loading conditions.

According to Harries and Petrou (2001), there are several options for detailing pile-to-pile cap connection. Some options are displayed in Figure 3.

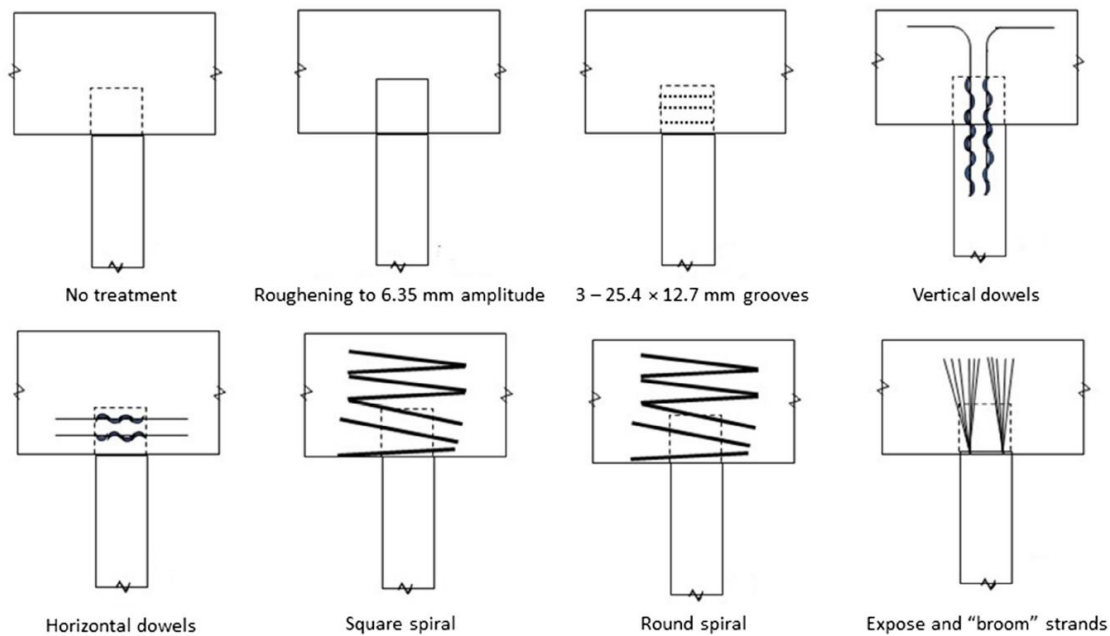


Figure 3. Different PTPC connections details (Harries and Petrou, 2001)

In many cases, however, designers rely solely on embedment of the pile into the cap, particularly with steel H-piles. This reliance on embedment eliminates the need for special reinforcing details, which can simplify construction and reduce costs. However, as Arockiasamy and Arvan (2022) emphasize, the degree of fixity achieved through plain pile embedment varies based on factors such as pile type, embedment depth, and interface conditions, requiring evaluation of these parameters to achieve desired moment capacities.

Studies by Arvan and Arockiasamy (2023) indicate that, for prestressed concrete piles, embedment depths between 1.2 to 1.5 times the pile diameter or width are often sufficient to develop adequate moment capacity, though additional research is needed to confirm similar guidelines for steel piles. Their findings support the cost-effectiveness of plain embedment connections, provided that the embedment depth and pile spacing are carefully designed to meet moment and ductility requirements.

Although a few analytical models and empirical equations exist to predict moment capacity based on embedment length, there is still great uncertainty, as laboratory data is still scarce, and field conditions introduce complexities not observed in laboratory settings. Rollins and Stenlund (2010) state that, under field conditions, load distribution

along the pile and pile cap rotation can significantly alter connection capacity, hence the need for further empirical data to inform code requirements. Additionally, recent studies (e.g., Shishlov et al, 2022; Arvan and Arockiasamy, 2023; Zapata et al, 2022; Harries and Petrou, 2001; Rollins and Stenlund, 2010; Xiao, 2003; Xiao et al, 2006; Shama et al, 2002) reveal that even short embedments - of around half the pile diameter - can achieve substantial moment capacity (up to 40% to 60% of the full moment capacity), challenging traditional design standards that call for larger embedment lengths.

The most desirable case for design is a pure fixed head condition, but this is rarely achieved in the field (Rollins and Stenlund, 2010). This assumption, in an analysis, would lead to underestimation of deflection values, and incorrect estimates of bending moments, when compared to real measurements. On the other hand, a design assumption of pinned condition could be extremely conservative and uneconomical in design, since it assumes large values of lateral deflection that do not happen in real cases.

Understanding the impact of pile embedment into the pile cap is important since this factor alone can greatly contribute to fixity behavior of pile-to-pile cap (PTPC) connections. According to Rollins and Stenlund (2010), relatively few tests have been performed on pile-to-pile cap connections, and significant uncertainty exists regarding appropriate embedment lengths to achieve a given moment capacity. Besides, Arockiasamy and Arvan (2022) demonstrated that plain embedment connections are highly cost-effective, as they eliminate the need for specialized equipment and extensive labor in the field. However, designers remain concerned about the ductility and strength capacity of these plain embedment connections.

2. Structural Standpoint: Mechanisms of Failure

In a pile-to-pile connection relying on plain embedment, there are several mechanisms of failure that can occur, such as connection bearing capacity, shear friction capacity of the connection interface, block shear rupture and strand slippage (specifically for prestressed concrete piles). However, all mechanisms of failure are also limited by the moment capacity of the pile itself (plastic moment of the pile).

According to Pertold et al. (2000a, 2000b), with a shallow embedment, the most likely failure mechanism is in the connection itself, with the occurrence of cap concrete

crushing. For deeper embedment, cap concrete crushing continues to be a problem, but the moment capacity of the connection is now greater. In consequence, the moment capacity of the pile itself is likely the limiting factor and must also be verified.

The following subsections will better describe each of these mechanisms of failure based on a review of available literature.

2.1 Structural Moment Capacity of Pile.

The purpose of this subsection is to serve only as a guide to the right tools to calculate the flexural strength of steel H-piles, steel pipe piles, and prestressed concrete piles. It does not detail equations or design guidelines.

- Steel H-Piles: AISC Steel Construction Manual (2023), Part 16, sections F3 and F6 for bending moments about strong and weak axis, respectively. Generally, plastic moment is calculated as $M_p = f_y \cdot Z$. Flange local buckling and lateral-torsional buckling also must be verified (Z is the plastic section modulus and f_y is the yield strength of the steel member, e.g. $f_y = 50$ ksi).
- Steel Pipe Piles: AISC Steel Construction Manual (2023), Part 16, section F8. Again, plastic moment is calculated as $M_p = f_y \cdot Z$. Local buckling also must be verified.
- Prestressed concrete piles: PCI Design Handbook (2010), chapter 5 and ACI-318-19 (2022). Additionally, as discussed further in section 2.3 ahead, ACI-318-19 (2022) also provides an equation for development length of the strands so that the prestress force is properly transferred.

2.2 Connection Capacity by Embedment

This section focuses on investigating the capacity of the connection itself.

2.2.1 Seminal Research

Regarding the determination of Pile-to-Pile Cap Connection (PTPC) capacity, the main works that support all studies that came afterwards were Marcakis and Mitchell

(1980) and Mattock and Gaafar (1982). Both developed analytical models, based on stress and strain distributions as seen in Figure 4, for determining shear and moment capacity of PTPC connections, and are considered very reliable, but conservative.

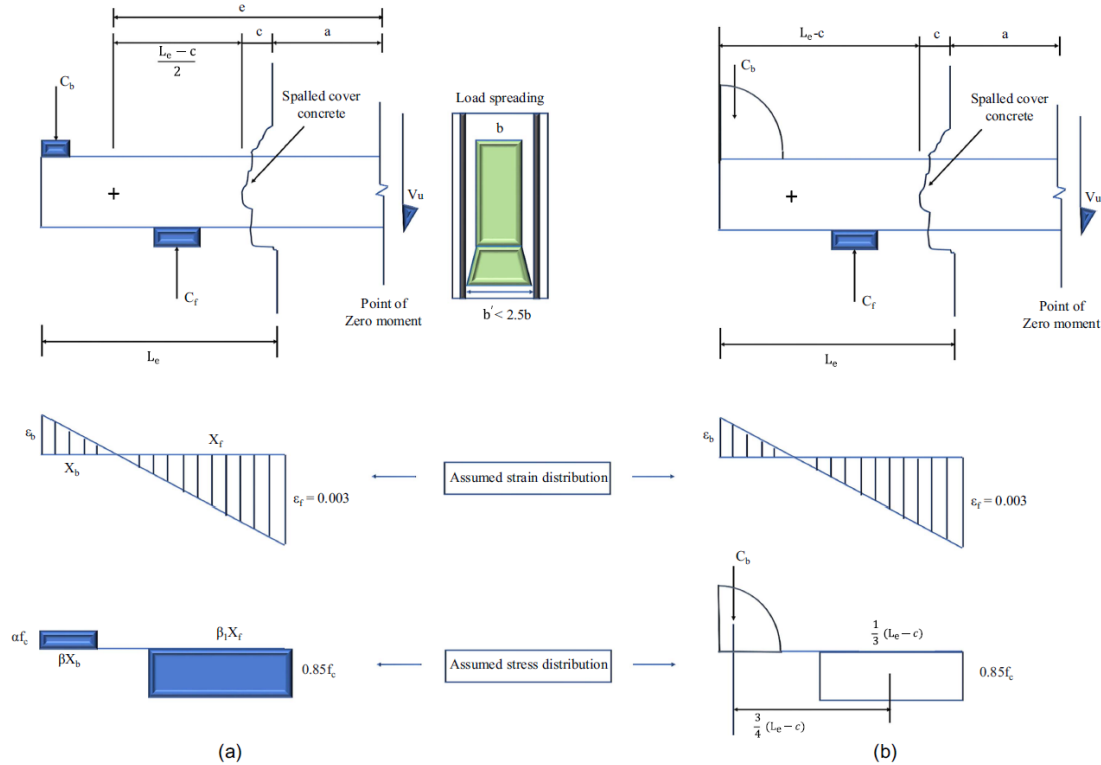


Figure 4. Strain and Stress distributions assumed by (a) Marcakis and Mitchell (1980) and (b) Mattock and Gaafar (1982) (Arvan and Arockiasamy, 2023)

2.2.1.1 Marcakis and Mitchell (1980)

Marcakis and Mitchell (1980) developed an analytical method for predicting the embedment capacity of steel members acting as supports for concrete beams. Their model was based on a series of experimental tests performed on 25 specimens, built as seen in Figure 5. All specimens were constituted of precast connections incorporating steel members, mostly a hollow square steel tube. Many aspects were changed for the study: different sizes of the steel tube, different axial loads applied to the concrete column, different eccentricities of shear force application points, presence of welded reinforcement at the connection, variation in the concrete clear cover and variations in the concrete

column dimensions, reinforcement, and properties. All tests were monotonically loaded to failure.

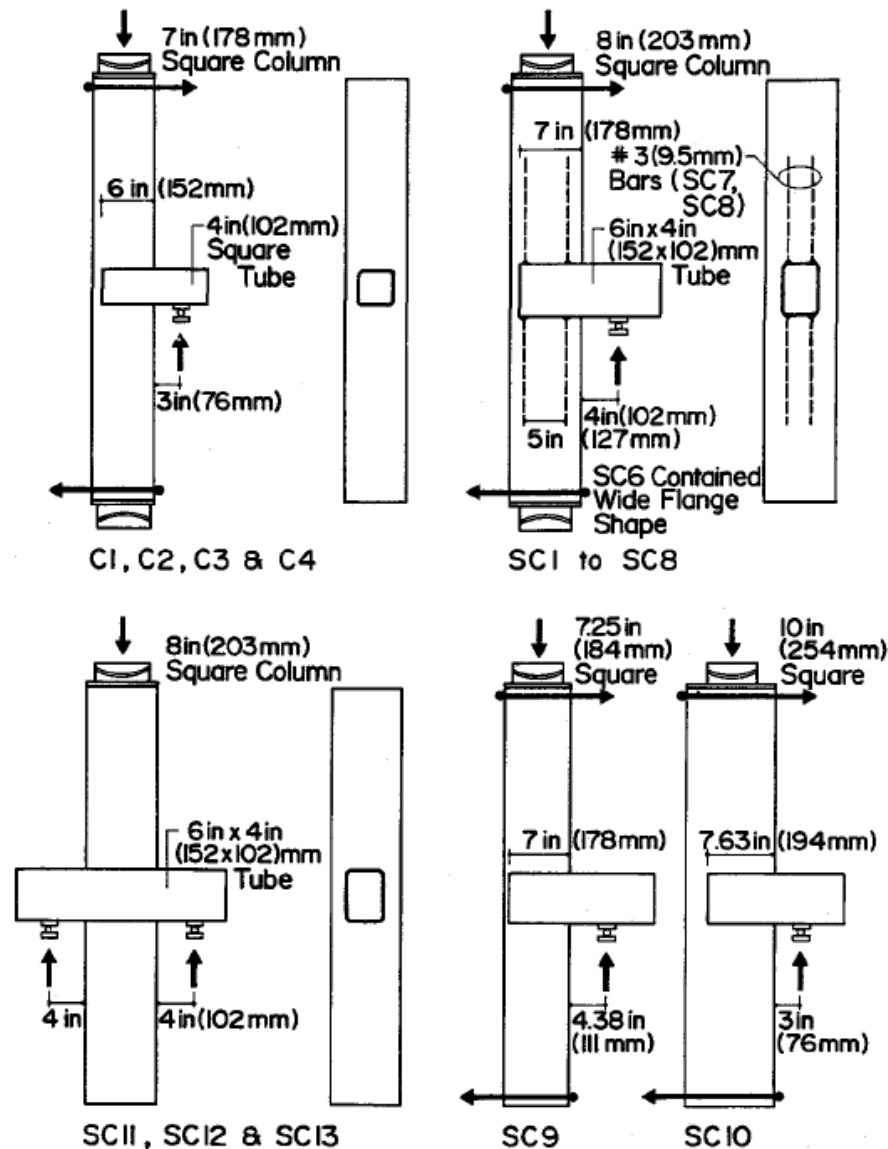


Figure 5. Details of test specimens tested by Marcakis and Mitchell (1980).

A solution in the form of design curves was developed, as seen in Figure 6. The curves consider the amount of reinforcement expressed in terms of a reinforcement index ω :

$$\omega = \frac{A_s}{b * l_e} * \frac{f_y}{f'_c} \quad (1)$$

f_y = yield steel tensile strength (steel member or steel reinforcement);

f'_c = concrete compressive strength;

A_s = reinforcement steel cross section area;

b = effective width, being the width of the confined concrete region, not exceeding 2.5 times the width of the embedded steel member;

l_e = length of embedded steel.

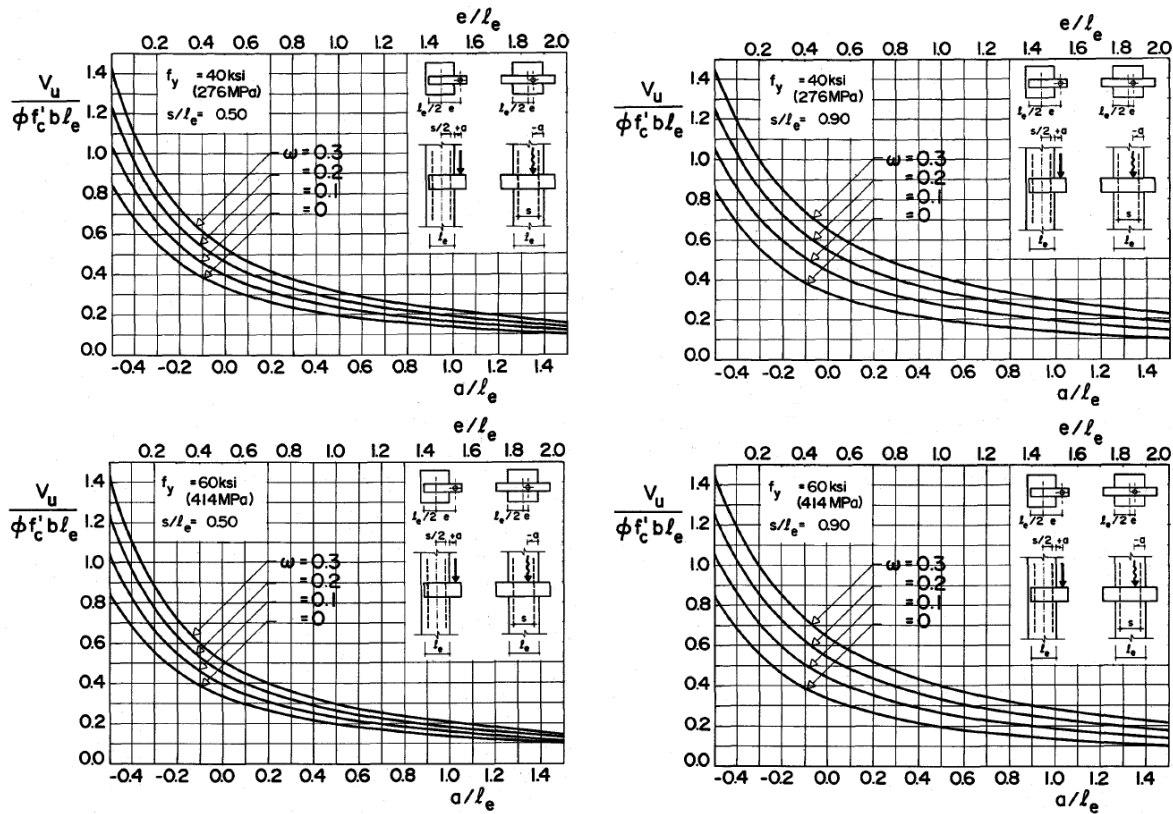


Figure 6. Design charts for evaluation parameters of embedded structural members (Marcakis and Mitchell, 1980).

However, for practical purposes, simplified expressions were also proposed - equations (2) to (4), these being most commonly used.

$$V_u = V_c + V_r \quad (2)$$

$$V_c = \frac{0.85 f'_c b l_e}{1 + 3.6 \frac{e}{l_e}} \quad (3)$$

$$V_r = \frac{2 \omega f'_c b l_e}{1 + \frac{6 \frac{e}{l_e}}{4.8 \frac{s}{l_e} - 1}} \quad (4)$$

V_u = Nominal shear capacity of a connection;

V_c = Nominal shear capacity of a connection without additional reinforcement;

V_r = Additional shear capacity provided by welded reinforcement;

e = eccentricity = $a + l_e / 2$, where “a” is the distance between the column face to the application point of the vertical load;

s = distance between symmetrically placed welded reinforcements A_s and A'_s (see detail in Figure 5, specimen SC1). If they are not symmetrical, it is recommended using 2 times the distance between center of embedment to the nearest welded reinforcement.

The embedded steel member must also be designed to resist the maximum shear, happening at the column face, and maximum moment, happening at a distance:

$$d_{max.mom.} = \frac{V_u}{0.85 f'_c b} \quad (5)$$

The maximum moment is:

$$M_{max} = V_u a + 0.5 * \frac{V_u^2}{0.85 f'_c b} \quad (6)$$

According to Marcakis and Mitchell (1980), the model assumes a linear strain distribution of 0.003 on the front face of the column (Figure 4). The authors concluded

that the model conservatively predicts the capacity of connections incorporating embedded steel members, both with and without additional welded reinforcement.

Harries et al. (1993) and Harries et al. (2000) performed experimental tests and suggested modifications to equation (3), considering the effects of concrete spalling at the face of the connection (Figure 7).

The modified equation, considering embedment only is:

$$V_c = \frac{0.85 f'_c b' (l_e - c)}{1 + 3.6 \frac{e}{l_e - c}} \quad (7)$$

e = eccentricity = $a+c+(l_e-c)/2$;

a = the distance between the column face to the application point of the vertical load

c = distance of concrete spalling, taken as the concrete clear cover to the outermost layer of transverse reinforcement;

b' = effective width, being the width of the confined concrete region, not exceeding 2.5 times the width of the embedded steel member;

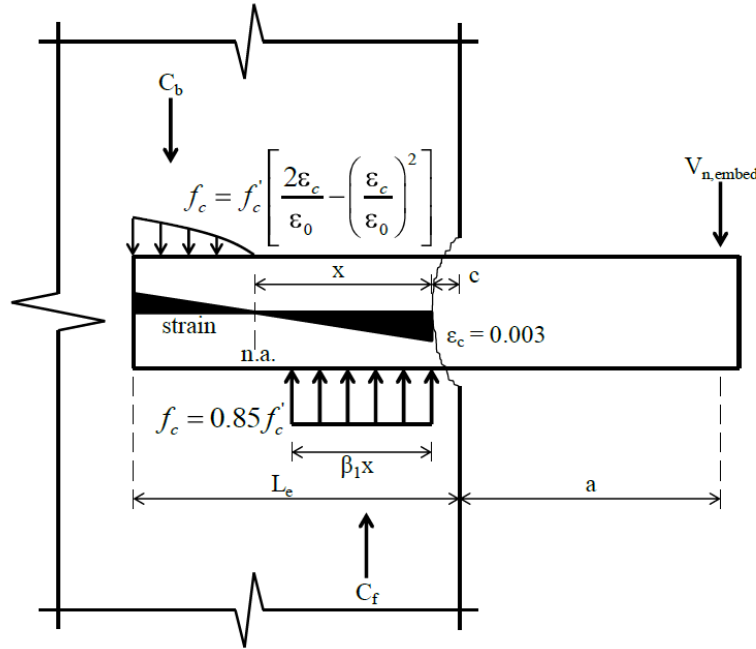


Figure 7. Modified Marcakis and Mitchell (1980) embedment model, considering concrete spalling (Motter, 2014).

2.2.1.2 Mattock and Gaafar (1982)

Mattock and Gaafar (1982) tested five steel sections (Rectangular shaped and I shaped), embedded into 10 in x 12 in concrete columns with four #6 bars as longitudinal reinforcement and #3 hoops spaced at 8 in as transverse reinforcement. All tests were also monotonically loaded to failure and the results were also used to calibrate an embedment strength design equation.

Mattock and Gaafar (1982) also assumed a linear strain distribution in the embedment region with a strain of 0.003 at the outer face. A solution in the form of an expression was proposed, considering only embedment:

$$V_u = 4.5 \sqrt{f'_c} \left(\frac{t}{b_f} \right)^{0.66} \beta_1 b_f L_e \left(\frac{0.58 - 0.22 \beta_1}{0.88 + \frac{a}{L_e}} \right) \quad (8)$$

V_u = Nominal shear capacity of a connection;

f'_c = concrete compressive strength, in MPa units;

t = concrete column width;

b_f = width of the embedded section;

β_1 = concrete stress block factor, defined in ACI 318-19;

L_e = length of embedment;

a = distance between the column face to the application point of the vertical load.

Similarly to Marcakis and Mitchell (1980) modified equation, Harries et al. (2000) also provide a modified equation for Mattock and Gaafar (1982), to include concrete spalling:

$$V_u = 4.5 \sqrt{f'_c} \left(\frac{t}{b_f} \right)^{0.66} \beta_1 b_f (L_e - c) \left(\frac{0.58 - 0.22 \beta_1}{0.88 + \frac{a + c}{L_e - c}} \right) \quad (9)$$

Equation (9) must also be used in MPa units.

The maximum moment at the connection happens at a section inside the connection, as seen in Figure 8.

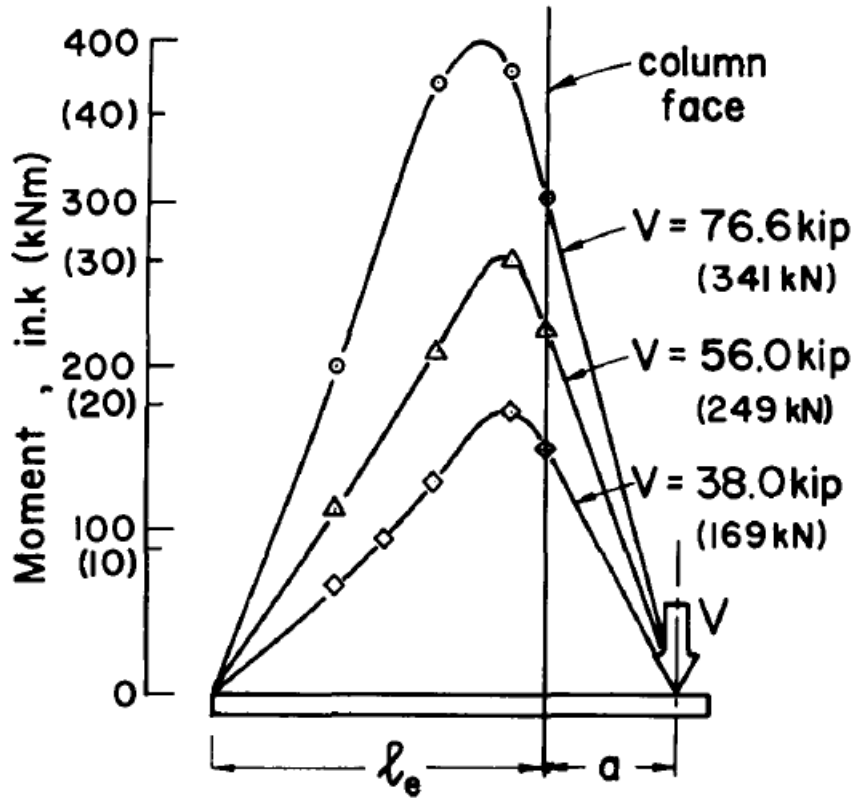


Figure 8. Variation of moment in the embedded section, specimen R5 (Mattock and Gaafar, 1982).

According to the authors, the maximum moment can be calculated as:

$$M_{max} = V_u a + \frac{V_u}{2 f'_b b_f} \quad (10)$$

$$f'_b = 4.5 \sqrt{f'_c} \left(\frac{t}{b_f} \right)^{0.66} \quad [\text{in MPa}] \quad (11)$$

2.2.2 Steel H Piles

This subsection investigates works that focus on connection capacity of steel H-piles.

Shama (2000) and Shama et al. (2002a, 2002b, 2002c)

These works present a comprehensive study on the performance of steel H-Pile-to-bent cap connections under lateral cyclic loading (Figure 9). Shama (2000) and Shama et al. (2002a, 2002b, 2002c), through both theoretical and experimental investigations, developed a plastic theory, which showed good agreement with the experimental results in order to predict the behavior of Pile-to-Pile Cap (PTPC) connections.

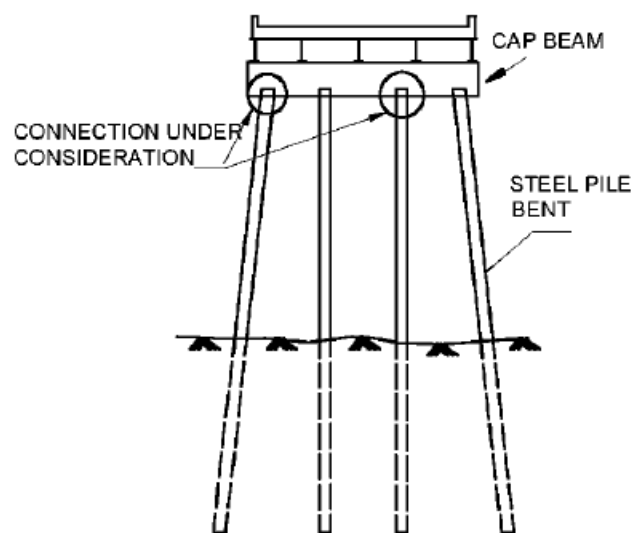


Figure 9. Typical geometry of deck bridge on steel pile bents, studied by Shama et al. (2002a).

Shama et al. (2002a), in their first paper, aimed to develop a plastic theory for assessing the performance of PTPC connections under lateral cyclic loading. This was complemented by an initial experimental program investigating steel pile bents subjected to similar loading conditions. In a second work, Shama et al. (2002b) proposed a retrofit strategy, followed by a second experimental program to validate this approach and the development of fragility curves for structures.

Previous works by Marcakis and Mitchell (1980), Mattock and Gaafar (1982), and Harries et al. (1993) have used the stress block approach in the embedded region of the pile to develop expressions for the theoretical resistance of PTPC connections. Following

their approach, Shama et al. (2002a, 2002b) made further progress in predicting the moment capacity of PTPC connections for H-Steel piles.

In this approach, the authors consider a single pile experiencing bending along its strong axis. The plastic hinges at two critical points are determined: one at the cap-pile interface and another at some distance below the soil surface. The point of inflection, where the moment is zero, is assumed to be midway between these two plastic hinges. Cohesive soil is also assumed, and Broms' (1964) theory is employed to determine the location of the lower plastic hinge.

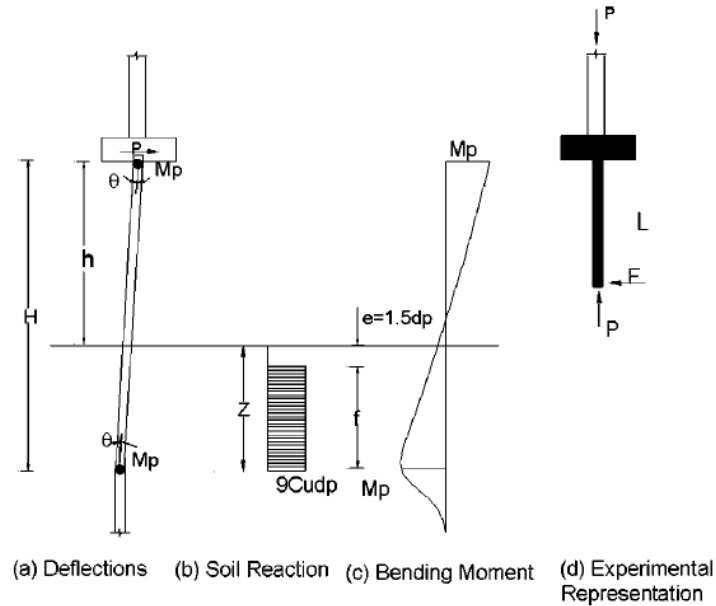


Figure 10. Soil structure interaction representation in plastic mechanism of pile bents (Shama et al, 2002a).

Figure 10 can be used to determine the expression for finding the lower plastic hinge location (Z):

$$Z = (h + e) * \left[\sqrt{1 + \frac{0.44 * M_p}{c_u * d_p * (h + e)^2}} - 1 \right] + e \quad (12)$$

Where:

$e = 1.5 \cdot d_p$ = depth where the lateral soil reaction equals zero, according to Broms (1964).

h = clear distance from the superior plastic hinge (at pile-cap interface) to the soil surface.

M_p = moment capacity of the steel pile section

c_u = undrained shear strength of soil

d_p = depth of the steel pile section

The authors, however, reached the conclusion that a typical depth to maximum moment usually lies in the range of $3 \cdot d_p$ to $9 \cdot d_p$. As an approximation, the authors suggest using $Z = 5 \cdot d_p$.

The theoretical model is structured around two parameters: the moment strength of the connection (M_j) and the moment capacity of the steel pile section (M_p). The connection efficiency, ρ , is defined as the ratio M_j/M_p , based on the stress distribution seen in Figure 12.

A ρ value greater than 1 indicates that the capacity of the connection is greater than the moment capacity of the pile itself. Therefore, plastification of the pile is expected. A ρ value less than 1 indicates that the cap concrete stress reaches its crushing capacity while the steel pile section still remains intact, having potential to absorb more energy and suffer more bending.

The authors presented an equation for connection efficiency ρ :

$$\rho = \frac{M_j}{M_p} = \frac{\frac{0.5\alpha\beta f'_c b_f d_p^2}{f_y Z_p} \left[\frac{f_y A_p}{f'_c b_f d_p} \left(\frac{P}{P_y} \right) + \frac{1}{2} \left(\frac{l_{emb}}{d_p} \right)^2 + \mu \left(\frac{l_{emb}}{d_p} \right) \right]}{\left[\mu + 2 \left(\frac{L}{d_p} \right) + \left(\frac{l_{emb}}{d_p} \right) \right]^2} \quad (13)$$

+

$$\frac{\left[\frac{f_y A_p}{f'_c b_f d_p} \left(\frac{P}{P_y} \right) + 4 \left(\frac{L}{d_p} \right)^2 + 2\mu \left(\frac{L}{d_p} \right) + 2\mu \left(\frac{l_{emb}}{d_p} \right) + 4 \left(\frac{L}{d_p} \right) \left(\frac{l_{emb}}{d_p} \right) + \frac{3}{2} \left(\frac{l_{emb}}{d_p} \right)^2 \right]}{\left[\mu + 2 \left(\frac{L}{d_p} \right) + \left(\frac{l_{emb}}{d_p} \right) \right]^2}$$

Where:

α = average concrete stress ratio

β = stress block depth factor

f'_c = concrete compressive strength

b_f = width of the H-pile section

d_p = depth of the H-pile section

f_y = yield stress of the steel

Z_p = plastic section modulus of the H-pile

A_p = cross-sectional area of the H-Pile section

P = Applied axial load

P_y = axial yielding load of the pile

l_{emb} = embedment depth into the concrete cap

μ = coefficient of friction between the steel and concrete

L = effective length of the pile specimen

α and β are stress block factors, dependent on the state of damage experienced by the concrete within the connection. Figure 11 shows a relationship adopted by the authors for α and β depending on the maximum concrete fiber strain.

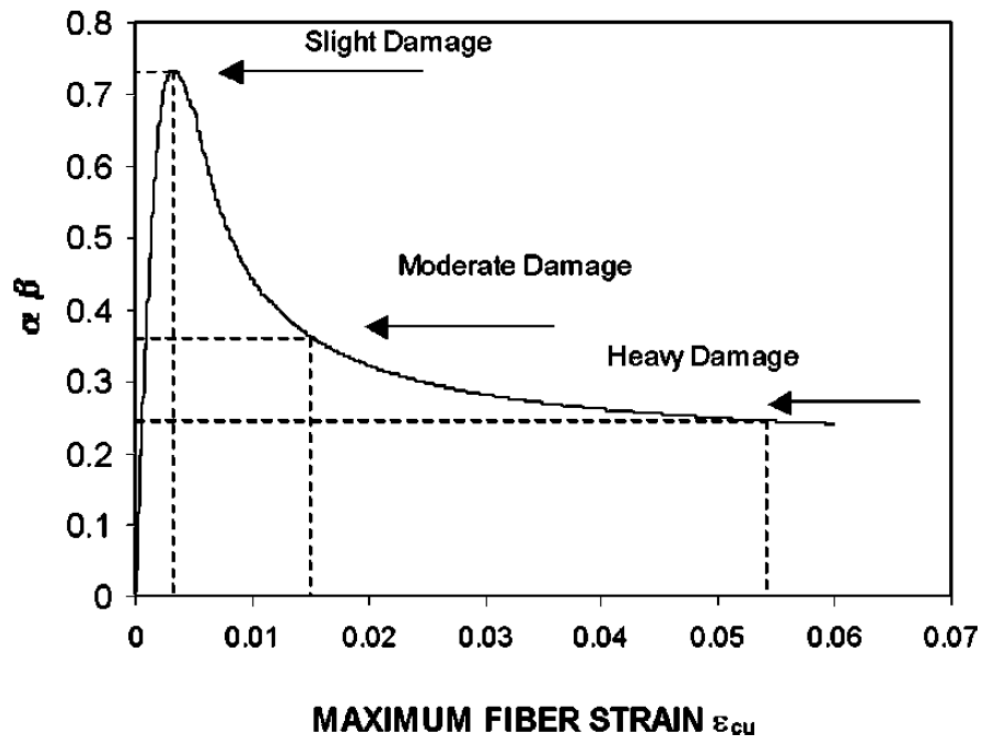


Figure 11. Stress block parameters depending on concrete strain (Shama et al, 2002).

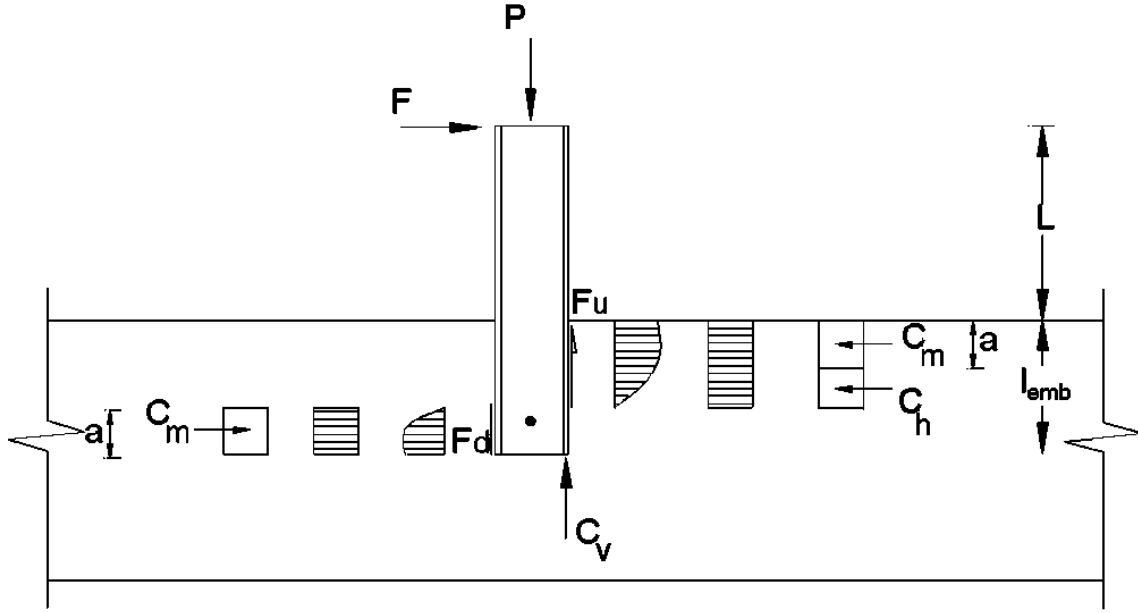


Figure 12. Adopted mechanism and stress distribution along the connection (Shama et al, 2002a).

In a more simplified form, the moment strength of the connection, M_j , will be calculated by multiplying force F by the distance from its application point to the neutral axis of the connection ($L + l_{emb} - a$). The authors, in Shama et al. (2002c), provided a direct expression to determine M_j :

$$M_j \leq \frac{f'_c b_f l_{emb}^2}{\left(6 + \frac{l_{emb}}{L^*}\right)} \quad (14)$$

Where:

L^* = distance from the point of application of the lateral load to the neutral axis of the connection ($L + l_{emb} - a$).

f'_c = concrete compressive strength;

b_f = width of the H-pile section;

l_{emb} = length of pile embedded into the concrete cap;

The results of the theoretical model indicate a clear relationship between the geometric parameters of the pile and its connection efficiency. Specifically, an increase in

the ratio of embedment length of the steel pile into the concrete cap (l_{emb}) to the depth of the steel pile section (d_p) leads to an increase in connection efficiency, as seen in Figure 13. This finding suggests that a deeper embedment of the pile into the cap improves the connection's ability to transfer loads, for the same pile cross-section.

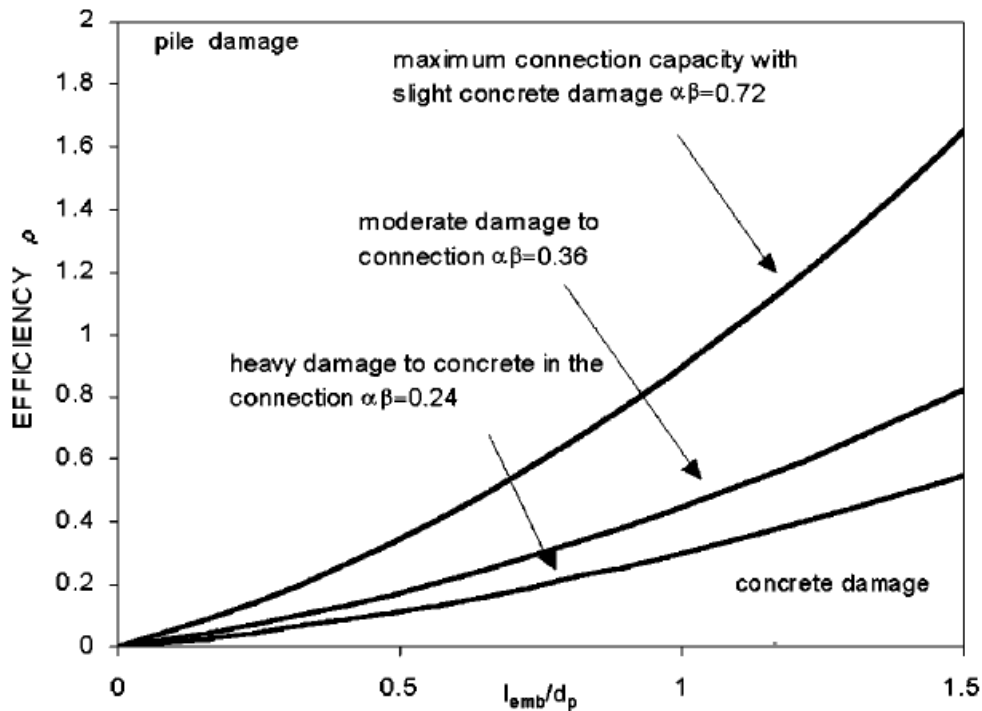


Figure 13. Connection efficiency in terms of stress block parameters (Shama et al, 2002a).

Conversely, as observed in Figure 14, an increase in the ratio of the unsupported length of the pile (L) to its depth (d_p) results in a decrease in connection efficiency, which asymptotically approaches a limit. The longer the pile extends without support, the less efficiently it can transfer lateral loads, up to a certain threshold where further increases in length have minimal impact on efficiency.

Additionally, the presence of a vertical load in the pile has been observed to slightly increase the connection's efficiency.

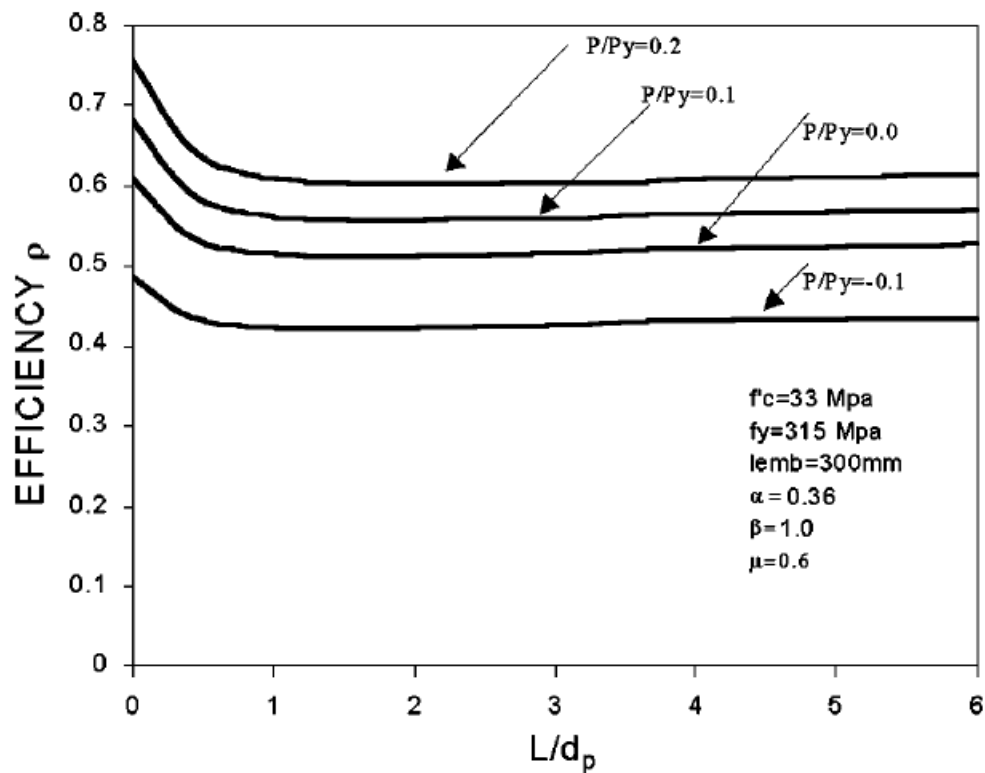


Figure 14. Effect of effective length on the connection efficiency (Shama et al, 2002a).

The experimental study involved testing HP 10x42 steel piles, embedded in the same pile cap, under cyclic lateral loads along both the strong and weak axes. The testing apparatus and instrumentation are detailed in Figures 15 to 17. Five tests were conducted, in three different specimens along strong axis and weak axis (S1, S2, S3, W1 and W2).

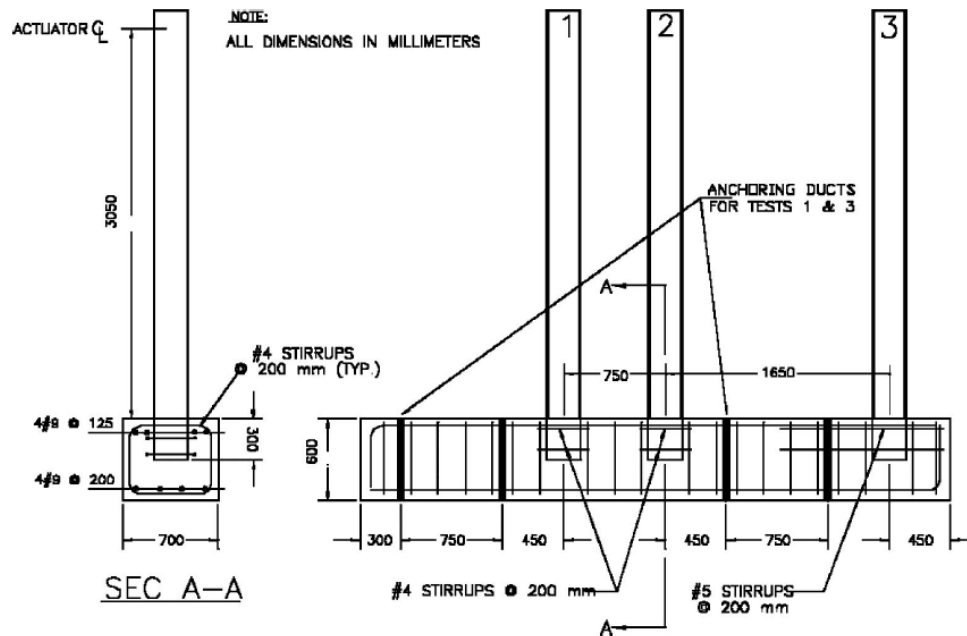


Figure 15. Geometry and reinforcement for strong axis (Shama et al, 2002a).

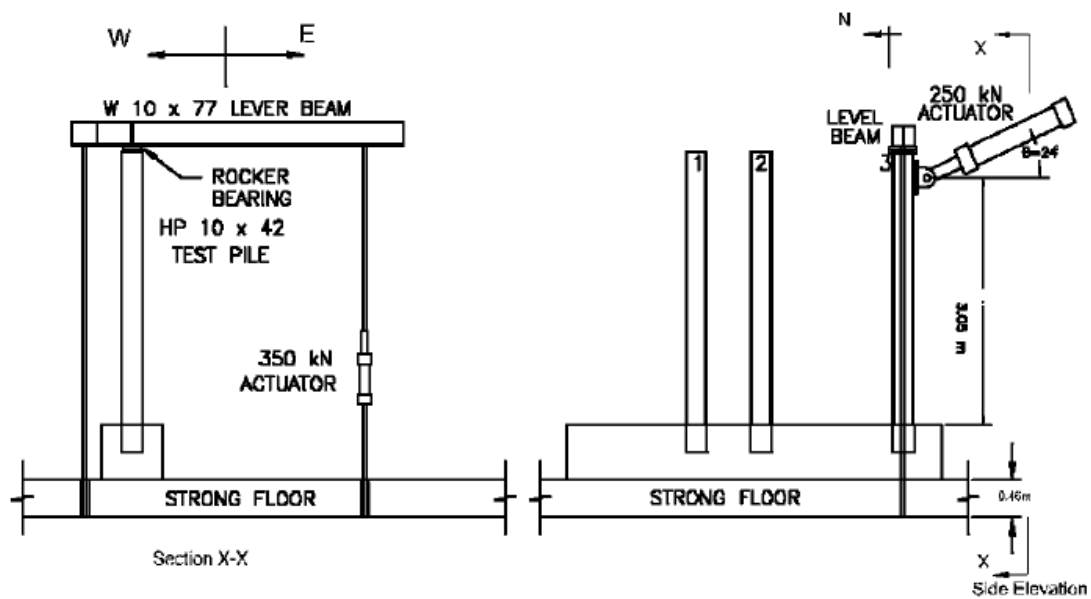


Figure 16. Test rig used in the experiments (Shama et al, 2002a).

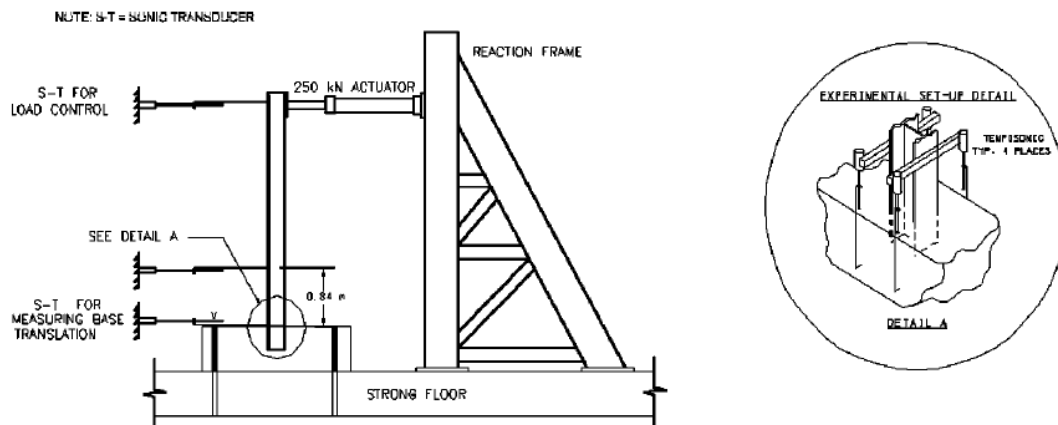


Figure 17. Instrumentation configuration used in the experiments (Shama et al, 2002a).

The results of the tests are described as follows:

- Specimen S1:** Tested under reverse cyclic lateral load without axial load. It underwent cycles at drift amplitudes from 60.5% to 65%. The test showed yielding of the section, damage concentrated in the pile and concrete cap beam, and significant strength loss at higher drift levels. The pile exceeded its yield moment and nearly reached its plastic capacity.
- Specimen S2:** Subjected to a constant axial load, representing the gravity load a bridge superstructure transfers to a steel pile. It was tested with drift amplitudes from 60.5% to 66%, showing elastic behavior up to 2% drift and significant moment capacity remaining after 5% drift. Failure in concrete was reached after ten cycles at 6% drift.
- Specimen S3:** Tested under variable axial load to study tension uplift effects. The axial load varied significantly, and the specimen behaved similarly to S1. It showed high ductility with limited cracking in the concrete cap beam. The test was not continued to ultimate failure due to test rig limitations.
- Specimen W1:** Examined the cyclic performance of interior piles oriented along the weak axis. It underwent cycles at drift amplitudes from 60.5% to 66%, showing elastic behavior up to 2% drift and yielding at 3% drift. The

steel performance was more critical than concrete behavior, with noticeable strength degradation at higher drifts.

- **Specimen W2:** Investigated the effect of tension uplift on exterior piles along the weak axis. It behaved elastically up to 2.5% drift, with yielding of flanges and local buckling at higher drifts. The connection reached its ultimate capacity at 7.5% drift, accompanied by significant strength loss and crack propagation.

The conclusions drawn from this study are significant. Connections with plain embedment may fail, in the strong axis, in a “non-ductile” manner, when the connection strength (M_j) is lower than the pile moment strength (M_p): $M_j < M_p$. On the other hand, while the adhesive bond between the steel pile and concrete remained intact and the connection strength (M_j) was greater than strength of the pile (M_p), ductile deformation of the steel controlled the failure behavior ($M_j > M_p$).

Under severe seismic loading, the connections fall in category of $M_j < M_p$ and are subject to damage, requiring retrofit strategy. Conversely, connections with bending about the weak axis exhibited a more ductile performance. The plastic theory and expression (14) developed in this study appear to be adequate for determining the connection's capacity.

In the second paper, Shama et al. (2002b) investigated a retrofit strategy to improve the PTPC connections of a pile bent.

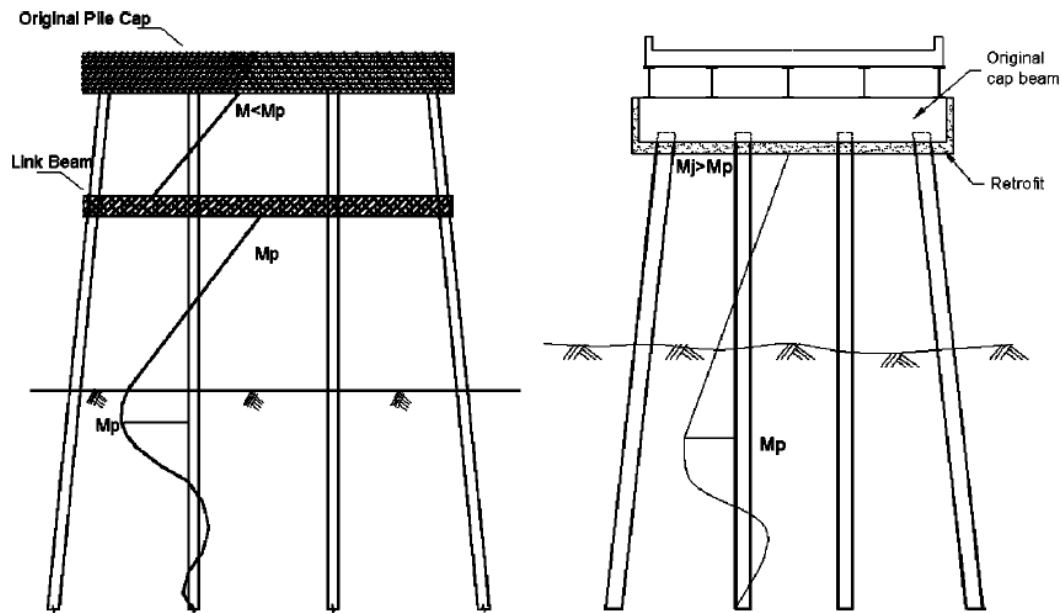


Figure 18. Retrofit strategies for steel pile bents: (a) link beam and (b) cap beam strengthening (Shama et al, 2002b).

This study focuses on cap beam strengthening, through overlay, a common retrofit strategy, as seen in Figure 18 (right-hand side). Following an additional experimental program, the authors found that retrofitted pile bent connections demonstrate superior performance compared to the initial experiments. An equation, considering the stress distributions shown in Figure 19, is suggested for determining the adequate embedment length of a H-Steel pile, into the retrofitted cap:

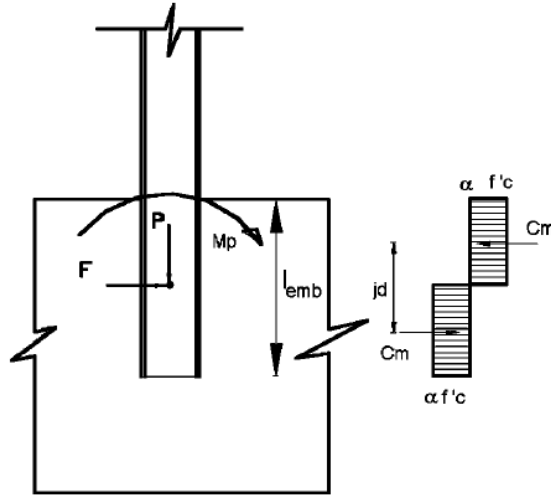


Figure 19. Mechanism adopted for connection retrofit (Shama et al, 2002b).

$$\frac{l_{emb}}{d_p} \geq 3.5 \sqrt{\left(\frac{f_{su}}{f'_c}\right) \left(\frac{t_f}{d_p}\right)} \quad (15)$$

Where:

f_{su} = the ultimate stress of the steel pile section

t_f = flange thickness of the steel pile section

d_p = depth of the steel pile section

l_{emb} = embedment of the steel pile into the concrete cap

f'_c = compressive strength of the concrete

This equation considers that $M_j / M_p \geq 1.6$ (Shama et al, 2002c).

Considering equation (15), a simple parametric study can be performed. Table 1 shows the results for all HP piles listed in AISC Steel Construction Manual (2023). Grade 50 steel was considered, $f_y = 50$ ksi and $f_{su} = 65$ ksi.

Table 1. Calculated embedment lengths, according to Shama et al. (2002c) equation.

HP Pile	Depth d_p (in)	Flange thickness t_f (in)	Ratio l_{emb}/d_p [for concrete compressive strength $f'_c = 4$ ksi]	Ratio l_{emb}/d_p [for concrete compressive strength $f'_c = 5$ ksi]	Ratio l_{emb}/d_p [for concrete compressive strength $f'_c = 6$ ksi]
18x204	18	1.130	3.5	3.2	2.9
18x181		1.000	3.3	3.0	2.7
18x157		0.870	3.1	2.8	2.5
18x135		0.750	2.9	2.6	2.4
16x183	16	1.130	3.7	3.4	3.1
16x162		1.000	3.5	3.2	2.9
16x141		0.875	3.3	3.0	2.7
16x121		0.750	3.1	2.7	2.5
16x101		0.625	2.8	2.5	2.3
16x88		0.540	2.6	2.3	2.1
14x117	14	0.805	3.4	3.0	2.8
14x102		0.705	3.2	2.8	2.6
14x89		0.615	3.0	2.6	2.4
14x73		0.505	2.7	2.4	2.2
12x84	12	0.685	3.4	3.0	2.8
12x74		0.610	3.2	2.8	2.6
12x63		0.515	2.9	2.6	2.4
12x53		0.435	2.7	2.4	2.2
10x57	10	0.565	3.4	3.0	2.7
10x42		0.420	2.9	2.6	2.4
8x36	8	0.445	3.3	3.0	2.7
Minimum			2.6	2.3	2.1
Maximum			3.7	3.4	3.1

As observed, the greater the concrete compressive strength, the lower the recommended embedment length of the pile. Additionally, for a given HP section, the greater the flange thickness, the greater the necessary embedment length. Generally, for usual values of cap concrete compressive strength (4 to 6 ksi), the recommended embedment length is rarely smaller than 2 times the pile depth.

This generally agrees with Castilla et al. (1984) recommendations. However, no specific mention to partial fixity is made by Shama et al. (2002a, 2002b, 2002c).

2.2.3 Steel Pipe Piles

This subsection investigates works that focus on connection capacity of steel pipe piles.

Steunenberget al. (1998)

Steunenberget al. (1998) conducted an experimental study on a 324 mm diameter steel pipe pile with a 12.7 mm wall thickness, which was subjected to lateral loading. This pile was welded to a thick steel plate measuring 635 mm by 610 mm with a 50 mm thickness, which was in turn attached to a reinforced concrete pile cap. The steel plate and the pile cap were connected using 30 studs, each 15 mm in diameter and 600 mm in length. The connection's capacity exceeded the pile's moment capacity, resulting in a ductile failure of the pile itself.

To predict the response of the specimen, the researchers used two approaches: a 2D non-linear analytical model and a 3D finite element (FE) analysis with ANSYS. The strength results from the experiment closely matched those predicted by the 2D non-linear model. This study provides a strong example of cases in which the plastic moment capacity of the pile is lower than the connection capacity.

Eastman (2011)

Eastman (2011) conducted experimental tests on three specimens consisting of pile cap and steel pipe pile, with different embedment lengths. The author then investigated the model of Marcakis and Mitchell (1980) and developed an alternative model.

The model, named Ultimate Strength Model by the author, is very similar to Marcakis and Mitchell (1980) as seen in Figure 20. The major difference is that it relies on an iterative process to determine embedment forces and includes an end bearing mechanism at the top of the pile-to-cap connection.

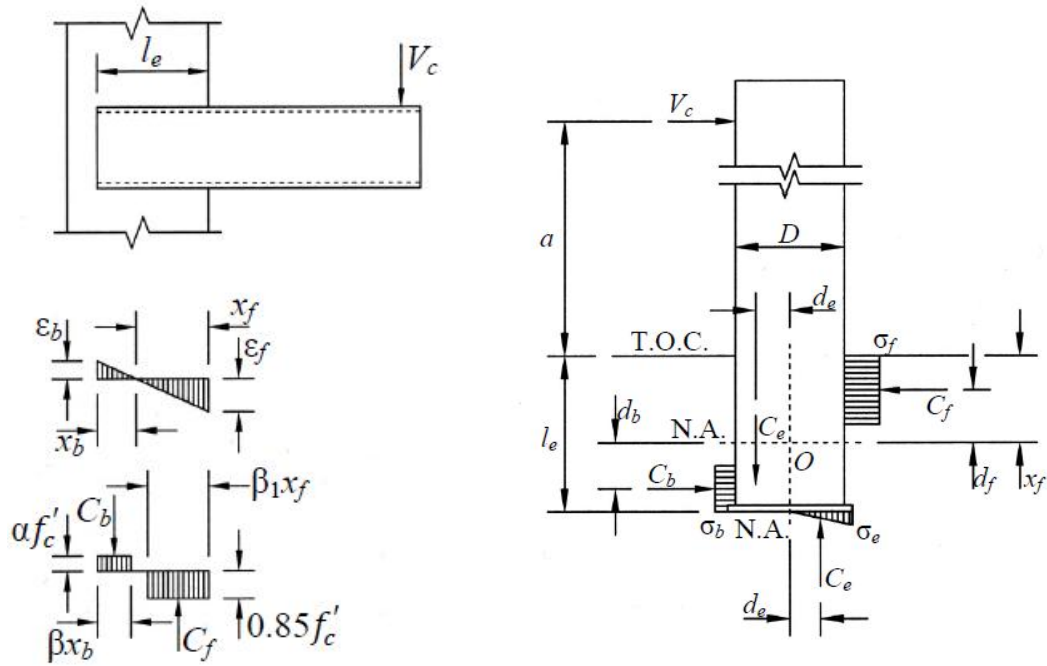


Figure 20. Assumed strain and Stresses (left) and applications to the Ultimate Strength Model (right) (Eastman, 2011).

Similar to Marcakis and Mitchell (1980), as seen in Figure 4, the forces at the front (C_f) and back (C_b) of the connection can be found by assuming equivalent stress blocks and maximum strain of 0.003 at the front of the connection. With these assumptions, an iterative process begins. Depth to neutral axis at the front (x_f) must be assumed and strain at the back must be calculated.

$$\epsilon_b = \frac{\epsilon_f x_b}{x_f} \quad (16)$$

ϵ_b = strain at the back of the connection;

ε_f = strain at the front of the connection, assumed to be ultimate at 0.003;

x_f = depth to the neutral axis, at the front of the connection;

x_b = distance from maximum back strain to neutral axis. By definition, it is equal to length of embedment (l_e) minus x_f .

β and α , the factors for, respectively, the length and the height of the back stress block, are determined by:

$$\beta = \frac{4 - \frac{\varepsilon_b}{\varepsilon_f}}{6 - 2 * \frac{\varepsilon_b}{\varepsilon_f}} \quad (17)$$

$$\alpha * \beta = \frac{\varepsilon_b}{\varepsilon_f} - \frac{1}{3} \left(\frac{\varepsilon_b}{\varepsilon_f} \right)^2 \quad (18)$$

Then, it is possible to find the magnitudes of the stress resultants C_f and C_b .

$$C_f = 0.85 f'_c \beta_1 x_f b \quad (19)$$

$$C_b = \alpha f'_c b \beta x_b \quad (20)$$

f'_c = concrete compressive strength;

β_1 = concrete stress block factor, defined as in ACI 318-19 (ACI 2019);

b = effective width equal to the width of the pile.

By force equilibrium, therefore:

$$V_c = C_f - C_b \quad (21)$$

For the end bearing mechanism, the force C_e acts at the end of the member as well as vertically toward the back of the connection. According to Eastman (2011), there are 3 possible ways of calculating C_e .

- 1st: Assuming $C_e = 0$
- 2nd: Assume that the magnitude of end bearing stress (σ_e) is equal to the stress at the back of the connection (σ_b), as seen in Figure 20.

$$C_e = \frac{1}{6} D^2 \sigma_e = \frac{1}{6} D^2 \sigma_b = \frac{1}{6} D^2 \alpha f'_c \quad (22)$$

D = pile diameter.

The resultant application point of C_e is at a distance d_e from the vertical neutral axis.

$$d_e = \frac{3\pi D}{32} \quad (23)$$

- 3rd: Assume that C_e is equal in value to the friction force between the pile and the concrete at the back of the connection, similar to Richards et al. (2011) (see section ahead 2.4.1).

$$C_e = \mu * C_b \quad (24)$$

μ = kinetic friction between steel, taken as 0.35 (75% of static friction $\mu = 0.47$, Baltay and Gjelsvik, 1990).

The resultant application point (d_e) of C_e is the same as equation (23).

Finally, after determining C_e , and using moment equilibrium, one can use the relationship below, and determine the new value of x_f .

$$V_c(a + x_f) = C_f d_f + C_b d_b + C_e d_e + C_e d_e \quad (25)$$

With the new value of x_f , new iterations of this process are repeated until the assumed initial x_f value matches the final x_f value from the moment equilibrium equation.

Therefore, moment capacity (M_u) of the connection would be given by:

$$M_u = V_c(a + x_f) \quad (26)$$

Stephens and McKittrick (2005) and Kappes et al. (2021)

Stephens and McKittrick (2005) reported that the embedment depth of the pile in the cap is one of the main parameters affecting the connection's capacity. Their work was based on performing analytical and experimental investigations on concrete-filled steel pipe PTPC connections subjected to extreme lateral loads, while keeping the same embedment length for all tested specimens.

Five half-scale models were built and tested. All of them had 8-in steel pipe piles (concrete filled) embedded 9 inches (or 22.86 cm) into the cap. The specimens had different parameters such as pipe wall thickness, reinforcement steel ratio, yield strength of the pipe steel as well as the cap concrete strength. The authors performed hand calculations, strut-and-tie models with the use of structural software and finite-element models, comparing all results to the experimental tests.

The authors stated that more experimental tests are necessary since the mentioned analytical tools, while useful up until a certain point, were insufficient to address various issues.

Arvan and Arockiasamy (2023) performed a finite-element numerical simulation of one the test specimens from Stephens and McKittrick (2005). The experimental load-displacement responses were compared to the numerical model by Arvan and Arockiasamy (2023), as seen in Figure 21.

According to the authors, the stresses induced as a result of axial load application compared well with the ones computed from hand calculations. Also, the lateral load results compared well for lower levels of lateral load. After a certain point, the FE model

appeared to be a lot stiffer than experimental behavior. The concrete cracking behavior was not completely predicted by the FE model and that could be the reason for this stiffer behavior.

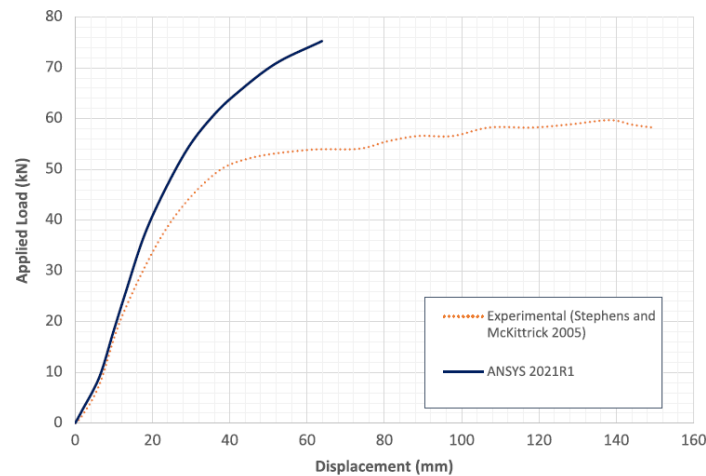


Figure 21. 8-in Steel pipe pile (concrete filled) behavior under applied lateral load: comparison between Arvan and Arockiasamy (2023) FE model and Stephens and McKittrick (2005) experimental result.

In continuation of Stephens and McKittrick's (2005) work, Kappes et al. (2013) and Kappes et al. (2016) kept investigating concrete filled steel tube (CFT) piles (8 5/8-inch diameter) connected to concrete pile caps. The study involved testing six connection specimens embedded up to 1.3 times the outer diameter and reinforced with U-bars at the connection level, with variations in the reinforcement details for each specimen (Figure 22).

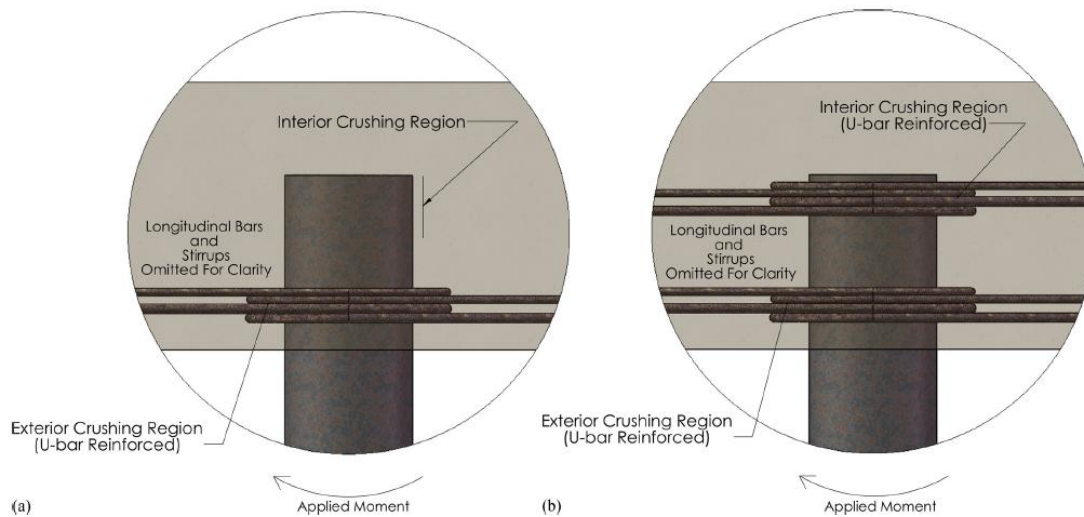


Figure 22. U-bar configurations (Kappes et al, 2016)

Test results showed that failure occurred in the concrete pile cap in five of the specimens, while the remaining specimen failed due to plastic hinging in the CFT at the face of the pile cap. Based on these findings and previous research, a moment-curvature-like analysis methodology was developed. This method proved effective in predicting the ultimate capacity of CFT-to-concrete pile cap connections.

Four critical limit states were identified in these connections: (1) plastic hinge formation in the CFT, (2) interior and exterior crushing of the concrete in the cap, (3) yielding of the longitudinal reinforcement in the cap, and (4) splitting of the concrete cap. The last three limit states caused a loss of fixity in the connection, leading to reduced energy dissipation.

These limit states, as well as the resulting cyclic degradation and loss of fixity, can be mitigated by selecting an appropriate embedment length and using a simple reinforcement scheme. U-bars encircling the CFT, along with tightly spaced transverse reinforcement in the connection region, were found to be effective.

The initial degradation observed in the cyclic response was likely caused by concrete crushing near the tip of the embedded pile. U-bars placed near the pile tip delayed the onset of this damage.

A preliminary design guide was developed, outlining the necessary embedment length, U-bar size and configuration, and transverse reinforcement details. Specimens

that followed this design methodology did not experience premature failure of the concrete caps.

$$L_{emb} = \sqrt{\frac{2 * M_u}{D * \beta_1 * \left(1 - \frac{\beta_1}{2}\right) * \phi_b * \alpha * f'_c}} \quad (27)$$

Where:

$\phi_b = 0.7$ (strength reduction factor for bearing on concrete, according to AASHTO (2020);

β_1 = stress block factor, according to ACI 318-19 (2022), usually 0.85;

D = diameter of the CFT pile;

$\alpha = 1.8$ (factor for amplification of the unconfined compressive strength);

Kappes et al. (2021) continued to develop a method of analytically determining the moment-capacity of a CFT pile, with or without U-bar reinforcement. The relative simplicity of the proposed analysis method, when compared to a full nonlinear 3D finite-element model, is beneficial considering that the method requires little computing effort and time.

In the proposed analysis methodology, the external moment and shear from the pile (M_{conn} and V_{conn}) are resisted in the cap by the U-bars and concrete immediately surrounding the pile, as shown in Figure 23. The embedded portion of the CFST pile is idealized as perfectly ridged. The reactive forces in the concrete consist of:

- Components acting normal to the surface of the pile,
- Friction forces acting parallel to the surface between the steel and the concrete, and,
- Interface shear at the top of the embedded pile.

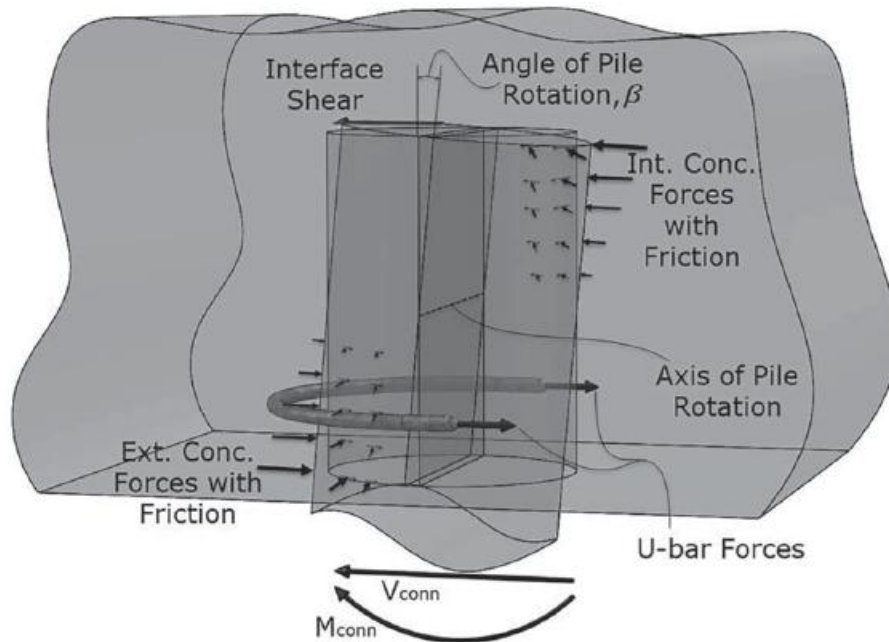


Figure 23. Imposed rotation and resultant internal forces and stresses (Kappes et al, 2021).

The authors then, based on the rotation, perform the following steps:

- **Compatibility relationship** is used to determine the strains in the concrete fibers and U-bar reinforcing steel.
- **Constitutive relationships** are then used to determine the subsequent stresses in the materials and ultimately the resultant concrete and steel forces.
- **Equilibrium** is then enforced on the connection through a double-iterative procedure in which both the rotation angle and the axis of the rotation location are varied until convergence is achieved. Increasing degrees of rotation are successively assumed and forces are calculated according to Figure 24, until the maximum capacity of the connection is mobilized.

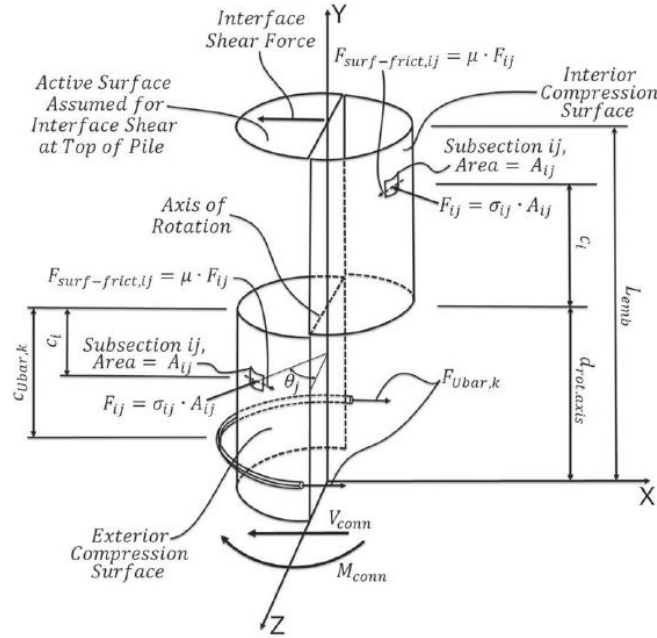


Figure 24. Effective differential areas and key dimensions for calculating forces resisting rotation (Kappes et al, 2021).

According to the authors, the internal forces and moments are calculated for an assumed angle of rotation (β), and an assumed rotation axis location $d_{rot,axis}$:

$$F_{conc_x-dir} + F_{frict_x-dir} + F_{Ubar,k} + F_{interface\ shear} = V_{conn} \quad (28)$$

$$M_{z_conc} + M_{z_frict} + M_{z_Ubar} + M_{z_shear} + (V_{conn} * d_{rot,axis}) = M_{conn} \quad (29)$$

Where:

F_{conc_x-dir} = resultant forces in the x-direction from the radial concrete stresses;

F_{frict_x-dir} = resultant forces in the x-direction from the friction stresses;

$F_{Ubar,k}$ = force from U-bar, k;

$F_{interface\ shear}$ = interface shear force component at the top of the pile;

M_{z_conc} = moment around the point of rotation resulting from the radial forces;

M_{z_frict} = moment around the point of rotation resulting from the friction forces;

M_{z_Ubar} = resultant moment from the U-bars;

M_{z_shear} = moment due to the interface shear force;

A further constraint is that the external shear (V_{conn}) and moment (M_{conn}) are related through an assumed inflection point located along the length of the pile (d_{pile_infl}).

$$M_{conn} = V_{conn} * d_{pile_infl} \quad (30)$$

If this condition is not met, the location of the center of rotation is adjusted until the system is in equilibrium. This entire process is carried out for increasing angles of rotation (β) while recording the moment, rotation, and location of the rotation axis.

The authors developed a spreadsheet to assist in the calculation of this methodology, where all parameters for cap, pile and reinforcement must be input. The spreadsheet then calculates the maximum moment, by varying the assumed rotation.

Berry et al. (2024) tried to validate this methodology. The authors performed a series of four experimental pile cap connections with CFT piles and the incorporation of U-bars. The authors concluded that, without U-bars the connection itself failed within the pile cap. This is consistent with the literature since the tested piles were embedded no deeper than 1.3 times the outer diameter, being consistent with partially fixed connections only. However, when including U-bars, connection capacity generally increased about 50%, and the failure mechanism changed to the formation of a plastic hinge in the pile itself.

As for the validation of the moment-rotation methodology for predicting the connection moment capacity, according to the authors, the methodology exhibited a tendency to overestimate by 10% the capacities of connections without U-bars and also underestimate by 10% those with U-bars.

Kidwell et al. (2018)

According to Kidwell et al. (2018), the most preferred form of failure of a PTPC connection is ductile, when yielding occurs in the reinforcement of the pile, producing a plastic hinge (rebars, strands or the steel pile itself). However, for the pile to reach

plasticity, the PTPC must be able to withstand a great amount of shear force. This agrees well to what was stated by Shama et al. (2002) ($M_i/M_p > 1$). Therefore, improving detailing is particularly important in plastic-hinge regions to produce ductile yielding and prevent brittle failure mechanisms.

The authors then, conducted a series of elastic and inelastic finite-element models using ANSYS to replicate the experimental behavior of both prestressed concrete piles and concrete-filled steel pipe piles from Harries and Petrou (2001) and Stephens and McKittrick (2005), respectively. Accurate modeling of pile-to-cap connections requires simulating large tensile cracks and concrete-strength degradation.

The model from Stephens and McKittrick's (2005) work is described here in this section, whilst the model from Harries and Petrou's (2001) work will be described further ahead, in section 2.2.4.

The FE model developed from Stephens and McKittrick's (2005) work aligned well with experimental data in the lower elastic region but was stiffer in the upper elastic and inelastic regions. The steel shell yielded at failure, but the concrete cap failed before the steel pile, showing more damage in the concrete.

Suppressed cracking capability in concrete elements led to stiffer model behavior. Contact elements indicated minimal rotation of the steel pile, suggesting high fixity. However, experimental piles showed more rotation and could be pulled from the cap after failure. Models for cast-in-shell steel piles were accurate in the elastic region but stiffer in the inelastic region than observed experimentally.

2.2.4 Prestressed Concrete Piles

This subsection investigates works that focus on connection capacity of prestressed concrete piles. Although this research does not focus on this kind of pile, the prior literature about them is still very informative regarding main findings, methodological choices, and analyses performed.

Harris and Petrou (2001)

According to Harries and Petrou (2001), PTPC connections are usually assumed fixed and must be able to transfer lateral forces to the pile while maintaining rigidity.

Although a pile may be designed to resist large force and moments, it is also necessary that the pile-to-pile cap connection must also be able to resist and transfer these forces.

The goal of the experimental program outlined in the paper is to demonstrate that no special detail is required for embedding prestressed concrete piles into pile caps, provided that the embedment length is adequately determined. The experimental setup included two 18-inch square by 18-foot-long piles, with an axial load of 200 kips, where the predicted nominal moment capacity of the piles was calculated be 285 ft-kips.

To determine the required embedment length for the piles, calculations were performed according to the methodologies proposed by Marcakis and Mitchell (1980) and Mattock and Gaafar (1981), so that the moment capacity of the connection would match the pile nominal moment capacity. Therefore, Pile 1 was determined to have an embedment length of 24 inches and Pile 2, 18 inches.

The experimental setup featured a cantilever beam in a horizontal position with a shear span of 146 inches, subjected to cyclic loads of ± 7 kips and ± 14 kips. This setup aimed to simulate the conditions under which the pile cap would provide the desired rigid end condition for the piles. The results indicated that the embedment of 18 inches and 24 inches were sufficient to develop the full capacity of the piles without the need for special detail.

The authors also performed an analytical study of Marcakis and Mitchell (1980) and Mattock and Gaafar (1982) equations for varying embedment lengths and square pile sizes, as seen in Figure 25.

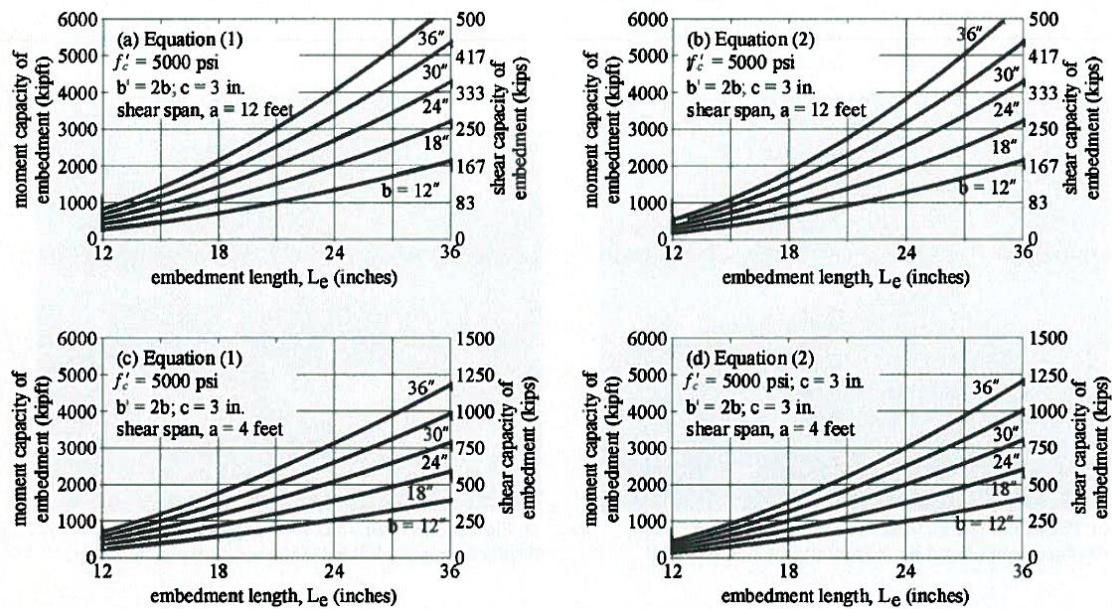


Figure 25. Embedment capacity predictions of varying pile sizes, with varying embedment lengths (Harries and Petrou, 2001). On the left-hand side, calculations according to Mattock and Gaafar (1981). On the right-hand side, Marcakis and Mitchell (1980).

Figure 25a and Figure 25b show the results for piles having a long shear span of 12 ft. Figure 25c and Figure 25d show the results for piles with a short shear span of 4 ft. The definitions of long or short shear span depend on how the cap-pile system is installed, as seen in Figure 26.

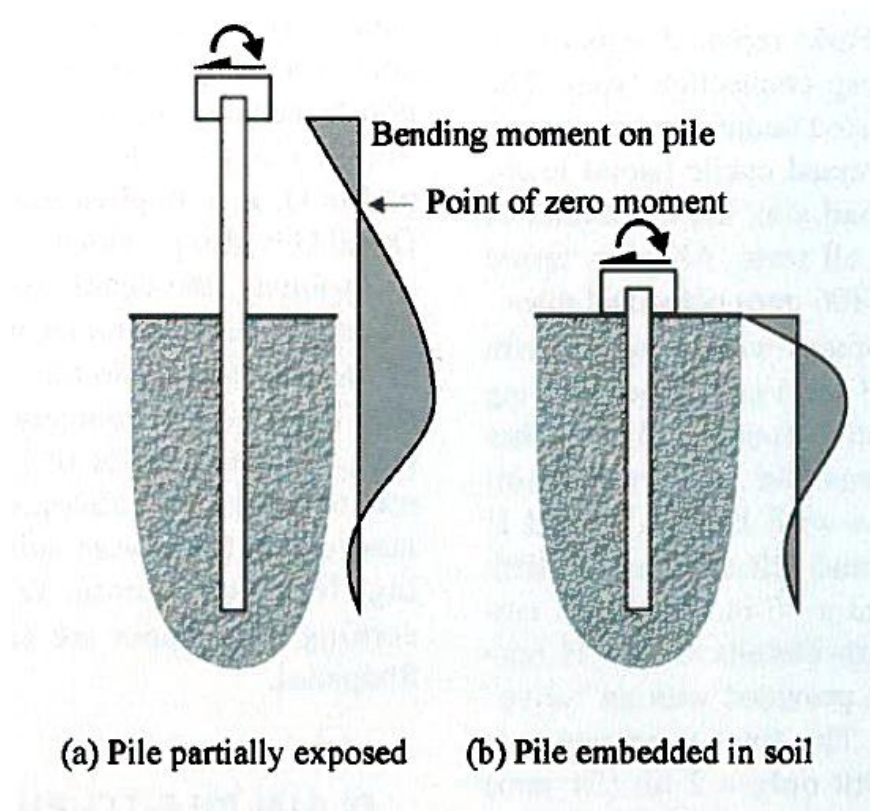


Figure 26. Bending of long piles due to horizontal loading (Harries and Petrou, 2001).

Results from Figure 25 show that calculated embedment capacities are lower for shorter shear spans, where the ratio of shear-to-moment at the pile-cap interface is high. As the shear span increases, embedment capacity also increases at a decreasing rate.

The authors proposed that a minimum embedment length equal to the width of the pile should conservatively result in sufficient capacity, offering a simplified yet effective guideline for the design and construction of PTPC connections.

Arvan and Arockiasamy (2023) also performed a numerical simulation, with ANSYSR21, of the experimental work by Harris and Petrou (2001). Their FEA model behavior was very similar to the experimental specimens, as seen in Figure 27.

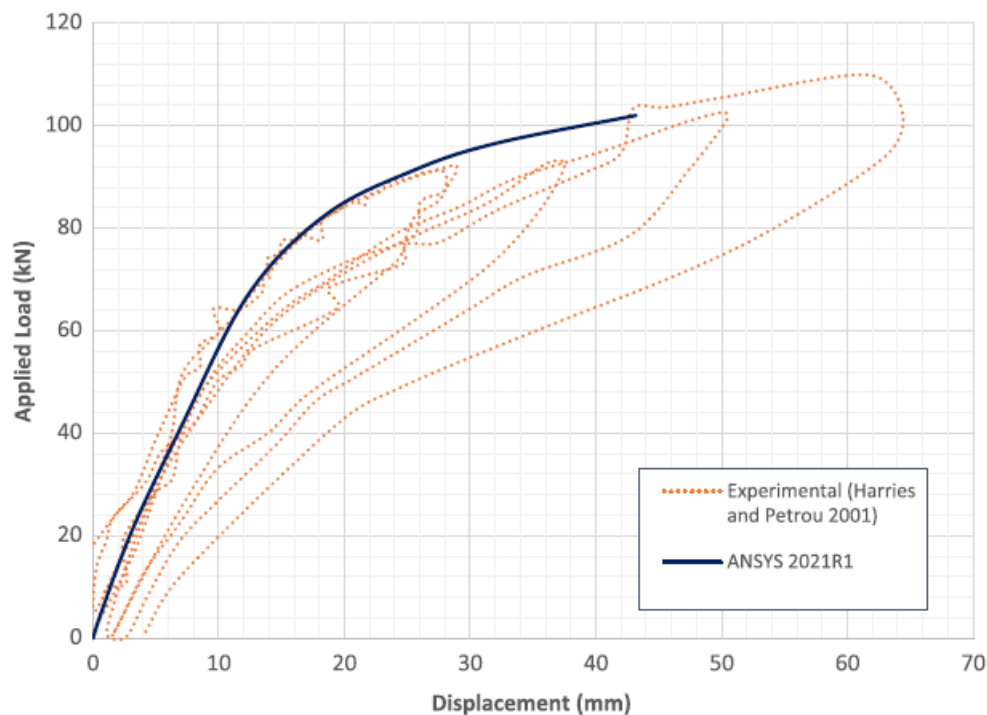


Figure 27. Experimental results for Harries and Petrou (2001) and comparison with Arvan and Arockiasamy (2023) FEA analysis (Arvan and Arockiasamy, 2023)

According to the authors, the model failed to converge at an applied load of about 102 kN (aprox. 22.9 kips) and corresponding moment of 377 kN*m (aprox. 278 kip-ft), which is quite close to the moment capacity obtained from the analytical model by Marcakis and Mitchell (1980) (386 kN*m or aprox. 285 kip-ft) and the experimental ultimate load value obtained by Harries and Petrou (2001) (111 kN or 24.9 kN). This means that, overall, FEA analysis of prestressed piles correlated well with the experimental data.

Xiao (2003)

Arvan and Arockiasamy (2023) summarized Xiao's (2003) work. Xiao (2003) investigated four 14-in prestressed square concrete piles with shallow embedment (3 inches, or approximately 0.21*pile width) into pile caps and anchorage dowel bars (Figure 28). The author aimed to demonstrate that shallow embedment depths are capable of

producing sufficient amount of moment and quantify the moment capacity by using Mattock and Gaafar (1982) and Marcakis and Mitchell (1980) equations.

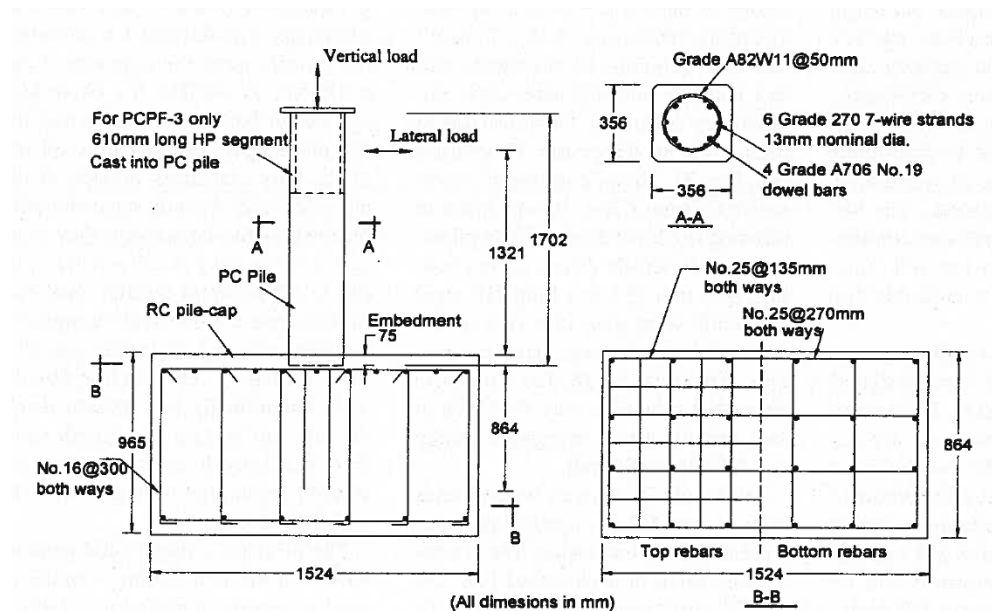


Figure 28. Pile and Pile cap assembly details (Xiao, 2003).

The study tested two loading conditions (cyclic and static loads), and the results indicated a significant moment capacity despite the shallow embedment depth. According to the author, the moment capacity can be calculated based on the combination of flexural resistance of the pile itself and embedment resistance (connection moment capacity).

The experimental findings showed that using both flexural and embedment resistances provides a realistic estimate of moment capacity, while using only flexural resistance gives a conservative estimate. Additionally, the calculation method proposed by Mattock and Gaafar (1982) provided a more conservative estimate than the method by Marcakis and Mitchell (1980).

Teguh et al. (2005)

Teguh et al. (2005a, 2005b and 2006) investigated the results from Harries and Petrou (2001) through finite element analysis. They showed that FEA is an excellent tool

for investigating the global and local behavior of PTPC connections. However, they also modeled a square prestressed concrete pile with exposed strands and confined with round reinforcement, embedded into the pile cap. According to Teguh et al. (2005a, 2005b and 2006), this configuration, when embedded 1.2 to 1.5 times the width/diameter of the pile, can produce adequate connection capacity.

Fuziol (2007) analytically investigated these previous results from Teguh et al. (2005a, 2005b and 2006) and Harries and Petrou (2001). Although Fuziol (2007) obtained slightly stiffer results, the results matched well with Harries and Petrou (2001) and it was concluded that the presence of the additional reinforcement, as suggested by Teguh et al. (2005a, 2005b and 2006), did not result in relevant increase on the connection capacity, when compared to plain embedment of the pile.

Sweigart (2010)

Sweigart (2010) initially conducted experimental tests on two interior and one exterior specimens of bent caps. The specimens consisted of 18-in square prestressed concrete piles, simply embedded 18 inches into representative bent cap segments (simulating interior or exterior configurations).

Results indicated that neither specimen achieved good capacity. The interior specimens failed because of strand slippage inside the embedded region. The exterior specimen failed because of poor connection moment capacity.

Another interior specimen with a 26-in embedment (1.5 times the width of the pile) was built and tested. This specimen showed superior results, mostly attributed to the greater embedment of the pile.

The author concludes that longer embedment is recommended as well as exploring special connection details. No specific value was recommended, though.

Kidwell et al. (2018)

As described previously in section 2.2.3, Kidwell et al. (2018) conducted a series of elastic and inelastic finite-element models using ANSYS to replicate the experimental behavior of both prestressed concrete piles and concrete-filled steel pipe piles from Harries and Petrou (2001) and Stephens and McKittrick (2005), respectively.

The model from Stephens and McKittrick's (2005) work was previously described here in section 2.2.3, whilst the model from Harries and Petrou's (2001) work is described here, in this section.

The FE model from Harries and Petrou's (2001) work accurately mimicked their experimental responses, showing significant inelastic deformation and higher displacement levels. Cracks in models were smeared, distributing damage over a larger area.

Contact elements showed significant shear transfer and minimal pile rotation within the cap. Embedment length significantly impacts response, with shorter embedments causing more damage near the pile-cap interface due to underdeveloped strands.

Models demonstrated the potential for plain embedment piles to achieve fixity, indicating areas of inelastic behavior and damage accurately. Contact models for embedment were accurate and useful for investigating shear transfer and bearing.

2.3 Strand Slippage for Prestressed Concrete Piles

This mode of failure applies to Prestressed Concrete piles, as they are most times reinforced with prestressed strands. As lateral load is applied and the pile bends, the strands in the tension area start to stretch, until they eventually "slip".

ACI 318-19 (2022) currently determines that the development length of pretensioned strands must be according to the equation below.

$$l_d = \left(\frac{f_{se}}{3000} \right) d_b + \left(\frac{f_{ps} - f_{se}}{1000} \right) d_b \quad (31)$$

f_{se} = effective prestress of prestressing strand (psi);

f_{ps} = nominal strength of prestressing strand (psi);

l_d = development length of prestressing strand (in);

d_b = nominal diameter of prestressing strand (in).

However, this expression was calibrated from tests on pretensioned flexural members and does not explicitly account for the conditions at an embedded pile-to-cap

connection. Some authors studied it and made a few suggestions for modifications, as observed in this section.

AASHTO (2020) also presented an expression for development length of pretensioned strands.

$$l_d \geq k * \left(f_{ps} - \frac{2 * f_{pe}}{3} \right) d_b \quad (32)$$

l_d = development length of prestressing strand (in);

k = Factor equal 1 for pretensioned panels, piling, and other pretensioned members with a depth of less than or equal to 24 in. Or 1.6 for pretensioned members with a depth greater than 24 in.

f_{ps} = nominal strength of prestressing strand (ksi);

f_{pe} = effective stress in the prestressing steel after losses (ksi)

d_b = nominal diameter of prestressing strand (in).

Shahawy and Issa (1992)

Shahawy and Issa (1992) conducted analytical and experimental studies on 19 14-in square prestressed concrete piles to study the effect of embedment depth to develop the ultimate flexural moment without strand end slippage, in the embedded region.

Pile caps were not built for this study. The piles were clamped by a reaction frame composed of a steel HP beam and a concrete support, as seen in Figure 29. Embedment lengths of the specimens varied from 36 in to 60 in, as well as the concrete strength, from 5.6 ksi to 7.8 ksi.

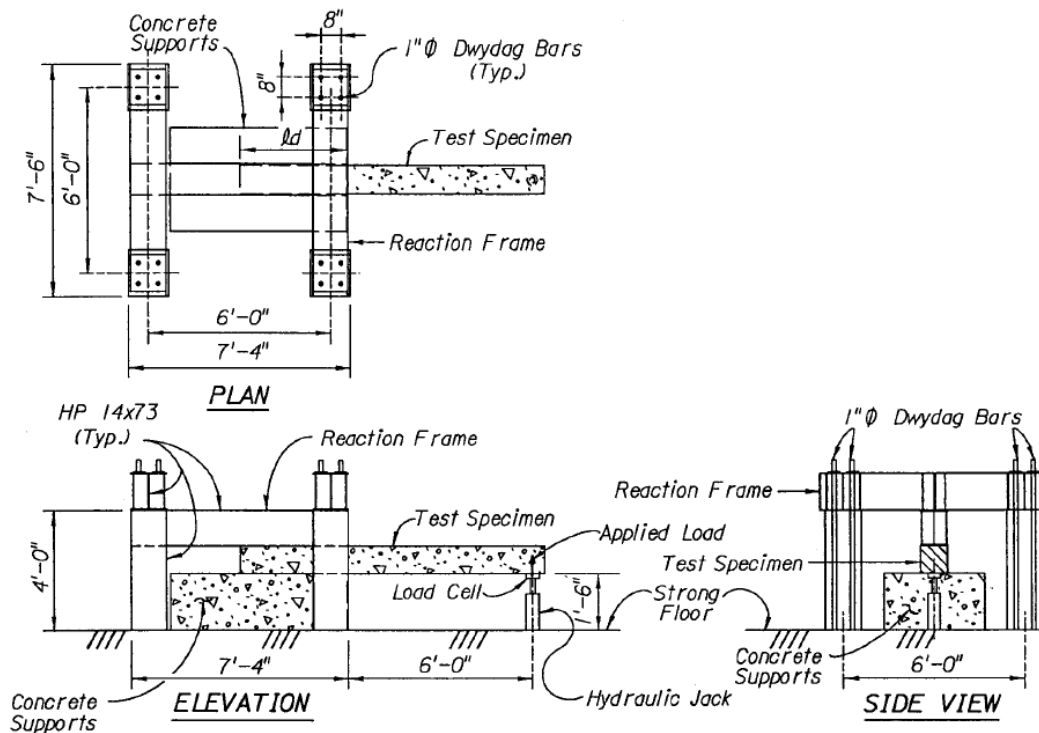


Figure 29. Details of test setup (Shahawy and Issa, 1992).

The analytical study was performed using a nonlinear material model to analyze the performance of the prestressed concrete piles and to develop a reliable computer program that can be used for future parametric studies.

The authors suggested some modifications for the ACI 318-19 equation for development length of prestressed strands.

$$l_d = \left(\frac{f_{se}}{3000} \right) d_b + \left(\frac{f_{ps} - f_{se}}{4 * \bar{u}_{ave}} \right) d_b \quad (33)$$

$$\bar{u}_{ave} = \frac{(P - T)}{\pi * l_e * d_b} \quad (34)$$

f_{se} = effective prestress of prestressing strand (psi);

f_{ps} = nominal strength of prestressing strand (psi);

l_d = development length of prestressing strand (in);

d_b = nominal diameter of prestressing strand (in).

u_{ave} = average bond stress (psi);

P = resisting steel strength (lbs);

T = resisting concrete strength (lbs);

l_e = available embedment length (in).

The study showed that there is a good agreement between the analytical and experimental results.

Issa (1999)

Issa (1999) tested 30-in square hollow-core prestressed concrete piles with a special connection detail. The pile would have a pipe inserted in its hollow core and be grouted (Figure 30).

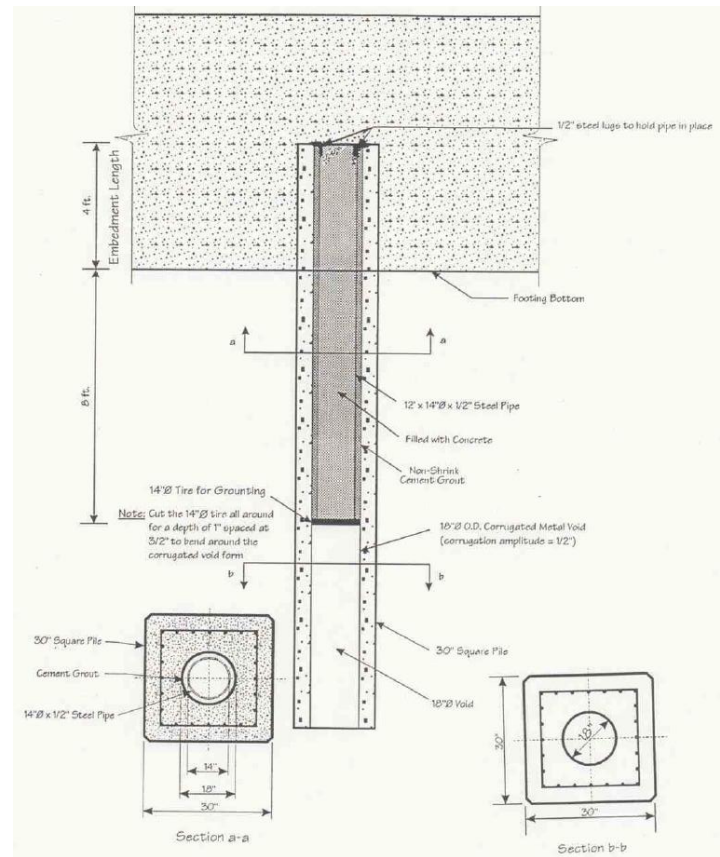


Figure 30. Recommended detail for pipe-pile to pile cap moment connection (Issa, 1999).

The author tested the piles, with and without the special pipe, and calculated their nominal moment strength, comparing them with the experimental results. The calculated nominal moment strength of the pile without the steel pipe was 1117 kip-ft. The insertion of the steel pipe would increase the calculated nominal moment to 1904 kip-ft (~72% higher).

The test resulted in a moment capacity of 1921 kip-ft, slightly above the calculations by the author. The author then suggests that an embedment of 48 in is adequate for developing the flexural strength of such piles, without any slippage of the strands in the embedded region.

One important point about this work is that the author calculates the nominal moment of the pile, with or without the pipe inserted into its hollow core. However, he

does not try to calculate the moment capacity of the connection (M_j) itself, in a similar fashion as Shama et al. (2002), or Marcakis and Mitchell (1980), or many others.

The author noted that the test failure happened due to the formation of a plastic hinge at the pile, in a ductile manner. As Shama et al. (2002) suggested, if $M_j/M_p < 1$, there is brittle failure, with the concrete of connection region in the cap failing first, but the pile still has potential for more bending. If $M_j/M_p > 1$, ductile failure of pile controls.

ElBatanouny et al. (2012)

ElBatanouny et al. (2012) and ElBatanouny and Ziehl (2012) performed experimental tests on four 18-in square prestressed concrete piles simply embedded 18 in, 22 in and 26 in into the bent cap. The authors focused on understanding how the equation given by ACI 318-19 code for development length of prestressed strands would compare to experimental results.

According to the authors, a significant difference in slipping stress was found between the experimental and predicted results for all specimens. They attributed this difference to the confining stress (also called “clamping force”) produced by the shrinkage of the cap concrete relative to the embedded pile.

Based on these tests, ElBatanouny et al. (2012) suggested modified equations to account for the shrinkage effect.

$$l_d = \left(\frac{f_{se}}{5000} \right) d_b + \left(\frac{f_{ps} - f_{se}}{1800} \right) d_b \quad (35)$$

f_{se} = effective prestress of prestressing strand (psi);

f_{ps} = nominal strength of prestressing strand (psi);

l_d = development length of prestressing strand (in);

d_b = nominal diameter of prestressing strand (in);

Larosche et al. (2014a, b) expanded on the this research by conducting experiments on full-scale three-pile bent specimen as well as four full-scale exterior pile bent cap connections, considering plain pile embedment for suitability in seismic regions

and compared to the current embedment practices by the South Carolina Department of Transportation (SCDOT) of 1 time the width of the pile. The results suggest that pile embedment depth of $1.3 \times D$ should be adopted for plain pile embedments, with a tolerance of ± 3 in.

Rakestraw et al. (2024)

Rakestraw et al. (2024) investigated embedment length required for pinned and fixed connections of prestressed concrete piles. According to them, there are significant discrepancies between current design recommendations for embedment and actual performance of the connection, regarding embedment length.

They focused on ten 18-in and 30-in piles embedded between 0.33 to 1.5 times the pile width into the caps. Moment-displacement results for all specimens are shown in Figure 31.

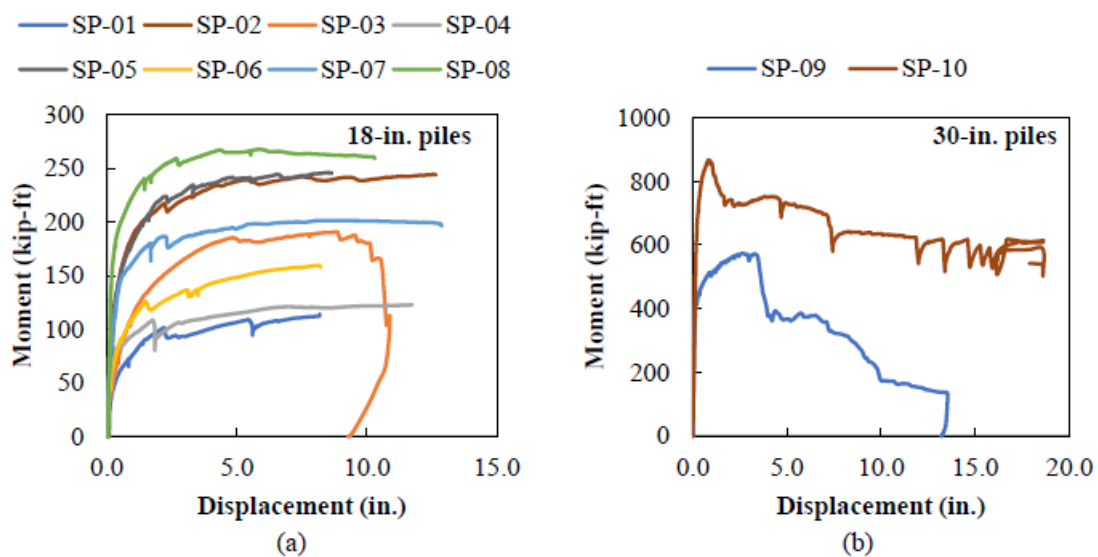


Figure 31. Moment versus displacement curves for (a) 18-inch and (b) 30-inch pile specimens (Rakestraw et al, 2024)

All specimens experienced a ductile failure mechanism, where there was significant deflection after the maximum load was reached. All 18-inch pile specimens

held close to the ultimate capacity while additional deflection was observed. The 30-inch pile specimens experienced a drop in capacity immediately after the ultimate load was reached. Specimens SP-01 through SP-09 all failed due to slipping of the strands. Specimen SP-10, however, failed due to punching shear failure of the edge of the cap, indicating that the connection itself failed first, before the strands could slip, likely due to the lower strength of the cap concrete. Table 2 shows a summary of the test results.

Table 2. Summary of failure moments and failure mechanisms for specimens with different embedment lengths (Rakestraw et al, 2024).

Specimen	Pile Size	Embedment	Service Moment (kip-ft)	Failure Moment (kip-ft)	Percentage of Pile Capacity	Failure Mechanism
SP-01	18"	6" (0.33d _p)	72	114.1	34%	Strand Development
SP-04	18"	9" (0.50d _p)	85.5	122.8	37%	Strand Development
SP-06	18"	12" (0.67d _p)	90	159.4	48%	Strand Development
SP-07	18"	18" (1.00d _p)	120	201.4	61%	Strand Development
SP-08	18"	27" (1.50d _p)	180	267.6	81%	Strand Development
SP-09	30"	12" (0.40d _p)	288	574.5	48%	Strand Development
SP-10	30"	30" (1.00d _p)	810	868.1	73%	Punching Shear

Results showed that even short embedments (0.33 and 0.5 times the pile width) can develop significant portions of the pile's moment capacity. The authors also noted

that, for prestressed concrete piles, capacity of the connection is more dependent on available development length of the strand than any other factor. Additionally, they found that additional shear reinforcement in the pile cap could prevent punching shear failures and improve the connection's capacity and ductility.

The authors also studied the equations from AASHTO LRFD Bridge Design Specification (2020) and ElBatanouny and Ziehl (2012) for calculating development length of the strands. They concluded that these equations are conservative, especially the one from AASHTO. The authors then proposed a slight modification for the k factor in AASHTO equation.

$$l_d \geq k * \left(f_{ps} - \frac{2 * f_{pe}}{3} \right) d_b \quad (36)$$

l_d = development length of prestressing strand (in);

$k = 0.6$;

f_{ps} = nominal strength of prestressing strand (ksi);

f_{pe} = effective stress in the prestressing steel after losses (ksi)

d_b = nominal diameter of prestressing strand (in).

One important observation from the authors is that the capacity of the connection did not appear to be dependent on the pile embedment length as a function of the pile size. The behavior of the connection appeared to be more dependent on the available strand development length provided by the pile embedment length.

The authors also observed that there is a linear relationship between rotational stiffness and embedment length, as seen in Figure 32.

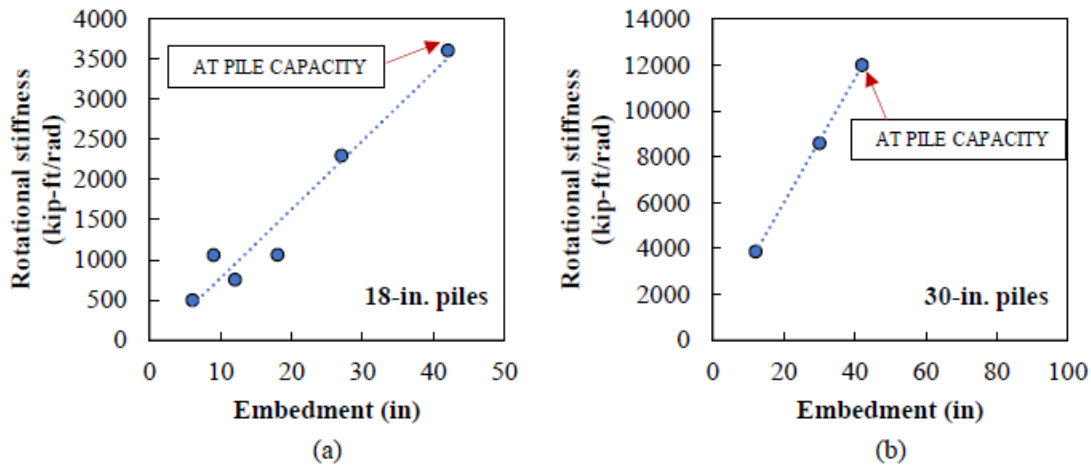


Figure 32. Linear relationship between rotational stiffness versus embedment length: (a) 18-inch piles, (b) 30-inch piles (Rakestraw et al, 2024).

2.4 Shear Friction of Interior Interface between Pile and Cap

Shear friction capacity at the interface between the pile and cap concrete is another mechanism that can control the capacity of the connection, as seen in Figure 33.

In a pile bent, this scenario is more critical at the piles that are subjected to tension forces, due to the lateral force overturning. In Rollins and Stenlund (2010) experiments, for example, three of four pile caps failed due to a pullout failure of the back pile. Two experiments had embedment lengths of 0.5 to 1 time the steel pipe diameter, with reinforcement details at the connection, and the third had plain embedment of 2 times the pipe diameter.

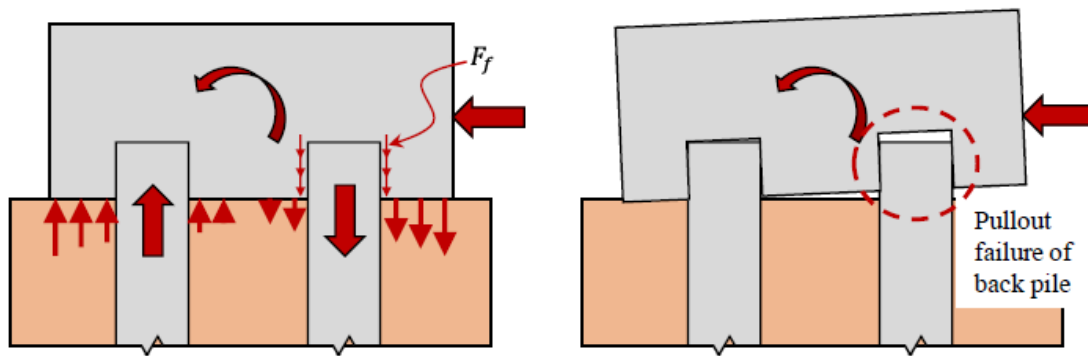


Figure 33. Pullout mode of failure (Rakestraw et al, 2024)

Calculating shear friction capacity is simple, but it depends a lot on the assumed coefficient of friction for the interface between the pile and the cap concrete. Castilla et al. (1984) investigated three different coefficients of friction between the cap and exterior surfaces of the pile in a parametric analysis: 0.4, 0.7, and 1.4. They found that increasing the coefficient of friction did not have a significant impact on the shape of the displacement curve but did decrease the maximum displacement and maximum rotation of the pile.

This type of failure would be expected in connections with shallower embedment lengths with a smooth interface surface between pile and cap.

2.4.1 Steel Pipe Piles

Eastman (2013) proposed a possible friction mechanism in his ultimate strength method for connection capacity, already described in this work in section 2.2.3.

This current section describes another method proposed by Richards et al. (2011).

Rollins and Stenlund (2010) and Richards et al. (2011)

Stenlund (2007) and Rollins and Stenlund (2010) conducted experimental and analytical studies. The authors conducted cyclic lateral loads on four pile caps each with two 12.75-in diameter, 40-ft long steel pipe piles. Two caps included a reinforced connection detail with 6-in or 12-in pile embedment, while the others relied exclusively on plain pile embedment of 12 in and 24 in.

According to the authors, the results indicated that the pile caps that relied on deeper embedment, without reinforcement, performed better than the caps with reinforcing connections and shallow embedment. However, the latter still developed at least 40% to 60% of the moment capacity of the pile, which are greater values than predicted by PCI design equations based only on embedment (Marcakis and Mitchell, 1980).

The analytic study consisted of hand calculations and two computer modeling programs, GROUP and LPILE, which results were very consistent with experimental values when the connections were considered “fixed-head”, as seen in Figure 34.

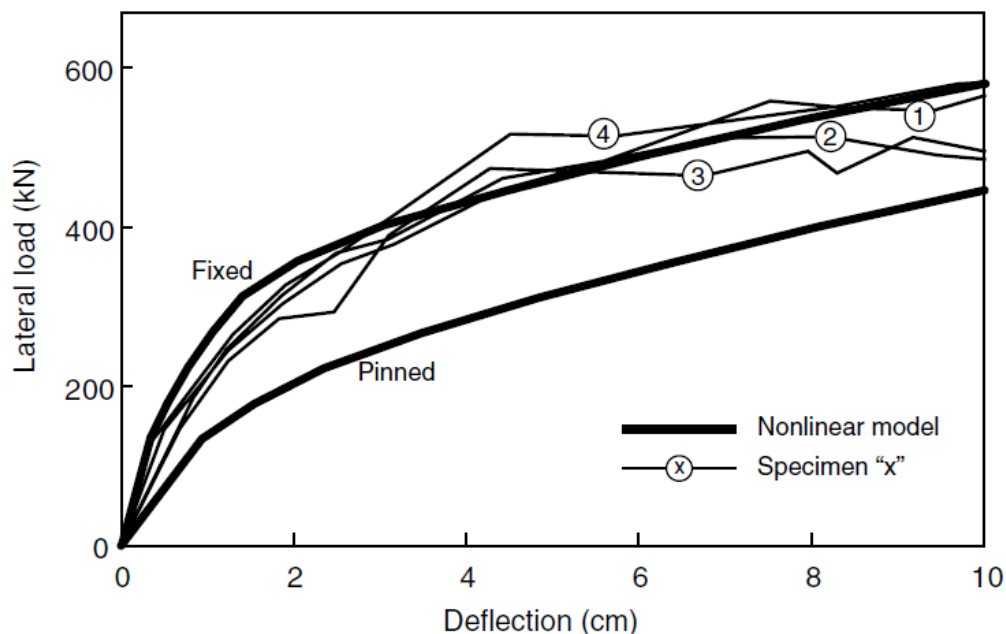


Figure 34. Load-deflection responses of all test specimens, and comparison with GROUP software analysis for fixed or pinned condition (Richards et al, 2011).

However, during software analysis, when assuming a “free-head” condition (or pinned connection), displacements were greatly overestimated when compared to experimental results, even for the shallow embedment test specimen. The authors believe that these results suggest that it is relatively difficult to create a truly pinned condition in

a Pile-to-Cap connection using embedment. Some connections that are assumed to be pinned may be, in reality, partially fixed.

This work had a few specimens that the calculated connection moment capacity was considerably less than the actual value reached in the experiment. Richards et al. (2011) explained that this may be due to two additional mechanisms. The first is dowel action from the two bottom grid bars that passed through the piles, which was considered in the calculations. The second is friction between the steel pipe and concrete.

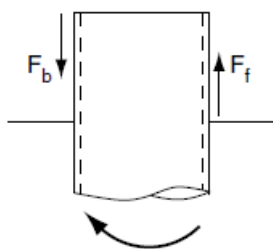


Figure 35. Friction mechanism that increases connection moment capacity (Richards et al, 2011).

Very few authors in literature consider this mechanism in their calculations. Richards et al. (2011), however proposed the following calculations.

As seen in Figure 35, the friction forces, F_b and F_f , can reach maximum values corresponding to the normal forces (C_b and C_f , as seen in Marcakis and Mitchell, 1980) multiplied by the friction coefficient for concrete poured against steel ($\mu = 0.47$, according to Baltay and Gjelsvik, 1990). In this calculation it is assumed that F_f will only develop to balance F_b , and that F_b has reached its theoretical maximum value.

$$F_b = \mu * C_b = \mu * 0.85 f'_c b_{eff} a_b \quad (37)$$

f'_c = concrete compressive strength;

b_{eff} = effective width of the compression block;

a_b = depth of the embedment stress block, assumed to be one-third of the embedment length.

Moment capacity due to friction, therefore, will be:

$$M_{friction} = F_b * d \quad (38)$$

d = pile diameter.

However, the authors state that this mode of failure requires a lot more experimentation and modeling to be fully understood.

2.5 Block Shear Rupture (or Punching Shear)

This failure mode occurs specifically for piles near the edge of a cap. In the case of a bent cap loaded along its long dimension, this failure mode is adequate for the exterior piles. For a bent cap loaded along its short dimension, all piles should be verified for this failure mode.

2.5.1 Steel H Piles

Research regarding block shear rupture was only conducted for steel H-Piles. This mode of failure is more concerning for steel H-piles being bent about the strong axis.

Xiao et al. (2006) and Xiao and Chen (2013)

Xiao et al. (2006) conducted experiments on HP-steel pile-to-pile cap connections with shallow embedment, and special details at the connection. Investigation was performed in a simulated specimen that represents a common configuration in California, of a single column bent bridge being supported by a footing with multiple piles, as seen in Figure 36.

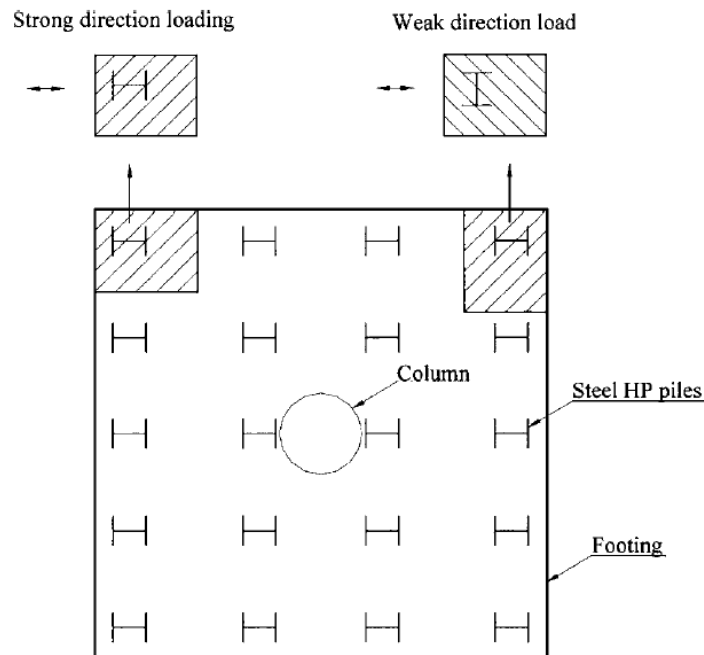


Figure 36. Common configuration of single column bent bridge, supported by pile footing (Xiao et al, 2006).

The tests were performed on the pile specimens that simulated being at the corner of a pile cap, as shown in Figure 36, both in weak and strong directions. The piles were HP 14x89 with a design service compressive capacity of 890 kN (ultimate capacity of 1779 kN). They were connected to the pile cap with a shallow embedment (approximately 0.35 time the pile width) and with the use of V-shaped bars. Five specimens were built, simulating the corner case, as shown in Figure 37.

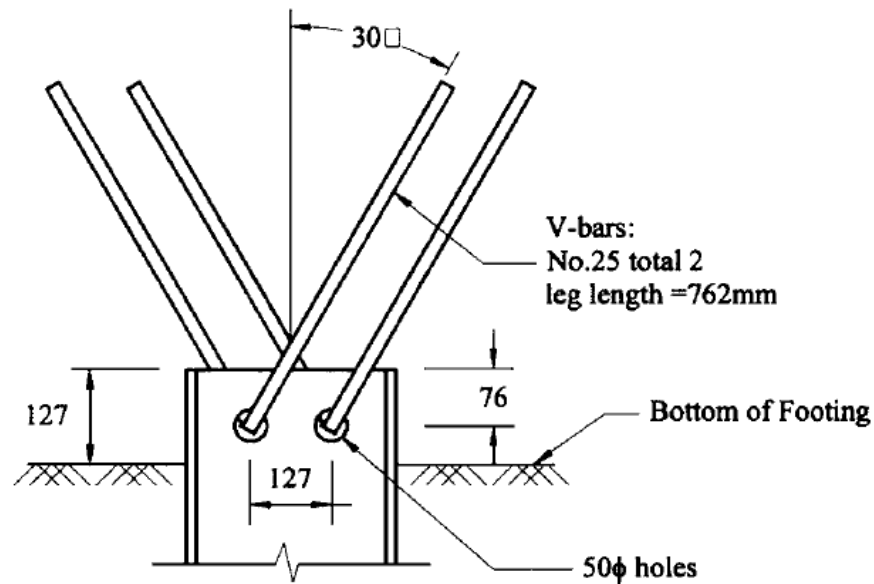


Figure 37. Details of typical anchorage of HP piles (Xiao et al, 2006).

Table 3 shows the testing matrix, detailing the parameters of the test, and how load was applied.

Table 3. Testing Matrix (Xiao et al, 2006)

Model specimens	Orientation of HP pile	Loading condition	Concrete strength f'_c (MPa)	Axial Load (kN)
HP-S-V	Strong direction	Cyclic vertical loading only	41.1	Varies
HP-W-V	Weak direction	Cyclic vertical loading only	37.9	Varies
HP-S-L	Strong direction	Cyclic lateral loading with constant vertical load	41.1	1,779
HP-W-L	Weak direction	Cyclic lateral loading with constant vertical load	41.1	1,779

HP-S-VL	Strong direction	Proportionally varied cyclic lateral and vertical loading	37.9	Varies*
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For the Cyclic Vertical Loading Tests (HP-S-V and HP-W-V), both piles behaved similarly. One pile had its orientation over the strong axis, and the other, over the weak axis. Both developed significantly high stiffness and capacity during the compressive loading but low stiffness and capacity under the tensile loading because of the rocking response of the cap. The failure of both specimens was caused by the rupture of the V-shaped anchor bars at tensile forces much lower than the ultimate design tensile capacity of 890 kN. Figure 38 shows the results for both specimens.

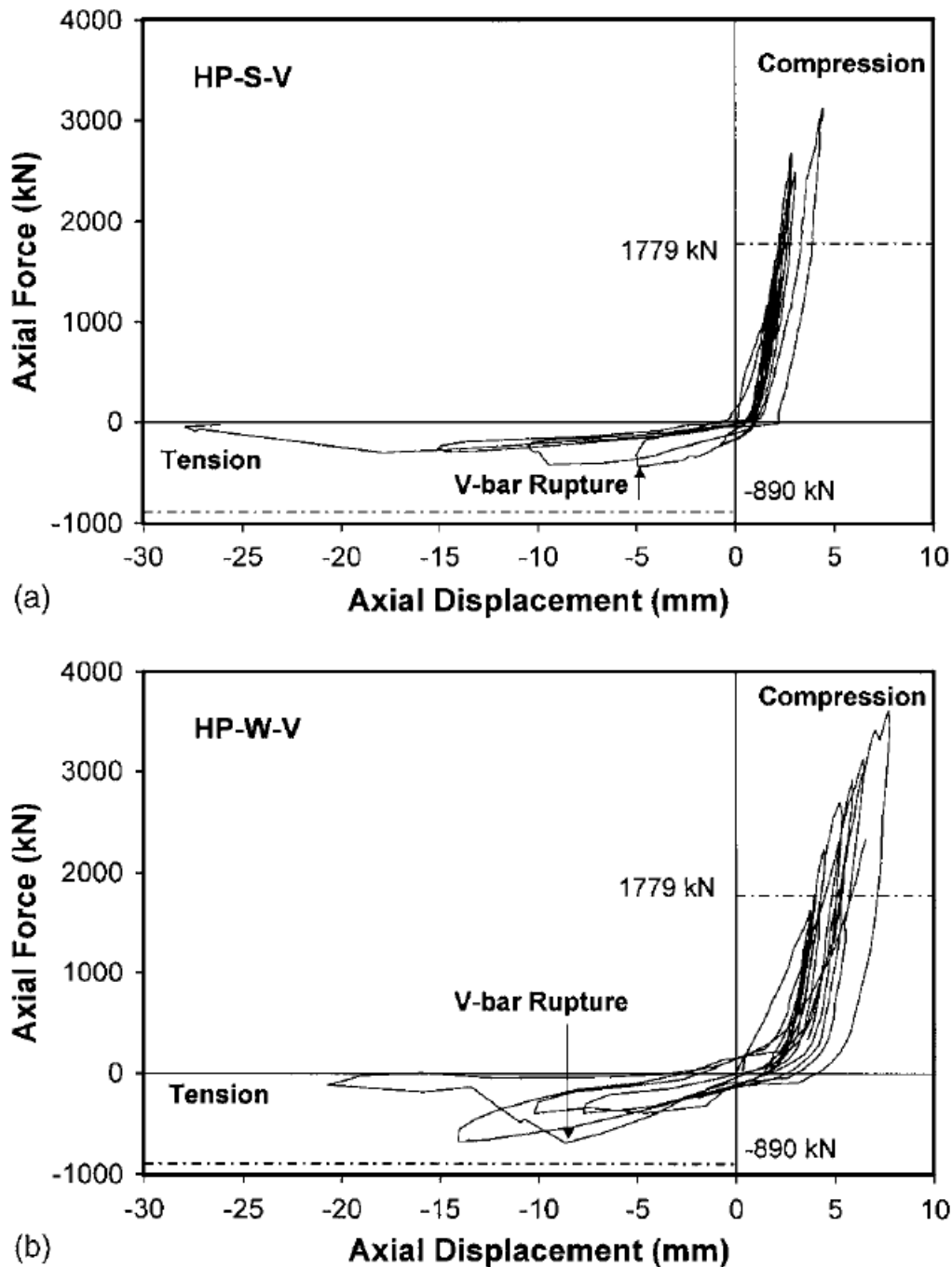


Figure 38. Vertical force-displacement response of Cyclic Vertical Loading Tests (Xiao et al, 2006).

For the Cyclic Horizontal Loading Tests (HP-S-L and HP-W-L), seen in Figure 39, under constant vertical load, the specimen tested in strong direction showed an unsymmetrical hysteric response, due to the pile cap rocking in the pull loading direction.

Cracks began near the pile end and spread toward the cap edges. At a 2% drift ratio in the pushing direction, a crack penetrated the cap, leading to a reduction in load-carrying capacity at subsequent peaks. During loading to a 4% drift ratio in the pull direction, one anchor bar ruptured.

The specimen tested in the weak direction showed a more stable hysteretic response, with symmetric loops in both push and pull directions. No cap rocking was observed, attributed to the small moment demand in the weak direction of the HP pile. Damage was minimal, with cracks localized near the pile end. Yielding of the HP-pile flanges near the pile end was indicated by strain gauge readings and surface line patterns from oxidized skin falling off. No anchor rupture occurred for this specimen.

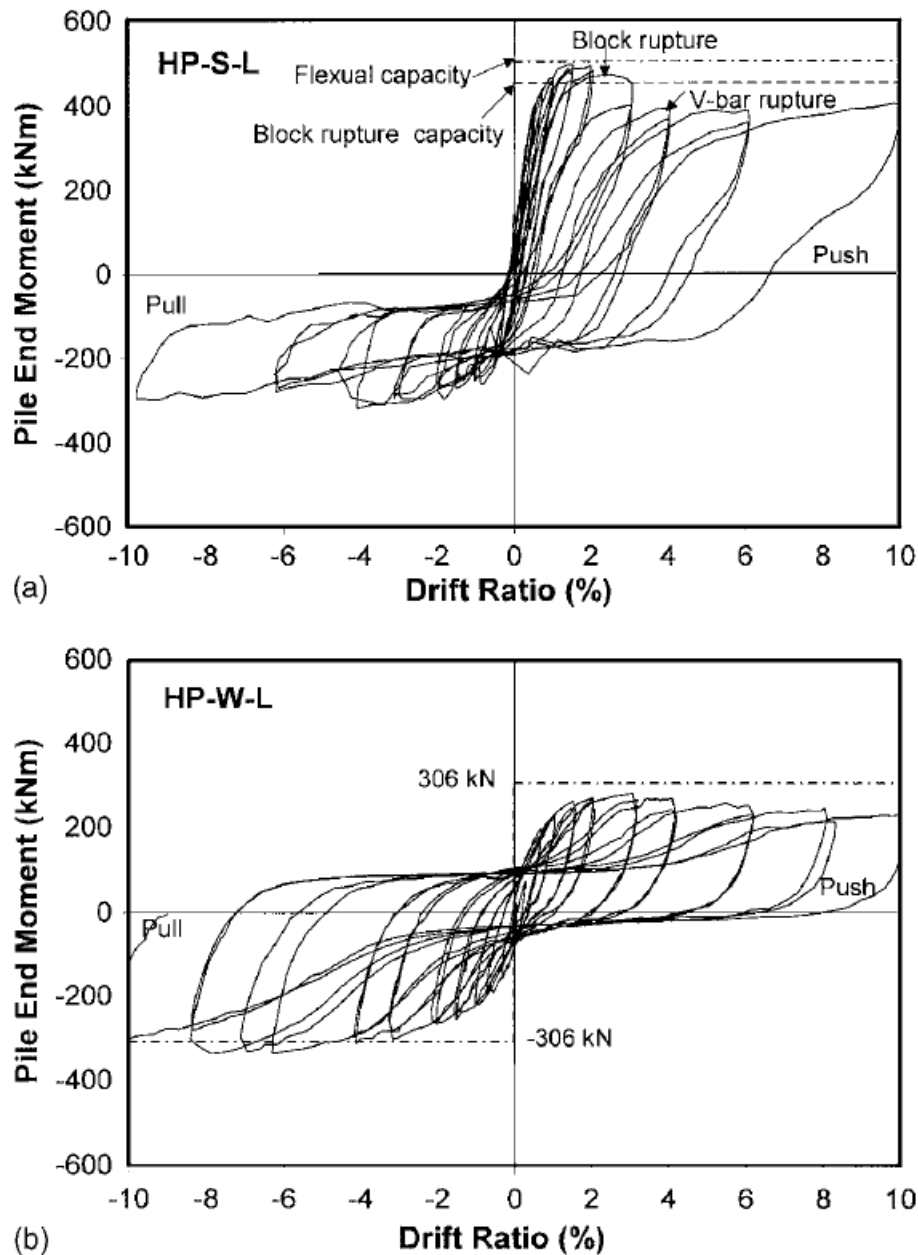


Figure 39. Pile end moment vs drift ratio plot for the Cyclic Horizontal Loading Tests, under constant vertical load (Xiao et al, 2006).

For the Cyclic horizontal/vertical loading test (HP-S-LV), in Figure 40, the behavior was hysteretic, but with lower carrying capacity. One anchor bar ruptured during pull loading at drift ratio of 8%, causing sudden degradation of load carrying capacity.

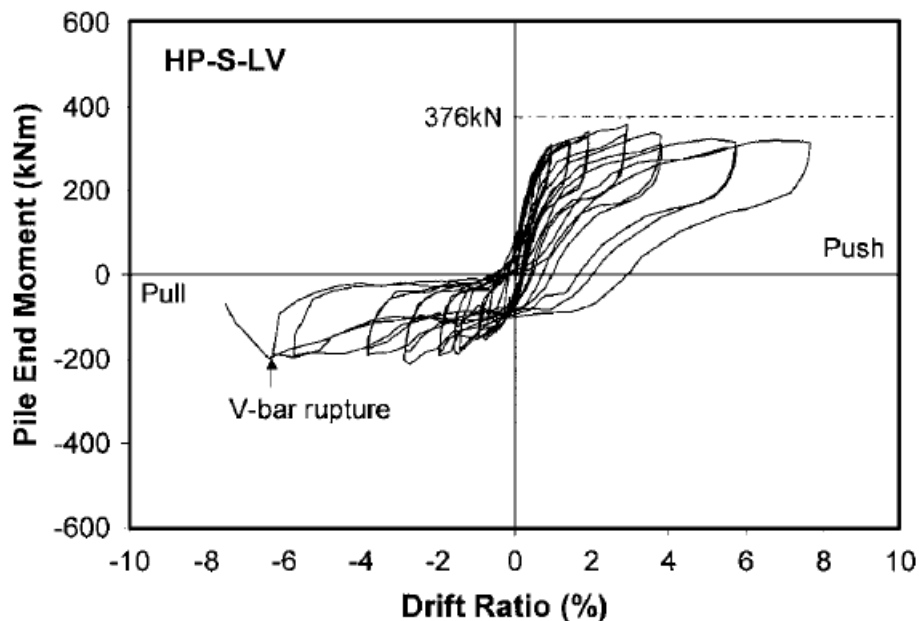


Figure 40. Pile end moment vs drift ratio plot for the Cyclic horizontal and vertical loading test (Xiao et al, 2006).

The test results clearly showed that, although shallow embedment is generally considered pinned, the tested connections developed considerable moment capacity. However, it is important to note that these tests had the influence of special detail connection, not plain embedment.

Xiao et al. (2006) recommend the use of Marcakis and Mitchell (1980) equation for determining the moment capacity of the connection due to the embedment of the pile. For specimen HP-S-L (Figure 39), for example, the predicted moment capacity due to embedment and the V-shaped bars was 504 kN.m. As seen in the results, the block ruptured at a certain load level, meaning that the embedment resistance disappeared, but only flexural resistance remained, at an end moment of about 400 kN.m.

The authors also found that, for piles near the edge of the footing and for strong axis bending, the concrete block rupture near the footing edge may control the capacity, since the block cannot rely on embedment mechanism anymore. The authors then proposed an equation for the block rupture strength (V_{rp}).

$$V_{rp} = c_e * (b_f + c_e + 2L_e) * f_t \quad (39)$$

Where:

c_e = distance from the face of the HP pile to the edge of the footing

f_t = concrete tensile strength

b_f = width of the pile flange

L_e = length of pile embedment

Important to note that this equation works for piles placed at a short distance from the edge (c_e) of the cap. For bigger distances, the block rupture strength equation will result in greater values. However, in these cases, other mechanisms will most likely control failure.

Xiao and Chen (2013) investigated further on this topic, performing experimental study on the use of vertical bars (instead of V-shaped bars as seen in Xiao et al, 2006) welded to steel H-piles, loaded monotonically. This work resulted in a slight modification of equation (39), proposed by Xiao et al. (2006) for block rupture.

$$V_{rp} = [(b_f + 2c_e) * \frac{(L_e + c_e)}{2} - b_f * L_e] * f_t \quad (40)$$

Where:

c_e = distance from the face of the HP pile to the edge of the footing

f_t = concrete tensile strength

b_f = width of the pile flange

L_e = length of pile embedment

Additionally, the authors analytically evaluated the connection capacity using equations from Marcakis and Mitchell (1980) and Mattock and Gaafar (1982), as well as the equations developed in Xiao and Chen (2006) for other mechanisms of failure (tensile capacity of welded bars, block rupture). Then, the authors focused on Finite element modeling of the piles using ANSYS to try to replicate the observed behavior.

The authors concluded that the load-displacement analysis demonstrated that the connections could sustain a certain level of moment, meaning they are not pinned, as many consider. They stated that, if using shallow embedment, the concrete block rupture failure may dominate the capacity. This means that using the concrete block rupture strength equation from Xiao et al. (2006) or its modification in Xiao and Chen (2013) may accurately predict the capacity of the connection, for this mode of failure. The finite element analysis (FEA) provided satisfactory results in capturing both the load-displacement response behavior and the load-carrying capacity of these connections.

Xiao et al. (2006) and Xiao and Chen (2013) did not provide any new recommendations or equations regarding steel H pile embedment, other than recommending the use of Mattock and Gaafar (1982) equation for bearing capacity.

3. Rotational Stiffness

Most pile groups are modeled as a fixed-head condition, meaning that the rotation of the top of the pile, inside the pile cap, is fully restrained. However, as observed by different authors (Rollins and Stenlund, 2010; Kappes et al. (2021) and others), full rotational restraint does not exist since the piles can deform and rotate inside the embedded portion of the cap.

As explained by Rakestraw et al. (2024), Figure 41 shows how one can use rotational stiffness in modeling partial fixity of a connection.

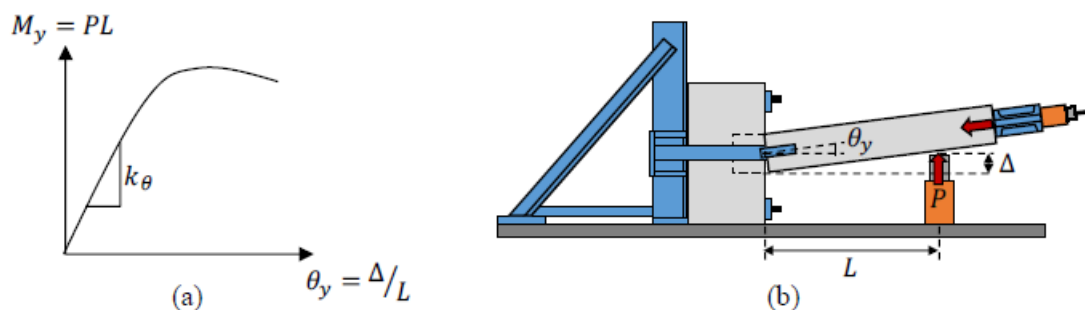


Figure 41. Determination of Rotational Stiffness (Rakestraw et al, 2024).

As seen in Figure 41, the rotational stiffness can be determined by plotting the moment versus rotation assuming rigid body kinetic rotation about the connection

between pile and pile cap. This can be calculated from experimental results or numerical modeling results. The rotational stiffness is then found based on the slope of the moment-rotation plot in the linear elastic region.

Mokwa and Duncan (2003)

Mokwa and Duncan (2003) also observed that rotation of pile cap itself influences the degree of fixity of piles. The authors proposed a practical method to estimate the moment restraint provided by the pile cap at the top of a pile group. Since fully fixed-head conditions are rarely achieved in reality, assuming a fully restrained pile head can lead to underestimated lateral deflections and overly conservative bending moment values in the cap. To address this, Mokwa and Duncan (2003) introduced methods for calculating a rotational restraint coefficient ($K_{M\theta}$) to improve deflection estimates of pile groups under lateral loads. This coefficient is entirely dependent on the axial stiffness of the piles.

Using data from testing in 1999, Mokwa and Duncan (2003) developed a procedure to estimate the moment restraint which would allow proper estimation of the actual pile head rotational stiffness which would be between the fixed and free conditions. The value of the rotational restraint coefficient, $K_{m\theta}$ is a function of the amount of movement required to mobilize the tensile and compressive loads in the pile. The amount of rotation is a function of the magnitude of the lateral load and rotational stiffness $K_{m\theta}$. Figure 42 shows the free body diagrams used to derive the equation for $K_{m\theta}$.

$$K_{M\theta} = \frac{\Delta M}{\Delta \theta} = \sum_{i=1}^n [K_{\Delta c} (x_i - a)^2] + \sum_{i=1}^n [K_{\Delta t} (x_i - a)^2] \quad (41)$$

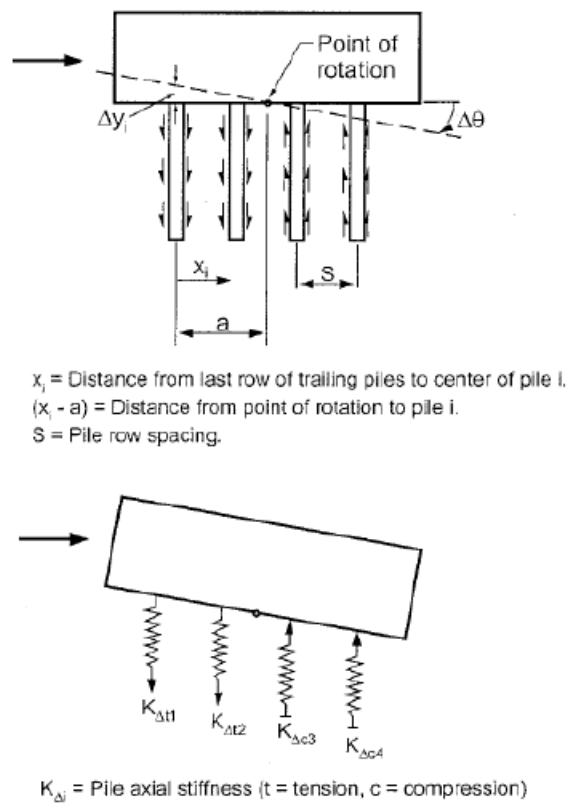


Figure 42. Free body diagrams showing rotational resistance relative to rotation (Mokwa and Duncan, 2003).

Figure 43 shows one of the load-deflection curves from testing and compares them to the curves predicted using fixed-head and free-head conditions as well as the rotationally restrained stiffness defined using equation (41) above. Clearly the degree of pile head fixity plays a substantial role in the computed load-deflection curves.

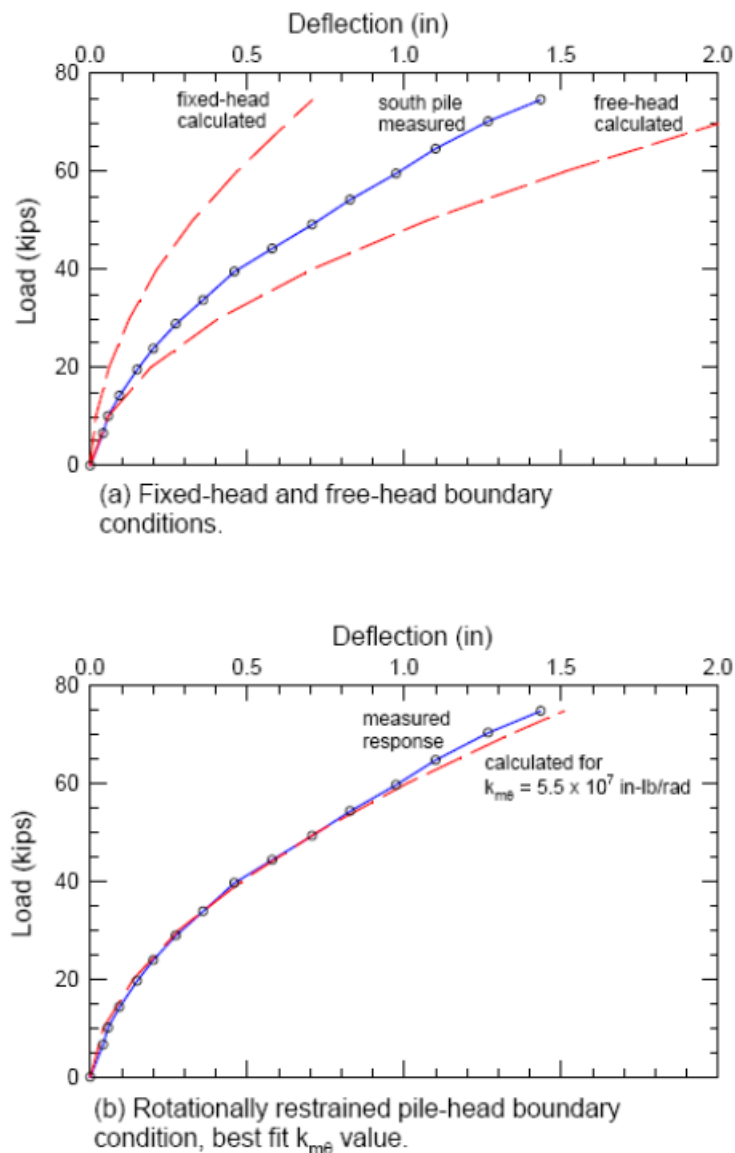


Figure 43. Load vs. Deflection curves comparing boundary conditions (Mokwa and Duncan, 2003).

Shishlov et al. (2022)

Shishlov et al. (2022) presented the following preliminary analytical model for determining partial rotational stiffness of a pile head (Figure 44).

The model considers a pile (with equivalent length to fixity equal to L) that contains gross cross-section properties (elastic modulus E and moment of inertia I) and is assumed to behave in a linear elastic manner. Also, the pile base is fixed, and the pile

head is assigned a rotational spring (with stiffness, k_r). The pile has an equivalent length to fixity equal to L , calculated by methods such as Davisson and Robinson (1965), etc.

Under application of a lateral load (F) acting at the pile head, the pile undergoes lateral translation and rotation until equilibrating reactions form, such as the moment in the pile head rotational spring, M_{top} and the moment at the fixed base, M_{bot} .

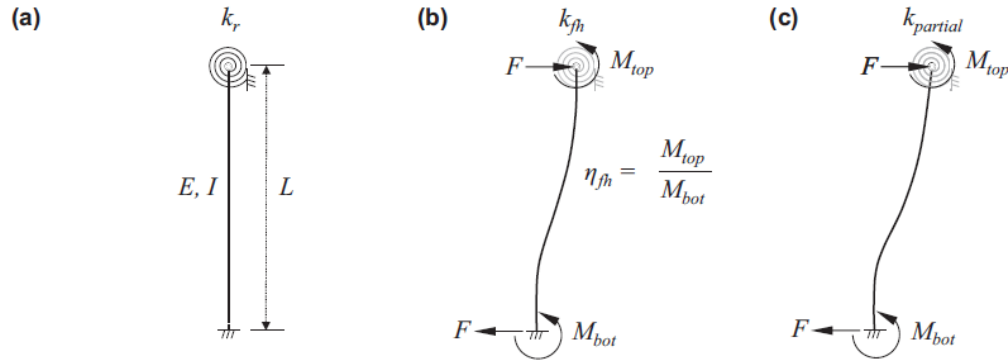


Figure 44. Analytical approach for idealized (linear elastic, representative, point-of-fixity) piles subjected to lateral loads: (a) idealized, representative pile, (b) laterally loaded pile under fixed-head conditions, (c) laterally loaded pile under partial fixity conditions (Shishlov et al, 2022).

The relationship between M_{top} and M_{bot} is given by η :

$$\eta = \frac{M_{top}}{M_{bot}} \quad (42)$$

If M_{top} is approximately equal to M_{bot} , then M_{top} is equivalent to a fixed head condition. Therefore, for fixed head condition, $\eta = \eta_{fh} \approx 1$, but never quite actually reaching 1.

Fixed-head rotational stiffness (k_{fh}) can be determined by plotting the moment-rotation results measured at the pile head from a lateral loading test, in assumed fixed-

head conditions. It can also be approximated, if using a hand method such as direct stiffness method:

$$k_{fh} = \frac{2EI}{L(\frac{1}{\eta_{fh}} - 1)} \quad (43)$$

For partial rotational stiffness (k_{partial}), a new term Λ is introduced, multiplying it by η_{fh} .

$$k_{\text{partial}} = \frac{2EI}{L(\frac{1}{\Lambda * \eta_{fh}} - 1)} \quad (44)$$

Where Λ is a decimal-fraction measure (from 0 to 1) of proximity to fixed head-conditions. Pinned-head conditions are converged on as Λ approaches 0 and fixed-head conditions are converged on as Λ approaches unity.

Shishlov et al. (2022) then demonstrated that, to achieve 50% fixity ($\Lambda = 0.5$, meaning 50% of η_{fh} , or that M_{top} is 50% of M_{bot}), only 1% of the fixed-head rotational stiffness (k_{fh}) is necessary, as seen in Figure 45.

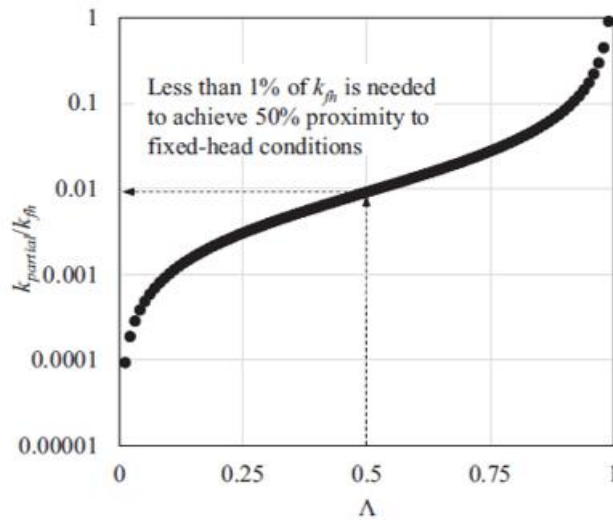


Figure 45. Plot of k_{partial} equation using demonstration-case parameters (Shishlov et al, 2022).

Then the authors moved on to a computational approach, which consists of iteratively scaling the rotational stiffness terms at pile heads throughout a pier foundation FE model, relative to fixed-head conditions, until a target response under lateral loading is achieved, as well as the associated level of partial fixity. Therefore, a target level of partial fixity must be known.

The authors define the factor Pile Head Rotational Stiffness (PHRS) as the ratio of partial fixity rotational stiffness relative to fixed head rotational stiffness. The computational approach algorithm then, requires two prerequisites: a specified target demand (displacement, or maximum pile head moment) and the percentage of fixity (between 0%, pinned-head, and 100%, fixed-head).

Then, a lateral load analysis is made for fixed-head condition and a certain magnitude of lateral displacement, for example, is obtained. The same lateral load analysis is performed, this time under pinned-head condition, and another magnitude of lateral displacement, for example, is obtained. After this, one can seek to achieve, for example, 50% of the displacement with the same lateral load, by iterating and changing the PHRS factor in the FE software.

In the case demonstrated by Shishlov et al. (2022), to achieve 50% of the displacement, a PHRS factor equal to 0.04 (or 4%) was necessary. This means that, to

achieve 50% of fixity, the necessary pile head rotational stiffness is only 4% of that corresponding to fixed-head.

The authors, then, performed a parametric study with several bridge pier configurations, and developed the Table 4 below for aiding engineers in the use with FE software and provide with initial estimates of PHRS factors.

Table 4. Summary Statistics of Computed Pile Head Rotational Stiffness (PHRS) Factors Corresponding to 25%, 50%, and 75% Proximity to Fixed-Head Conditions (With Respect to Lateral Displacements, Relative to Fixed-Head Conditions) (Shishlov et al, 2022).

Lateral Displacement relative to the fixed-head lateral displacement (%), also called degree of fixity	Load magnitude applied, relative to pushover load (%)	Mean PHRS factor	Standard Deviation
25	10	0.014	0.013
	25	0.020	0.011
	50	0.028	0.013
	Average:	0.021	0.013
50	10	0.039	0.036
	25	0.047	0.028
	50	0.060	0.028
	Average:	0.049	0.032
75	10	0.087	0.079
	25	0.101	0.070
	50	0.111	0.056
	Average:	0.100	0.070

Although this work is very useful for practical applications in design-oriented FEA software, there are some caveats. The concept of rotational stiffness can become quite complicated to grasp as one approaches fixed or pinned conditions. Approaching, pinned

conditions would mean a rotational stiffness equal to zero, which does not exist. On the other hand, approaching fixed conditions means that rotational stiffness would tend to infinity, which does not exist as well. Therefore, a very small change in fixity near fixed conditions could mean a very large change in the rotational stiffness.

Zapata et al. (2022)

According to Zapata et al. (2022), incorrect assumptions about the level of fixity of a PTPC connection can lead to undesirable outcomes. Assuming a greater level of fixity than reality can lead to great displacements in the structure. On the other hand, assuming lower levels of fixity than reality (pinned, for example) can lead to overdesign and construction of bulkier structural members in order to compensate for the assumed lack of moment transfer, leading to greater and unnecessary costs.

Zapata et al. (2022) conducted an investigation to understand the behavior of PTPC connections considering varying pile embedment into the pile cap, with or without presence of dowels. In this study, FEA models were created to replicate the results of Harries and Petrou (2001) as well as a parametric study with 131 different pile-to-cap connection models was performed, using prestressed concrete piles with different widths (18 in, 24 in and 30 in). Results indicated that, for shallow pile embedment, cracking and high concrete stress were observed at the PTPC interface. As for deeper embedment, cracking and high concrete stress were observed on the tension face of the pile, suggesting that the pile reached its moment capacity, rather than the connection.

The authors also modeled entire bridges to understand how typical designs behave depending on the fixity assumption. For a bridge bent model, the authors used an approach with spring rotational stiffness (k_θ). The stiffness of the rotational spring was different for each embedment length, and was calculated by plotting the moment compared with rotation, assuming rigid body kinematic rotation about the pile-to-cap connection. The rotational stiffness was then found based on the slope of the moment-rotation plot, in the linear elastic region. As an example, the authors determined a certain rotational stiffness $k_\theta = 54,610$ kip-ft/rad (or 74,040 kN-m/rad) for an 18-in diameter pile embedded 4.5 in ($0.25 \times \text{diameter}$).

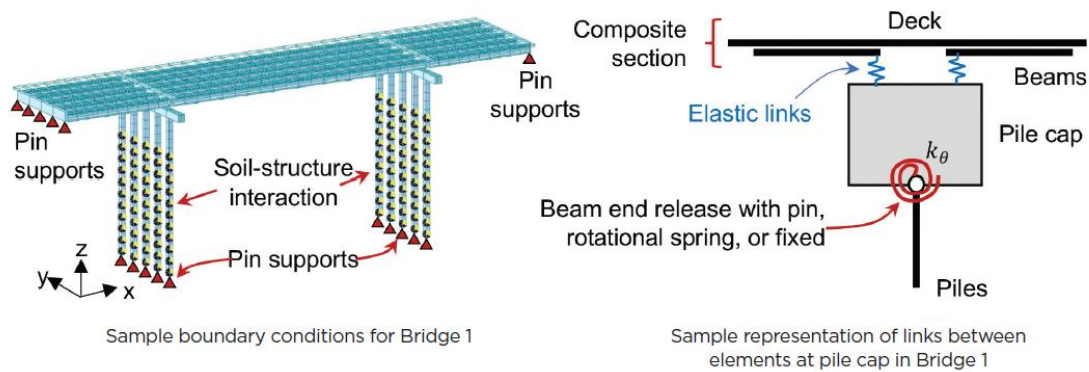


Figure 46. Example of a bridge model by Zapata et al. (2022).

One developed model, as seen in Figure 46, is a bridge supported by bent caps, and prestressed concrete piles directly embedded into them. The author analyzed the bridge not only when fully constructed, but during construction phase. This way, the PTPC connections were analyzed during each stage of construction.

During girder and deck placements, the type of pile-to-cap connection had little impact on the structure's behavior. The moment demand during these stages was minimal, around 10% of the piles' full moment capacity. The key requirement during these stages was ensuring enough moment capacity to counteract the moment from the uneven placement of the support bearing on the cap.

With the fully constructed bridge, the superstructure's moment and shear demand were not affected by the type of pile-to-cap connection. However, the moment transferred to the piles was significantly influenced by the connection type. A fixed connection led to about 43% more moment from the superstructure into the piles compared to a rotational spring connection (partially fixed).

The study found that the moment capacity of pile-to-cap connections is influenced by the embedment length, with longer embedments increasing capacity until the pile's limit is reached. Shallow embedments can still develop significant moments, especially with interface reinforcement. The normalized moment response was consistent across different pile sizes. Additional axial compression improved connection performance. Pile concrete strength had minimal impact on shallow embedments but slightly increased capacity for deeper ones. On the other hand, the pile cap concrete strength influenced

shallow embedment performance, while pile cap size and confinement reinforcement had little effect. There was minimal difference in performance between piles with different prestressing strand diameters.

Hammett (2017)

Zareian and Kanvinde (2013) explored the fact that many practicing engineers consider connections as rotationally fixed. But this assumption is problematic. Rodas et al. (2017) mention that experimental research into rotational stiffness of connections is still scarce. Additionally, they presented a method to characterize rotational stiffness of deeply embedded column base connections.

Building on this concept, Hammett's (2017) primary goal in this research was to develop a theoretical relationship between an applied moment and the corresponding rotational displacement for steel piles embedded in concrete. According to the author, in the steel-cap connection, the embedded portion of the steel pile rotates when subjected to an externally applied moment. This behavior can be affected by the following factors:

- Pile embedment length
- Concrete compressive strength
- Pile section properties (geometric and material)
- Bending stresses in the concrete due to frame action

In this research, the author varied the factors affecting connections and studied 96 different connection scenarios. For this, the author developed a few analytical methods to evaluate the behavior. Initially, he introduces the concepts of Pile stress transformation length (L_{pst}) and pile stress transformation block (PTSB), as seen in Figure 47.

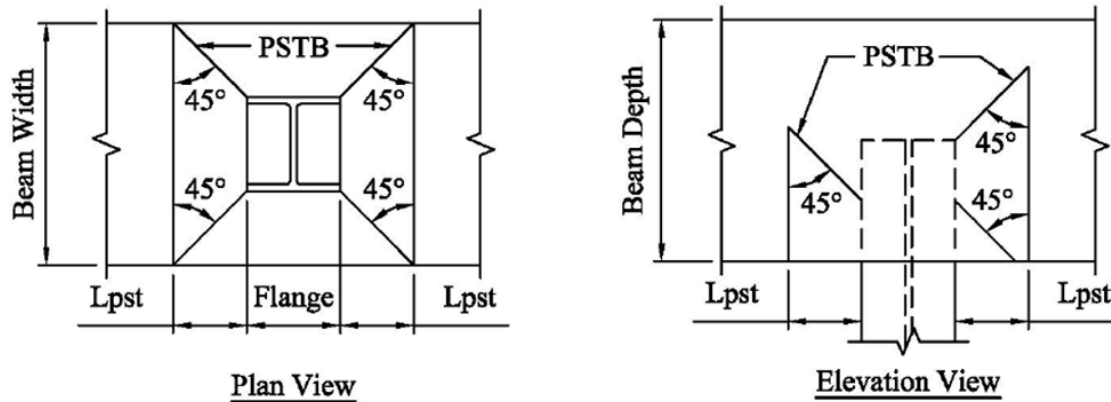


Figure 47. Illustration of PTSB and L_{pst} at Embedded Steel Pile Connection (Hammett, 2017).

The author states that simple, two-dimensional analysis methods can be used to model the flow of stresses from the localized pile bearing stress at the face of the pile to the idealized bending stresses assumed to exist at a distance L_{pst} .

He develops four different methods using a few assumptions:

- Non-linear behavior of concrete.
- Tensile bending stresses in concrete are limited to the modulus of rupture.
- Compressive stresses in concrete are limited to the peak stress (no post-break behavior is allowed in the analysis).
- Performance of steel pile is also based on nonlinear stress-strain curve, but the limiting condition for connection evaluation is the yield moment of the pile. Therefore, in practice, the steel pile will not exhibit significant non-linear behavior.

Hammett (2017) adopted stress-strain curves from Karthik and Mander (2011), as seen in Figure 48 below.

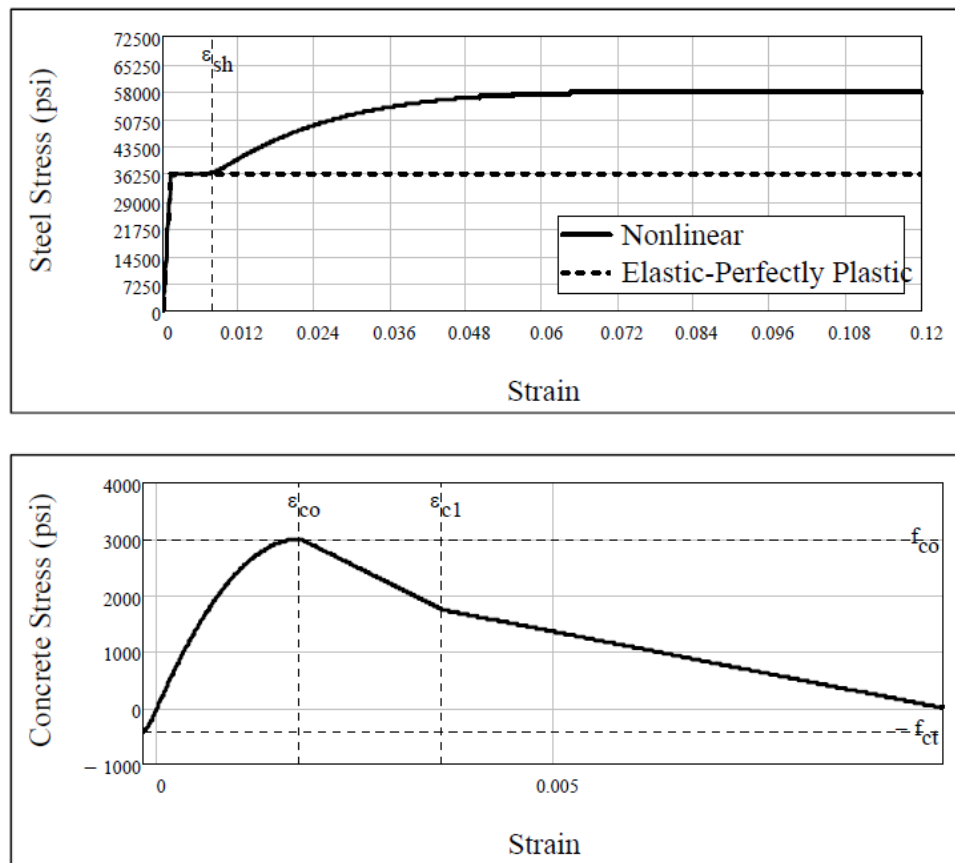


Figure 48. Representative Stress-Strain Curves for Steel and Concrete Components (Hammett, 2017).

According to the author, the concrete within the PSTB volume is considered unconfined because of its proximity to the edges of the beam and because of the lack of confining reinforcement present in typical pile-to-cap connections in ALDOT bridge bents.

For his analytical models, Hammett (2017) also assumed that the PSTB concrete and its interaction with neighboring concrete elements happen through 1-D elements: Nodes and lines (called links) that transmit tension and compression forces acting along their length to model the load sharing between the PSTB and the surrounding cap concrete.

In this work, Hammett (2017) adopted the following assumption regarding the PSTB concrete and its interaction with the neighboring concrete elements: 1-D elements were considered - nodes and lines (called links) to transmit tension and compression

forces acting along their length to model the load sharing between the PSTB and the surrounding cap concrete.

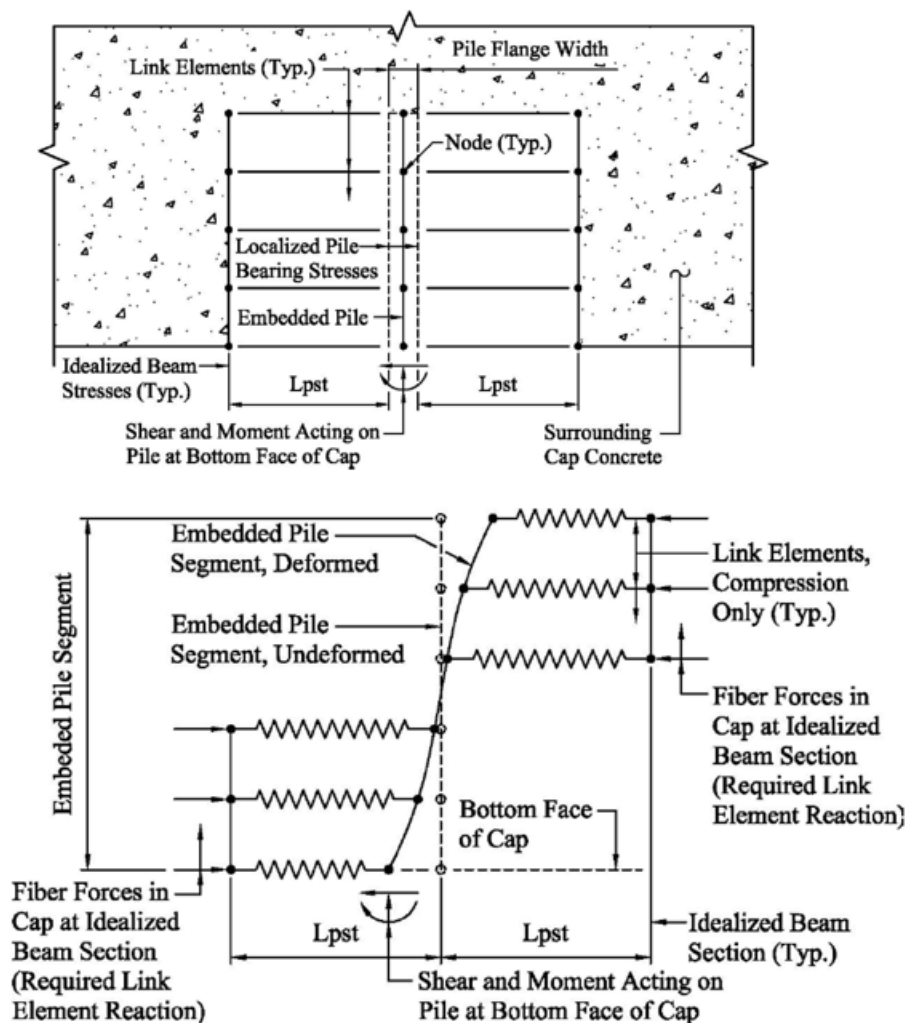


Figure 49 below exemplifies the assumptions for methods 1, 2 and 3. Method 4, on the other hand, assumes that the embedded pile segment rotates as a rigid body in response to the applied load.

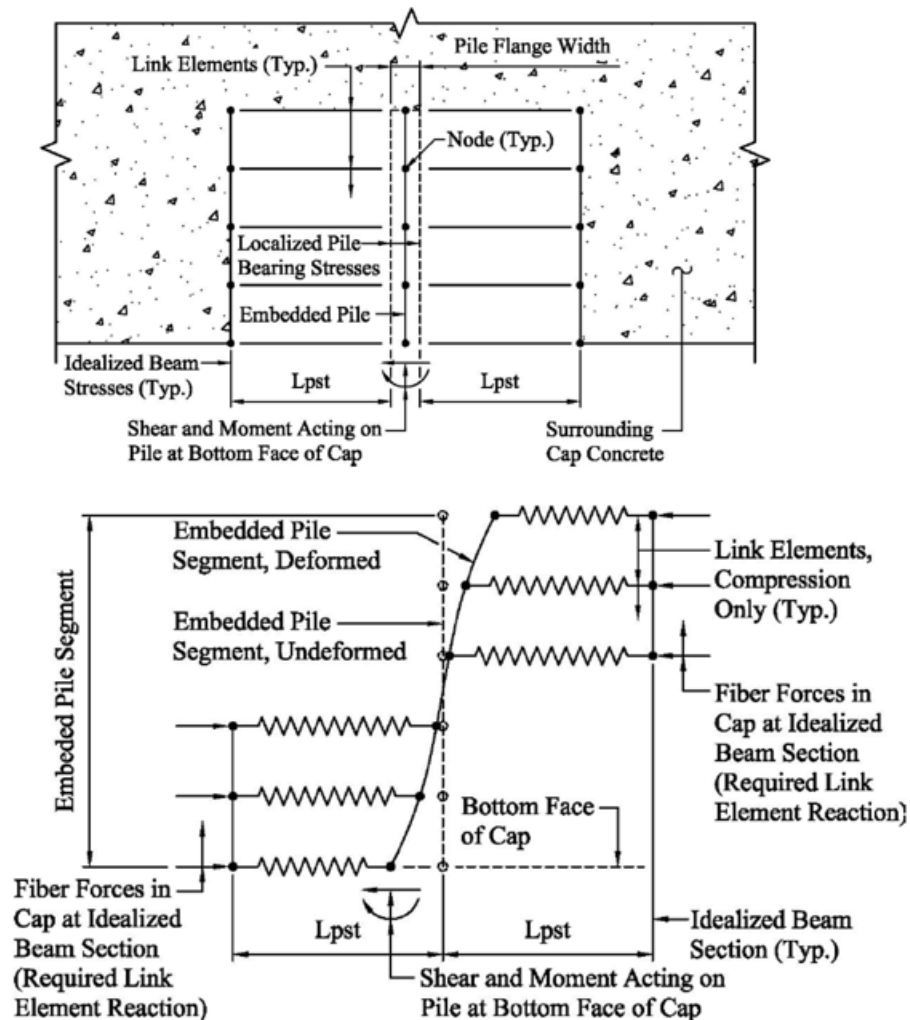


Figure 49. Connection model (top) and Structural Analysis model (bottom) used for Methods 1, 2 and 3 (Hammett, 2017).

The author describes how each method works. Method 1 is called “Link element with linear Traction force - peak at pile face”. The 1-D linear link assumes the presence of a linear traction force along its length. The linear traction force is zero at the end of the PSTB area, and is at its peak at the pile face. This means that most of the load sharing between the PSTB and the surrounding cap happens very close to the pile face. This characteristic minimizes deflections at the pile face and maximizes connection stiffness (Figure 50).

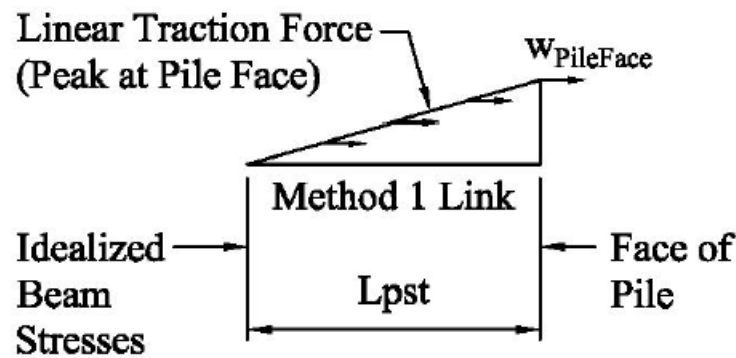


Figure 50. Method 1: Link element with linear Traction force - peak at pile face (Hammett, 2017).

Method 2 is called “Link Element with uniform traction force”. The 1-D linear link assumes the presence of a uniform traction force along its length. This means that the load sharing between the PSTB and surrounding cap concrete occurs uniformly along the link element. For this reason, deflections and connection stiffness are considered intermediate (Figure 51).

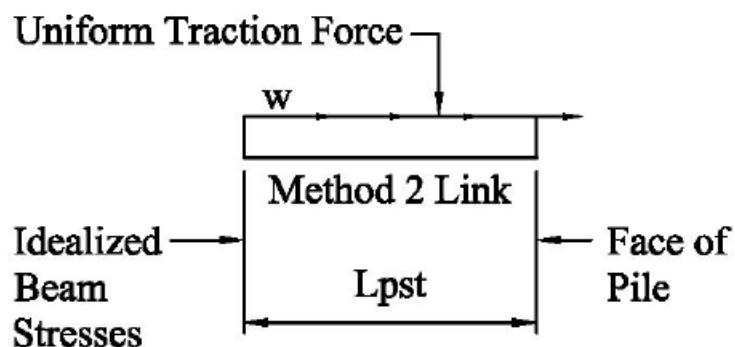


Figure 51. Method 2: Link Element with uniform traction force (Hammett, 2017).

Method 3 is called “Link Element with Linear traction force - Peak at End of PSTB area”. This method is the opposite of method 1. Again, the 1-D link element assumes the presence of a linear traction force along its length. However, in this method, the linear traction force is at its peak at the end of the PSTB area, and is zero at the pile face. Therefore, most of the load sharing between PSTB and surrounding concrete happens at the end of the PSTB area. This maximizes deflections at the pile face and minimizes connection stiffness (Figure 52).

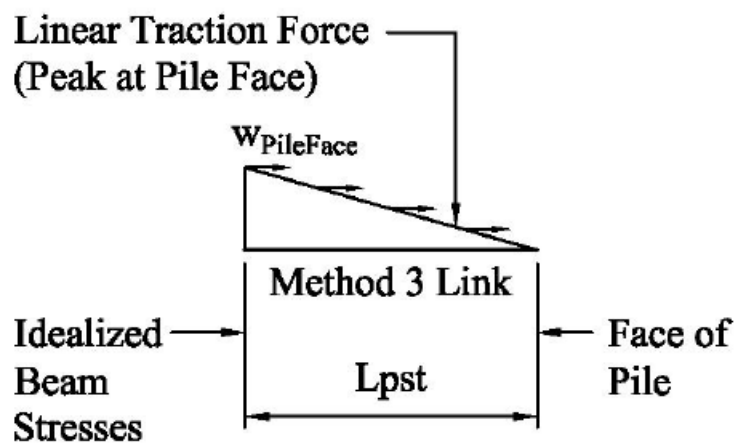


Figure 52. Method 3: Link Element with Linear traction force - Peak at End of PSTB area (Hammett, 2017).

Method 4 is called “Rigid Body Behavior - Deformations by Strain Density”. This method assumes that the embedded pile segment rotates as a rigid body in response to the applied loads, without deformation. Therefore, the neutral axis shifts away from the centroid due to the applied shear (Figures 53 and 54).

In the calculation process, the author used a strain density approach. Compressive strains on both sides of the pile face and idealized beam strains over the pile stress transformation length (L_{pst}) are used to calculate "strain density" and determine displacements.

According to the author, the calculation is initiated by determining the internal shear and moments. They are derived by integrating stress concrete equations from Karthik and Mander (2011) model. In the equations, σ_{co} is the design (peak) compressive strength, ϵ_{co} is the strain that corresponds to the compressive strength, ϵ_{bf} is the strain at the bottom face of the cap, Y_{RB} is the location of the neutral axis measured from the bottom face of the cap, d_p is the pile depth and x is the distance from the neutral axis.

In this method, the Neutral axis location (501 steps) and strain values at the cap face (1500 steps) are systematically varied. This generates a matrix of 751,500 combinations. The combination with the smallest residual force (i.e., mismatch in internal vs external equilibrium) is considered the solution.

The translational displacement of the nodes of the pile centerline is calculated by comparing the area between the rigid-body strain profile and the idealized beam strain. This area represents the displacement at a given node (top of pile and bottom of cap). Then the rotation of the embedded part of the pile is obtained from the arctangent of the relative displacement between the top and bottom of the embedded pile segment divided by its length.

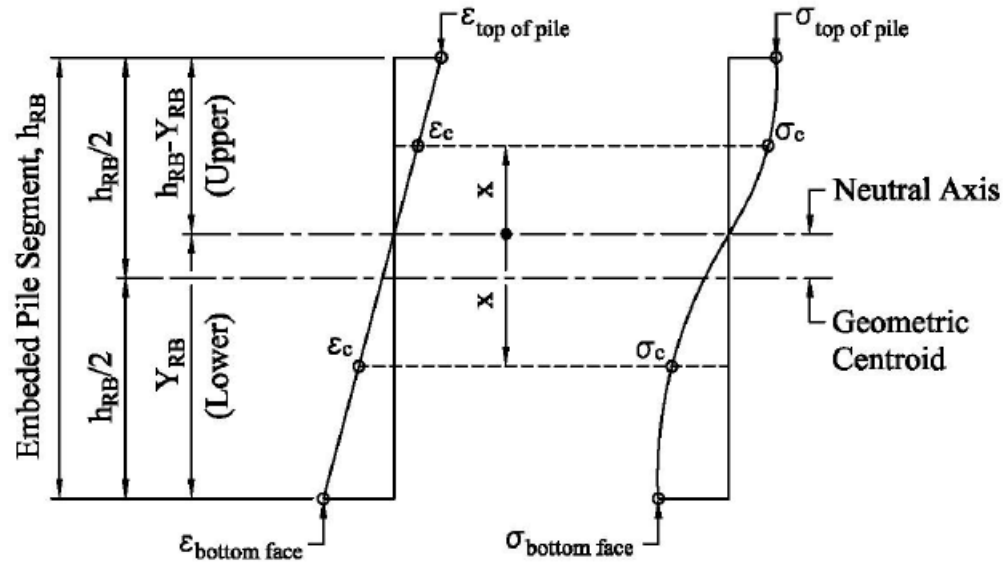


Figure 53. Method 4: Rigid Body Behavior - Deformations by Strain Density (Hammett, 2017).

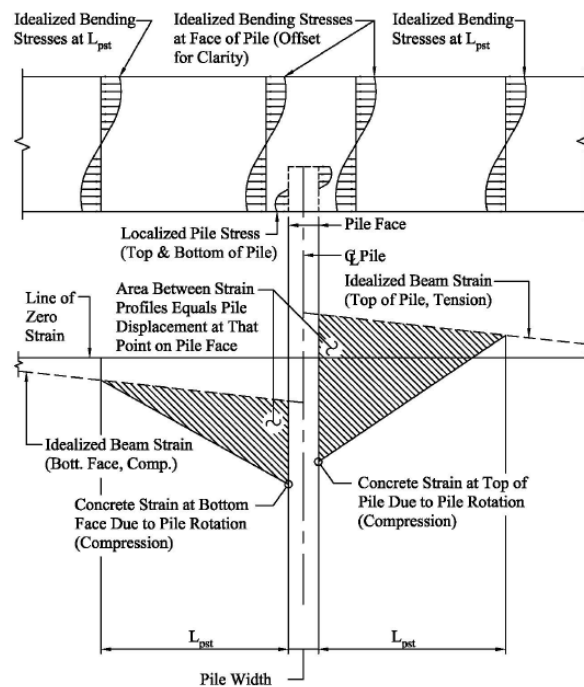


Figure 54. Illustration of Geometry Used for Strain Density Calculations in Method 4 (Hammett, 2017).

The results show how various factors influence the maximum moment supported by embedded steel pile connections. Increased embedment depth and higher concrete compressive strength generally lead to higher maximum moments, though increases plateau when the pile already reaches its yield moment. Larger pile sections significantly boost maximum moment capacity, with up to 97% increases between successive section sizes. Bending stresses in the concrete cap have negligible effect due to their relatively small magnitude compared to the stresses in the pile stress transfer block (PSTB). Finally, the choice of analysis method strongly impacts the maximum moment, with Method 4- assuming rigid body behavior-producing the highest values due to neglecting pile deformations. In contrast, Methods 1, 2 and 3, which model pile flexibility via different link elements, yield lower maximum moments, with Method 3 providing the highest among them. The results show the importance of accounting for modeling assumptions when evaluating pile-cap rotational performance and moment capacity. Comparison with full-scale tests shows Method 3 overestimates stiffness by 11% and 36%, suggesting it reasonably predicts realistic rotational stiffness values.

In conclusion, the author stated that Method 3 was the one that provided a better comparison of rotational stiffness, when compared to results in Rodas et al. (2017). As for predicting the flexural capacity of the connection, method 4 has the best results. When trying to determine neutral axis, all 4 methods appeared to be unable of accurately determining its position.

The author believes that, with a few adjustments, Method 3 is the best method to accurately predict rotational stiffness and flexural capacity of the connection. The author states that future works should include full-scale testing to facilitate necessary adjustments to method 3 and to also propose a simple multiplier (between one-quarter to one-third, multiplied by the embedment length) to determine the neutral axis location.

Tryon (2016)

The work by Tryon (2016) introduces the continuum model to calculate the rotational stiffness of embedded wide flange column connections, focusing on shallow embedment but extendable to all depths. Validated by experiments from Barnwell (2015) and Grilli and Kanvinde (2015), the model reasonably predicts elastic stiffness using

normalized plots of rotational stiffness vs. embedment length, showing diminishing returns beyond twice the column depth, consistent with previous studies (Castilla et al, 1984).

The research demonstrates that no connection is perfectly rigid, even at full fixity, and that shallow connections still exhibit measurable stiffness useful for design. In this work, the authors modeled the base plate as a rotational spring, allowing quantification of its contribution, on top of the embedment stiffness itself.

Importantly, the thesis extends Hetenyi's beam-on-elastic-foundation theory to vertical columns surrounded by concrete. For this, it proposes a modulus of subgrade reaction for concrete between 300-600 kips/in³, with 500 kips/in³ as a conservative value. The model is specific to wide flange steel shapes and should not be applied to other geometries or materials without further validation. Future work should include experimental testing and finite element modeling to refine and verify the assumptions.

4. Soil Structure Interaction

Most research on the pile head fixity problem has traditionally focused on the structural standpoint, mainly examining how the pile head's connection to the pile cap affects overall stability and load transfer. This approach often considers the pile head as either a fixed, pinned, or intermediate connection based solely on structural assumptions.

However, geotechnical aspects also have great influence in determining pile-to-cap connection capacity. The interaction between the pile, pile cap, and surrounding soil influences the connection behavior and, consequently, the distribution of forces and moments. Factors such as soil type, stiffness, density, depth of the pile embedment into the ground, and the pile's bearing conditions impact how the pile and cap interact under loads. Ignoring these geotechnical considerations can result in incorrect definition of pile head fixity, leading to overestimation of deflections or underestimation of moment transfer, for example.

A geotechnical factor that must be considered is scour, which can also influence fixity of the pile head inside the pile cap by influencing the overall structural behavior.

As scour removes soil around a pier or pile, the ability of the soil to resist lateral and vertical loads is reduced. As the unsupported length of the pile increases due to scour,

the pile behaves more like a cantilever, increasing the bending moment at the pile head. Even though the pile head is embedded into the cap with the assumption of fixity, excessive moment due to scour-induced load redistribution can lead to overstressing or cracking in the cap-pile interface, which might reduce the fixity.

Furthermore, the flexibility of the pile increases as the soil support decreases due to scour. This increased flexibility can cause more rotation at the pile head, which the pile-to-cap connection must accommodate. Over time, this could degrade the connection or reduce its effectiveness in maintaining full fixity.

According to Rollins and Stenlund (2010), most of the previous research has been conducted exclusively under lab conditions with the load applied at some distance from the base of the pile cap. Under field conditions the load is not all applied at one location but is distributed along the length of the pile due to soil-pile interaction. This factor, which may affect connection capacity, has not been extensively investigated.

Castilla et al. (1984)

Castilla et al. (1984), in a two-part study, analytically evaluated the degree of rotational fixity associated with different HP Steel piles, with different embedment lengths. The work's main objective was to determine the required pile embedment length to achieve a condition of full fixity of the pile-to-pile cap connection.

A common rule of thumb for steel HP piles design, at the time, was analyzed by the authors: embedment of 1 ft or less was considered pinned connection, embedment between 1 ft and 2 ft was considered partially fixed and embedment greater than 4 ft was considered fully fixed.

The authors performed an analytical study considering a cap-member system and another considering a cap-member-soil system. The work involved studying two types of steel piles: HP 14x117 and HP 14x73, considering embedment of 2 ft and 4 ft into the concrete cap, as seen in Figure 55.

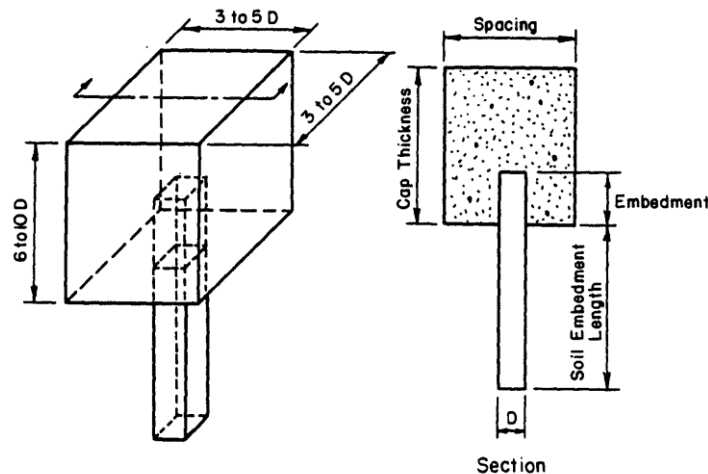


Figure 55. Model parameters and ranges of Castilla et al. (1984) study.

For the structural analysis (cap-member study), the authors developed finite element models, using ANSYS finite element computer program. The analysis focused on displacement and stress distribution, concluding that a moderately fine mesh was adequate for the study. Additionally, the impact of pile embedment depth on rotation at the concrete cap was analyzed, revealing that embedment depths greater than approximately $2D$ approached a limiting rotation value, suggesting partial fixity for shallower depths. The study also noted that concrete's low tensile strength and cyclic loading degradation were not fully represented in the models, leading to the incorporation of an interface element in subsequent analyses.

A parametric study was also conducted in this cap-member model to determine if other parameters would significantly affect the degree of fixity of the pile section. Four key parameters were investigated: coefficient of friction between the pile and concrete cap, effective width of the concrete cap, pile spacing, and concrete cap thickness. Results showed that increases in the coefficient of friction and concrete cap width significantly reduced pile displacement and rotation. Pile spacing and concrete cap thickness had minor impacts on pile behavior.

The authors, then, concluded that a pile embedment length equal to or greater than twice the pile width/diameter must be adopted for developing a condition of full fixity. As for partial fixity, the only statement made was that an embedment of 1 ft (or

approximately 1 time the width of the studied pile) developed about 61% to 83% of a fully fixed moment (4-ft embedment), indicating partial fixity.

In the second part of the study, the authors conducted a detailed parametric analysis focusing on how various factors influence the fixity of piles embedded in the concrete cap, using finite differences software COM622. This analysis examined the impact of the embedment length of the pile in the cap, the stiffness and type of soil, the shear force at the cap/soil interface, and the stiffness of the pile itself. The primary metric for assessing fixity was the rotational response of the pile section at the point where it meets the concrete cap.

The results showed that as the embedment/pile width ratio is increased, the rotation of the pile at the cap/soil interface decreases with an asymptotic behavior. After a L_e/B ratio of, approximately, 1.75 to 2, rotation would barely change, as seen in the example in Figure 56. Additionally, this figure shows how an increase in soil stiffness (coefficient of soil subgrade reaction, η_h or modulus of subgrade reaction, K_h) leads to a decrease in the rotation of the pile at the cap/soil interface for the same level of pile embedment. The stiffer the soil, the less rotation at a fixed condition. As observed, clayey soils tend to show less rotation than sandy soils.

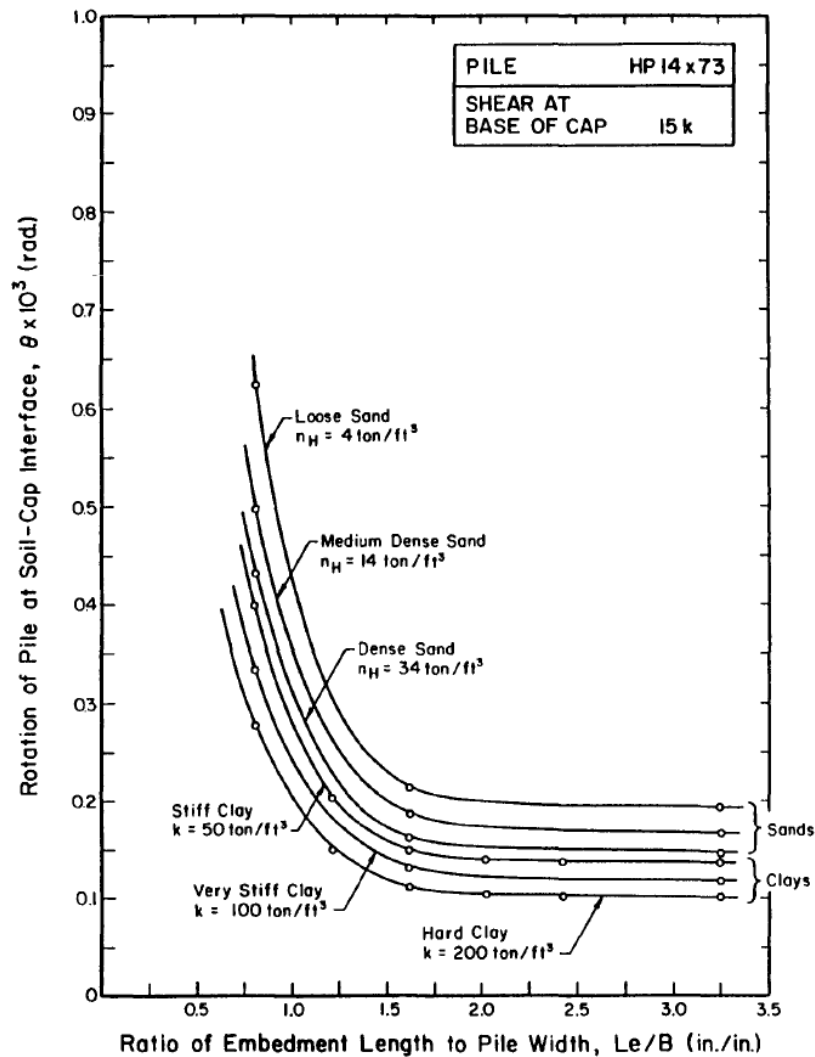


Figure 56. Effect of L_e/B ratio on pile rotation at soil/cap interface, HP 14x73 pile with 15 kip shear at base of cap (Castilla et al, 1984).

Also, as seen in Figure 57, a higher shear force at the cap/soil interface correlates with an increase in pile rotation and fixity. For a ratio of pile embedment length to pile width equal to 1.25, for example, the 5-kip shear curve is almost approaching its own asymptotic line, whilst the 25-kip shear curve is still far from it.

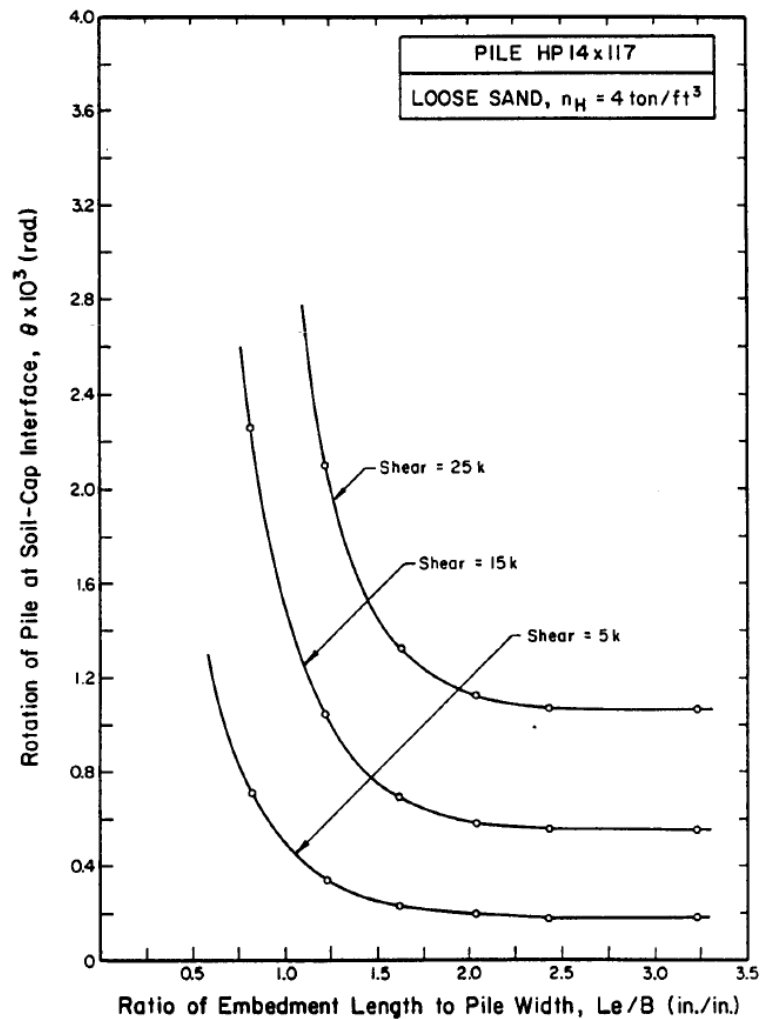


Figure 57. Effect of L_e/B ratio on pile rotation at soil/cap interface, due to change in shear at soil/cap interface, HP 14x117 pile in loose sand, $\eta_h = 4 \text{ ton/ft}^3$ (Castilla et al, 1984).

Finally, as the stiffness of the pile itself increases, the rotation at the interface decreases, as observed in Figure 58.

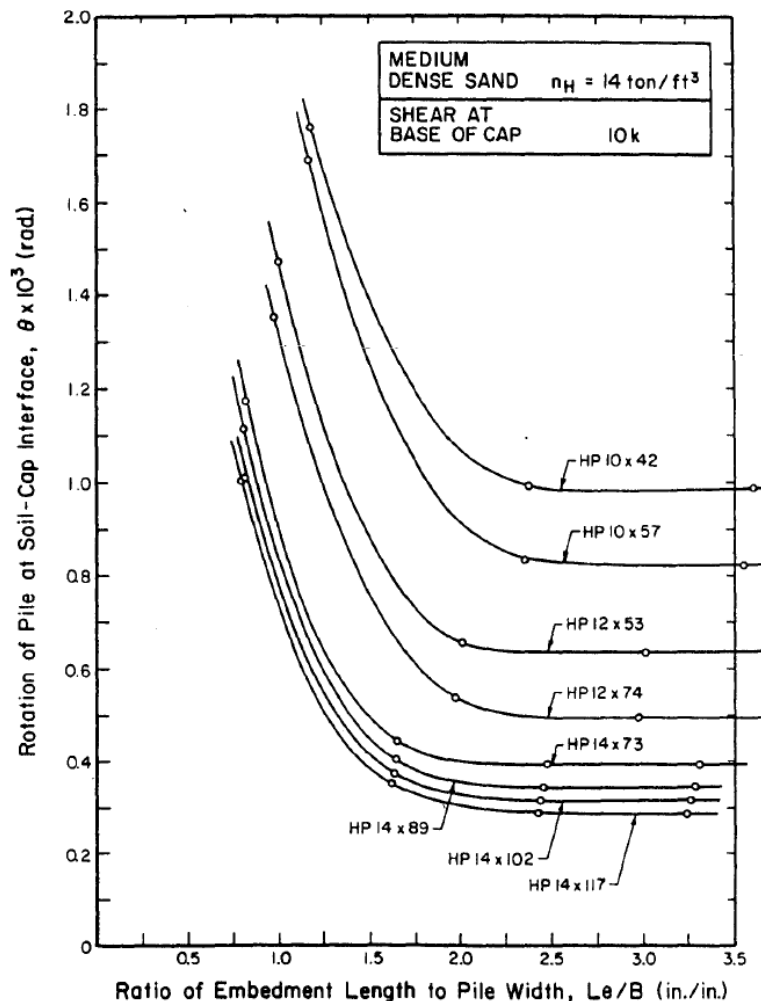


Figure 58. Effect of L_e/B ratio on pile rotation at soil/cap interface, for several pile sections (change in stiffness), in medium dense sand with 10-kip shear at the base of cap (Castilla et al, 1984).

The authors also used typical design shear forces at the cap base to compute maximum moment and cap displacement for different embedment lengths (1, 2, 3 and 4 ft) of the two types of steel piles (HP 14x117 and HP 14x73). It was found that the computed moments for 1 ft embedment were significantly lower than those for 4 ft embedment (61% to 83% of the 4-ft embedment moments). This suggests that the conventional “rule of thumb” that an embedment length of 1 ft or less results in a simply pinned pile may not be accurate, indicating that such piles might be already partially fixed.

Cheng and Sriitharan (2020)

Cheng and Sriitharan (2020) performed half-scale testing of a prefabricated column-pile cap-pile system at a cohesive soil site. In this setup, the system consisted of a precast bridge column, a precast pile cap, and eight steel piles, which were subjected to combined axial and lateral loads. The connections between components were made using socket connections preformed in the pile cap with corrugated steel pipes.

Under service loading conditions, no damage was observed in the test units. However, under seismic loading, plastic hinges formed in the column, allowing the system to maintain fixity while showing good structural behavior at the column and pile connections. The authors reported that flexible foundations accounted for 40% of the column's top displacement under service loads.

They concluded that, for effective performance, the embedment length of the column should be equal to its diameter to allow plastic hinge formation, while the pile embedment length should be 1.5 times the pile depth to maintain fixity in the pile socket connection.

5. Summary of Embedment Length Research

As observed in the previous sections, although many authors investigated the issue of pile to cap connection and degree of fixity, very few systematically associated embedment lengths to degrees of fixity. Most were simply concerned with making embedment recommendations to develop an assumed fixed-head condition. Others indicated that a certain shallow embedment length could not be considered pinned (as formerly considered), but no indication of degree of fixity was made.

Table 5 summarizes the literature reviewed so far as to what each author states regarding embedment length.

Table 5. Embedment recommendations from literature.

Research	Embedment recommendations	Notes
Steel HP Piles		

Castilla et al. (1984)	2*Pile depth for fixed head condition	• Work purely numerical.
Shama et al. (2002)	2-3*pile depth for fixed head condition (for usual values of concrete strength: 4, 5 and 6 ksi)	• Experimental and analytical work. • The authors themselves did not make any specific recommendations. The recommendations made here come from a simple parametric analysis of the authors' equation.
Cheng and Sritharan (2020)	1.5*pile depth for fixed head condition	• Experimental work. • Embedded portion of pile was reinforced with corrugated steel socket.
Steel Pipe Piles		
Rollins and Stenlund (2010)	2*pile diameter of plain embedment for fixed-head condition (or 1*pile diameter with special rebar reinforcement)	• Experimental work. • For 12-in diameter piles. • Authors suggest that, for diameters other than 12 in, a conservative estimate can be made with Marcakis and Mitchell (1980) equation.
Prestressed Concrete Piles		
Shahawy and Issa (1992)	50 in (or ~4.1 ft or ~3.5*pile width) for fixed-head condition	• Analytical and experimental work. • Necessary embedment for development of the prestressing strands.
Issa (1999)	48 in (or 4 ft or ~1.6*pile width) for fixed-head condition	• Analytical and experimental work. • Necessary embedment for development of the prestressing strands. • Embedded region reinforced with steel pipe, inserted into the central void.
Harris and Petrou (2001)	1*Pile width for fixed-head condition	• Experimental work. • Minimum of 1 ft.

Teguh et al. (2005) and Fuziol (2007)	1.2 to 1.5*pile width for fixed-head condition	- Numerical analysis. - Development of exposed strands with round reinforcement around the strands.
Larosche et al. (2014a, b)	1.3*pile width (± 3 in) for fixed-head condition	- Experimental work. - Plain embedment of the prestressed concrete pile.

6. Summary of Embedment Recommendations from Departments of Transportation

According to Zapata et al. (2022) and Rakestraw et al. (2024), twenty-four states specify a required pile embedment length into the footing or pile cap. Of these, three states (Florida, Minnesota, and Wisconsin) require an embedment length of 0.5 or 1.0 feet for pinned connections. Six states (Florida, Wisconsin, Colorado, Illinois, New Hampshire, and Oregon) specify an embedment length for fixed connections, ranging from 1.0 to 4.0 feet. In Colorado and Oregon, the required embedment length is calculated based on the pile's plastic moment capacity about the strong axis, concrete compressive strength, and pile width. The remaining 17 states that mandate an embedment length do not indicate whether the detail results in a pinned or fixed connection behavior.

AASHTO (2020) recommends a minimum embedment of piles into pile caps to be no less than 1 foot. For special cases, 6-in embedment is allowed.

The Table 6 summarizes all southern states (as defined by the United States Census Bureau) recommendations concerning pile embedment into the pile cap.

Table 6. Embedment details from DOTs

State	Description	Source
South Atlantic Division		
Delaware, DelDOT	-Dowel bars used for connection with precast piles - For Steel H-piles, a minimum embedment length of 1 foot is required	DelDOT (2017)

Maryland, MDOT SHA	- Drawings show that, for any kind of pile, it must be embedded 1 ft into the cap	MDTOSHA (2020)
District of Columbia, DDOT	No specifications	DDOT (2023)
West Virginia, WVDOT	- Minimum embedment length of 1 ft.	WVDOT (2016)
Virginia, VDOT	- For prestressed concrete piles only: -----top of the pile must be embedded a minimum of 6 in into the cap and, additionally, the strands must extend 34 in beyond the pile, into the cap.	VDOT (2024)
North Carolina, NCDOT	- For bent caps smaller than 4 ft, recommended embedment length of 12 in, for all kinds of piles. - For bent caps larger than 4 ft and Integral end bents, recommended embedment length of 24 in, for all kinds of piles. - For general pile footings, abutment and retaining walls, recommended embedment length of 9 in for Steel H-piles and 12-in prestressed concrete piles, and embedment length of 12 in for Steel pipe piles and prestressed concrete piles larger than 12 in.	NCDOT (2023)
South Carolina, SCDOT	- Embedment length equal to the pile with. For steel pipe piles, 1.25 ft minimum.	SCDOT (2006)
Georgia, GDOT	- Any kind of pile: ----- Piles up to 20 inches in width or diameter: embed the pile 1 foot into the pile bent	GDOT (2024)

	---- Piles larger than 20 inches: embed the pile 1.5 ft in to the pile bent	
Florida, FDOT	- Specifications only for prestressed concrete piles. - 4-ft embedment for prestressed concrete piles to achieve full bending capacity. No mention is made about the connection capacity or required embedment for full fixity. In this case, moment capacity or strand development length will control failure. - For pinned conditions, the guidelines specify a 1-ft embedment and strands development length must follow AASHTO LRFD Bridge Design Specification (2020).	FDOT (2023) and AASHTO (2020)
East South-Central Division		
Kentucky, KYTC	No specifications	KYTC (2024)
Tennessee, TDOT	-Specifies for Steel H-piles and pipe piles: ---- Use of Marcakis and Mitchell (1980) equation for determining pile embedment, with a minimum of 1 ft	TDOT (2023)
Mississippi, MDOT	No specifications	MDOT (2010) and MDOT (2016)
Alabama, ALDOT	No specifications	ALDOT (2023)
West South-Central Division		
Arkansas, ArDOT	- For footings and non-integral bent caps: -----Any kind of pile smaller than 18 in, minimum embedment of 12 in. -----Any kind of pile larger than 18 in, minimum	ArDOT (2020)

	embedment of 15 in. - For integral bent caps: -----Any kind of pile, minimum embedment of 18 in.	
Oklahoma, ODOT	A few drawings show piles embedded 10 in into caps, without specifying if it is a requirement or not.	ODOT (2016)
Texas, TxDOT	- 1-ft embedment for 16-in and 18-in square concrete piles and Steel HP14 and HP16 piles - 1.5-ft embedment for 20-in and 24-in square concrete piles and Steel HP18 piles.	TxDOT (2019)
Louisiana, LaDOTD	No specifications	LaDOTD (2020)

Chapter 3: Bridge Bents

In the previous chapter, a comprehensive literature review on behavior of pile-to-cap connections was performed. However, literature focused almost entirely on individual behavior of pile-to-cap connections. This research, though, aims to investigate how this can be applied to pile-to-bent cap connections.

1. Background

A bridge bent is an intermediate support between bridge spans that transfers and resists vertical and lateral loads from the superstructure to the foundation (Caltrans, 2022). The bent cap beam supports the longitudinal girders and transfers the loads to the bent columns/piles (Figure 59).

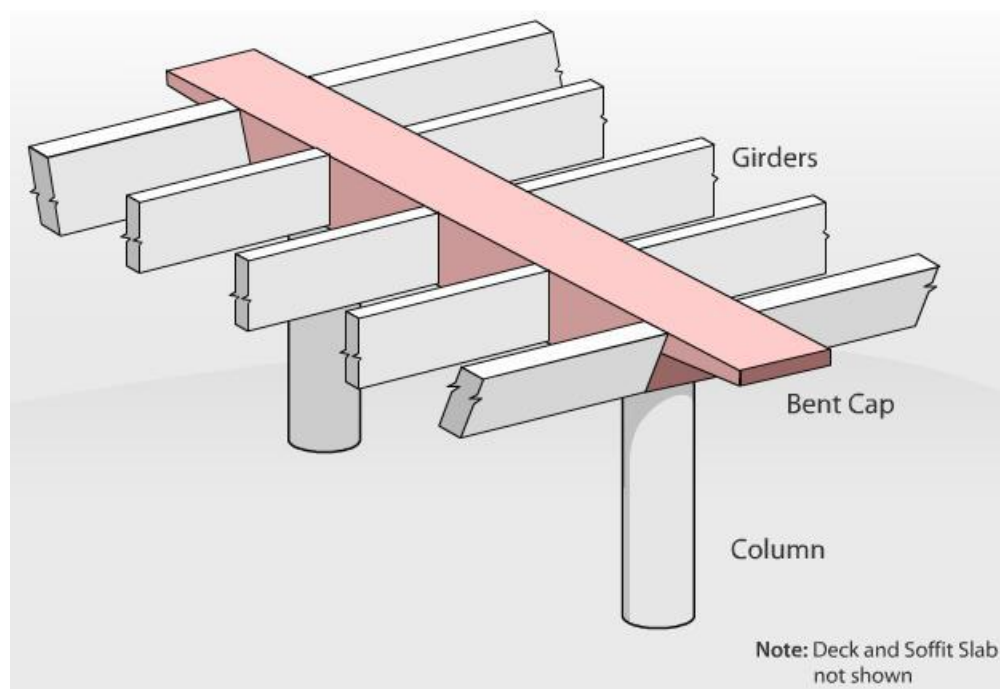


Figure 59. A typical integral bridge bent (Caltrans, 2022).

The construction of bridge pile bents, supported by driven piles, typically involves the following steps, illustrated in Figure 60.

- a) Piles are driven to a sufficient elevation below ground level, based on calculated end bearing and side friction capacities.
- b) After all piles have been driven, the tops of the piles are cut off, so all the pile heads are at the same elevation above the ground.
- c) Formwork and reinforcement are placed around the piles to construct the cast-in-place bent cap.



Figure 60. Construction steps of a pile bent: a) piles driven into the ground; b) piles heads cut off and formwork placed; c) formwork and reinforcing cage; d) finished pile bent after pouring concrete and removing formwork (Skinner, 2019)

In the state of Alabama, bridges may be supported in different ways: single column piers (usually called “hammerhead” pier), multiple columns connected to footings (for longer than 50-ft bridge spans) or pile bents (for shorter than 50-ft bridge spans) (ALDOT, 2023).

According to Marshall et al. (2017), the multi-span bridge with simply supported precast-prestressed concrete girders and intermediate bents consisting of driven steel piles and a reinforced concrete bent cap is widely used in the state of Alabama. A typical pile bent can be seen in Figure 61.



Figure 61. Typical Pile Bent with Galvanized Steel HP Piles and Reinforced Concrete Cap (Marshall et al, 2017).

Marshall et al. (2017) state that typical intermediate pile bents are constructed with either precast concrete or steel piles, which are driven into the ground using hydraulic, diesel, or air hammers. The exterior piles are often battered at a 1.5:12 slope. Steel HP piles are among the most used sections in pile bent substructures. These piles have flanges and webs of equal thickness, which helps reduce the driving stresses they endure (Bowles 1996).

To prevent section loss, steel HP piles are often encased in non-structural concrete, with encasements shaped either circular or square. Alternatively, ALDOT has used galvanized piles for corrosion protection, as seen in the bent in Figure 61, which consists of galvanized steel HP piles. A key element of a typical pile bent is the cast-in-

place reinforced concrete bent cap, poured after the pile driving process. The pile tops are typically embedded at least 12 inches into the bottom of the bent cap.

2. Load Testing of Bridge Bents

This section describes a series of historic load tests performed in pile bents with steel H-piles.

2.1 Marshall et al. (2017) and Anderson et al. (2018)

A series of load tests were conducted in bridge bents in the state of Alabama. Marshall et al. (2017) conducted 7 lateral load tests in different bridge bents. Four tests were conducted on a bridge in Alabama County Road 9, Macon County (Shorter, AL). Two tests were conducted in a bridge in US Highway 331, south Montgomery, AL. The two final tests were performed on two bridge bents, specifically built to be tested to failure, in the Auburn University National Experimentation Site (AUNGES) (Anderson et al, 2019), Lee County, Opelika, AL.

Their work was motivated by a previous observation that steel pile bridge bents with battered piles and no sway bracing usually experimented higher than expected forces by ALDOT engineers. The purpose of this work, then, was to investigate the behavior of steel pile bridge bents under lateral loading conditions. A few observations regarding pile embedment could be drawn from this work.

Macon County Road 9 Bridge, Shorter, AL

The first test on Alabama County Road 9, Macon County (Shorter, AL) was conducted on Bent 5 of a multi-span bridge over Old Town Creek (Figure 62). The bridge consists of six 40-foot spans, each supported by four HP14x89 steel piles, 31.5-ft long, spaced 8 feet apart, with the two exterior piles battered at a 1.5:12 slope. Some of the piles were galvanized, while others were encased in 30-inch diameter non-structural concrete. The piles had a clear height of 5 feet, and each pile was embedded one foot into the cast-in-place bent cap (approximately $0.85 \times \text{pile depth}$).



Figure 62. Field test setup on bridge over Old Town Creek, Alabama County Road 9, Macon County (Shorter, AL) (Marshall et al, 2017).

The bent was instrumented with electrical resistance strain gauges to measure axial forces and bending moments on both the steel piles and the concrete encasements. The strain gauges were placed on both sides of the web for weak axis bending and at different sections above and below grade to provide a moment versus depth profile. Additionally, draw wire sensors (wirepots) were installed on the bent cap to measure total lateral deflection during the test. This comprehensive instrumentation helped monitor strain, load, and displacement throughout the test.

A static lateral load test was performed. The bent was loaded incrementally, first in 10 kip increments up to 60 kips and then in 5 kip increments up to a total lateral load of 75 kips. Load increments were held for five to ten minutes, and the system was then unloaded in two to three increments. Hydraulic jacks (with a 60-kip capacity) were used to apply the lateral load, and high-strength steel threaded rods anchored the system.

The results of the test showed that the bridge bent exhibited high stiffness. The piles experienced bending moments with the highest values at the pile heads, and these moments dissipated as depth increased, largely due to load transfer from the encased concrete to the ground. Strain profiles revealed that the steel piles and concrete encasements did not behave fully composite, which could be attributed to cracks in the concrete caused by the steel pile restraint. The lateral deflection was minimal, with the bent deflecting only 0.2 inches at the maximum load of 75 kips. This stiffness was attributed to the short free height of the piles and the significant contribution of the concrete encasements to the overall structural stiffness.

Regarding the pile embedment into the bent cap, the connection between the pile and the bent cap displayed substantial moment capacity, contributing significantly to the structural stiffness of the bent. Despite the non-perfect composite action between the steel and concrete, the bent performed well under lateral loads, demonstrating behavior consistent with a fixed-head condition where the maximum bending moment occurs at the pile cap connection.

Three additional lateral load tests were, later, performed in the same pile bent, under different conditions. The first was after the placement of a bridge deck roadway on top of the girders, but no extra gravity loads were applied. The second was with the addition of gravity load (ALDOT standard truck) centered over the roadway, above the tested pile bent. The third (final test) also had a gravity load, but the truck was positioned at the edge of the roadway (also above the tested pile bent). According to the authors, the pile bent also behaved extremely stiff for the three cases. It was observed that the addition of gravity load did have an effect on the magnitude of the bending moments at the instrumented pile sections six inches from the bottom of the cap. The maximum bending moments were higher in each of the loading conditions with additional gravity load compared to the lateral load test without additional gravity load which is to be expected due to the increased flexural stiffness of the piles due to the larger compressive force.

Regarding connection capacity, a simple analysis can be performed based on the findings of Marshall et al. (2017).

In the four lateral load tests performed - (1) pile bent with girders only, (2) pile bent with the bridge deck but no additional gravity load, (3) pile bent with the bridge deck and a centered gravity load, and (4) pile bent with the bridge deck and an off-centered gravity load - it was consistently observed that Pile 4 experienced the highest moment demand at the instrumentation level of 1.5 ft (6 inches below the cap)

For the lateral test involving the pile bent with girders only, Pile 4 registered a maximum measured moment of 171 kip-ft at the instrumentation level of 1.5 ft (6 inches below the cap), close to the bottom of the cap.

The structural moment of the HP14x89 pile, calculated along its weak axis, is 253 kip-ft (limit state of flange local buckling), calculated according to AISC manual (2023). This calculation conservatively ignores the contribution of the concrete encasement. Additionally, applying the equation by Marcakis and Mitchell (1980), the pile-to-cap connection is determined to have a moment capacity of 329 kip-ft. This indicates that the pile reaches its structural moment capacity before the connection itself reaches its full capacity. In this case, Pile 4 developed approximately 67% of its structural moment capacity.

The pile bent shows behavior characteristic of a fixed-head condition, as the shape of the bending moment profile during the load test closely matches the theoretical profile for a pile with fixed-head behavior, where the maximum bending moment occurs at the top of the pile. This behavior was consistent across all four piles. However, no more conclusions can be drawn since no failure was achieved.

For the other three tests (pile bent with no additional gravity load, with centered gravity load, and with off-centered gravity load) the same pattern emerged for Pile 4, as well as for the other piles. The maximum moment achieved by Pile 4 was 49%, 59%, and 52% of its plastic moment, respectively, in these tests.

US 331 Bridge, Montgomery, AL

Two lateral load tests were conducted on the southbound bridge over Catoma Creek on US Highway 331, south of Montgomery, AL (Figure 63). These tests aimed to assess the bridge bent's performance under combined gravity and lateral loads, as well as lateral load only.

The bridge comprised pile bents, each with six driven HP10x42 steel piles, 40-ft long, encased in 16-inch square concrete encasements. The bents included a trapezoidal reinforced concrete cap identical in geometry to the cap used in the Macon County load tests, with approximately 11 feet of clear height from the cap bottom to the ground line. The two exterior piles were battered at a 1.5:12 slope, and each pile was embedded 1 foot into the bent cap (approximately $1.2 \times \text{pile depth}$).

Instrumentation consisted of strain gauges installed on three sections of the concrete encasements above ground level to capture axial forces and bending moments in the piles. Due to limited access, only the concrete encasements were instrumented, not the steel piles directly. Additionally, draw wire sensors (wirepots) were placed on the bent cap to measure total lateral deflection during the tests.

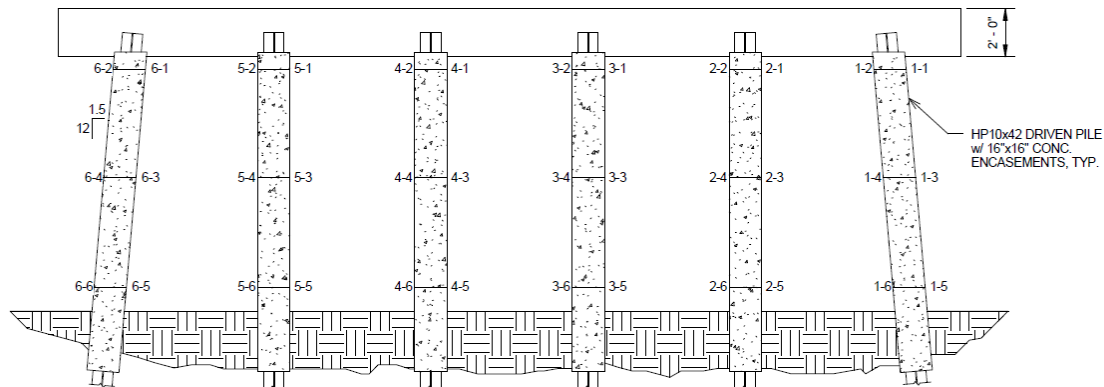


Figure 63. Test bent on southbound bridge over Catoma Creek, on US Highway 331, south Montgomery, AL (Marshall et al, 2017).

The southbound bridge bent was tested using the northbound bridge as a reaction frame. Two tests were performed: a combined gravity and lateral load test that included a live load (ALDOT standard truck) positioned directly above the test bent at the edge of the roadway, and a lateral-only test, which included only the bridge deck and girders, without any live load. Both tests applied lateral load in 10-kip increments up to a maximum load of 90 kips.

The combined gravity and lateral load test showed that overturning behavior occurred, where the exterior pile on the jack end experienced significant axial tension, while the opposite exterior pile exhibited axial compression. Interior vertical piles primarily resisted through flexure, contributing minimal axial load. In the lateral-only test, the exterior piles again carried most axial load due to their batter, with a noticeable difference between certain piles: Pile 6 had greater compressive force than the tensile force in Pile 1, which demonstrates the effects of overturning.

Bending moment profiles during the combined load test initially increased due to the live load but shifted from positive to negative as lateral load rose, particularly around the 30-50 kip range. The observed maximum moments were relatively low, around 14.5 kip-ft, located six inches below the cap. The disparity in stiffness between the steel piles and concrete encasements caused non-composite behavior, limiting the steel piles' contribution to bending moment. In the lateral-only test, bending moments matched those from the combined load test, peaking at the bottom of the cap with values between 11 and 20 kip-ft.

In the reaction bent, bending moments were lower than in the test bent, with the highest axial compression occurring in the exterior battered pile (Pile 7). Axial forces in the reaction bent also indicated overturning behavior, with Pile 7 experiencing the most significant compression relative to the other piles.

Deflection measurements reflect the bent's stiffness, showing less than 0.1 inches of deflection in both tests. This minimal deflection reflects the multiple piles' cumulative effect and the load-sharing with adjacent bents. The load-deflection response remained linear at service-level loads, with only a slight stiffness increase when additional gravity loads were applied, suggesting that service-level gravity loads had little impact on lateral stiffness.

Bending moment profiles suggested a fixed-head condition at the pile-to-cap connection, as maximum moments concentrated near the cap's bottom and decreased linearly along the pile height. The inflection point's location near the ground surface supports the fixed-head assumption. While direct monitoring of the steel piles was limited, the test response closely aligned with theoretical fixed-head behavior for pile bents.

Again, in a similar manner as Macon County results, regarding connection capacity, a simple analysis can be performed based on the findings of Marshall et al. (2017).

For both tests (combined lateral and gravity load test, and lateral-only test) pile #5 was the most demanded, concerning bending moment at the maximum lateral load level. Considering that pile HP 10x42 has a structural moment capacity of 81 kip-ft, is possible to observe that pile #5 developed approximately 19% and 24%, respectively, of its own structural capacity. The connection capacity, calculated according to Marcakis and Mitchell (1980), is approximately 331 kip-ft. This means that the pile will yield long before the connection itself.

Again, it is possible to conclude that the pile bent exhibits behavior characteristic of a fixed-head condition, as the shape of the bending moment profile during the load test closely matches the theoretical profile for a pile with fixed-head behavior, where the maximum bending moment occurs at the top of the pile. However, again, no failure was achieved.

AUNGES, Lee County, Opelika, AL

Two more bridge bents were built for the sole purpose of loading to failure. The bents each featured four HP12x53 piles, 35-ft long, driven 20 ft into the ground. The bents were 3-ft x 3-ft in cross section and were designed to have 9 feet of clear pile length above the ground level. The concrete used was tested and had a compressive strength of approximately 5,200 psi to 5,500 psi.

The two test bents were built with specific configurations. One bent with entirely vertical piles, each embedded 18 inches into the pile cap (Figure 65). The other bent followed ALDOT standard practice: it included two vertical piles and two battered piles angled at a 1.5/12 slope (Figure 64). The piles were embedded 12 inches into the cap. The pile bents were aligned to bend along the weak axis under lateral loading, with caps designed to resist structural failure. Strain gauges and inclinometers were installed along various pile depths to capture strain data and deflection.



Figure 64. AUNGES battered test bent (from left to right: piles 1, 2, 3 and 4), in Lee County, Opelika, AL. Horizontal load applied from right to left. (Marshall et al, 2017).



Figure 65. AUNGES vertical bent test (from left to right: piles 8, 7, 6 and 5), in Lee County, Opelika, AL. Horizontal load was applied from left to right (Marshall et al, 2017).

The piles were instrumented with strain gauges for measuring axial and bending moment at three cross sections along the depth of the piles: 6 in below the cap bottom, 42 in below the cap bottom and 84 in below the cap bottom. Additionally, an inclinometer was used to measure the deflection of the piles along their lengths.

The battered load test was performed up to a maximum lateral load of 140 kips, during which several key structural responses were observed as the load progressively increased.

At 100 kips, flexure cracks began to form in the bent cap, originating above Pile 2 and extending toward Pile 1. This cracking indicated the significant bending stress within the cap as it resisted the applied lateral forces. As the load increased to 110 kips, localized yielding became evident in the pile flanges at the base of the bent cap, where bending moments were highest. Mill scale flaked off from the surface, signaling the onset of steel yielding under stress.

By 130 kips, additional shear cracks had appeared alongside the initial flexure cracks, particularly around Pile 2, where combined bending and shear stresses intensified. These cracks followed the reinforcement path within the cap, illustrating the way the load was distributed through the bent structure. At the maximum applied load of 140 kips, significant buckling developed in the flanges of Piles 1 and 3, between 5 and 7 inches below the bent cap. This buckling indicated that these sections had reached their load capacity, undergoing plastic deformation, a critical point that marked the potential onset of structural failure.

At this 140-kip load, the lateral load test reached its limit as deflection increased dramatically to approximately 6.75 inches, with a residual drift of 4.5 inches after load removal. The jack's stroke capacity was fully utilized, preventing any further loading. The figures 66 to 69 illustrate the lateral load test results, axial loads, bending moment diagrams, and inclinometer profiles. It is important to note that, as the bent approached failure, the gauges in the upper section began to fail, due to the buckling in the pile flanges. Therefore, the plots only present results up until the 80 kips lateral load stage.

The results show that the piles achieved moments very close to the plastic moment capacity ($M_p \approx 129$ kip-ft), which is far below the connection capacity, calculated according to Marcakis and Mitchell (1980), of approximately 186 kip-ft, for a 12-in embedment.

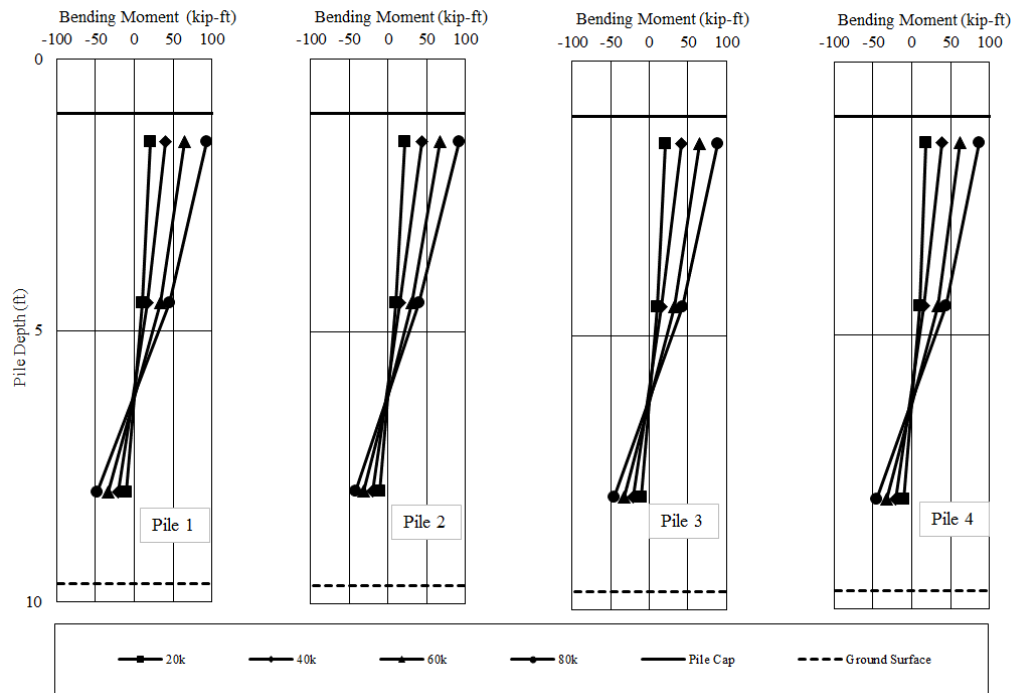


Figure 66. AUNGES battered bent test, Bending moment diagrams (Marshall et al, 2017).

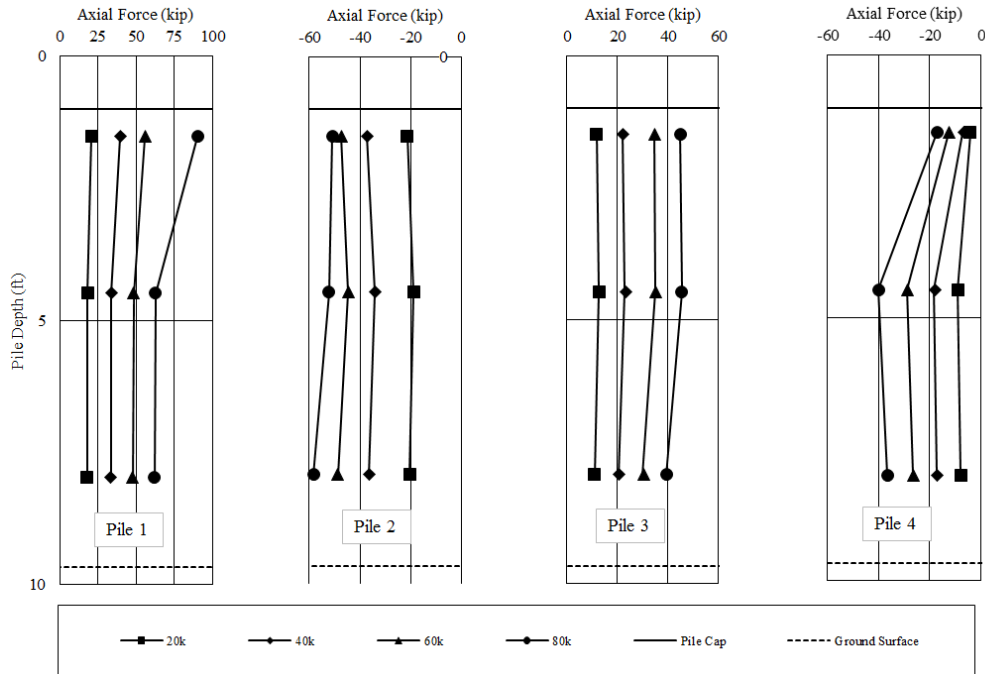


Figure 67. AUNGES battered bent test, Axial results (Marshall et al, 2017).

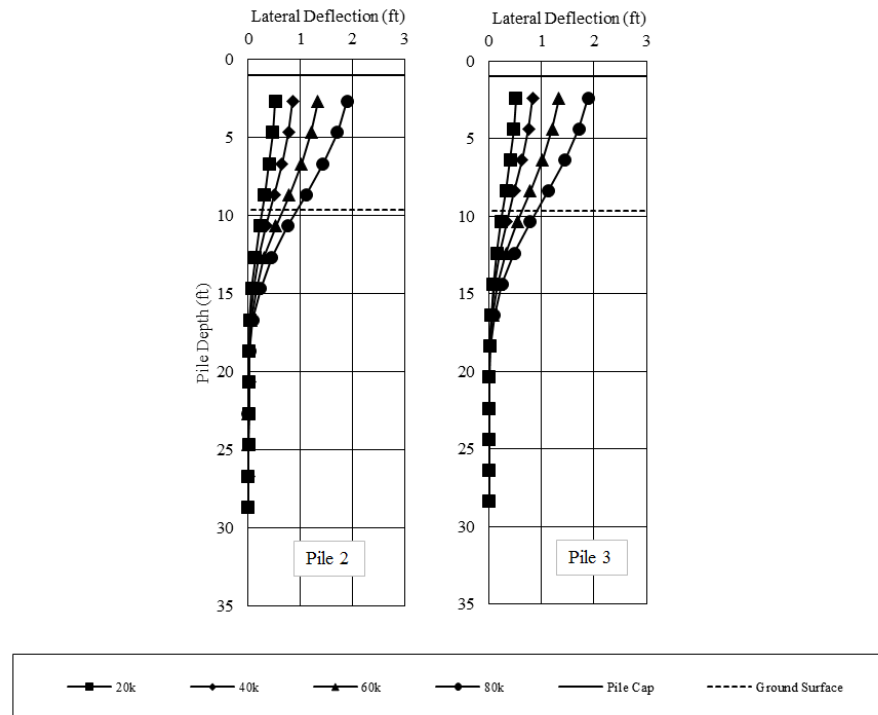


Figure 68. AUNGES battered bent test, Inclinator results (Marshall et al, 2017).

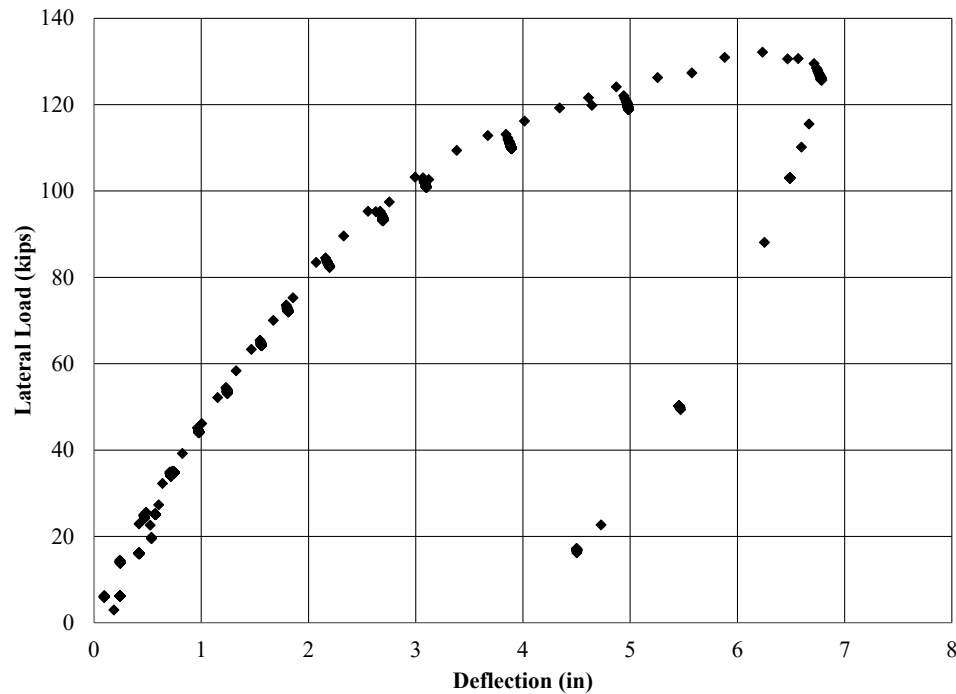


Figure 69. AUNGES battered bent test, Wire Pot load deflection behavior (Marshall et al, 2017).

The vertical bent load test proceeded with 10-kip increments until structural failure or the jack capacity was reached. Inclinator readings were captured at every 20-kip load increment and at each unloading step to monitor residual deformations. The test reached a maximum load of 140 kips, at which point the piles experienced localized yielding, especially in the flanges up to 5-7 inches below the bent cap. The deflection of the bent cap increased nonlinearly once yielding began, and some strain gauges failed under excessive strain (especially after the 80-kip stage). Ground cracks opened up around the bent, indicating substantial lateral energy transferred into the soil, and residual drift was observed.

The test results included moment and axial force profiles, inclinometer readings, and load-deflection behavior captured by wire pots (see figures 70 to 73).

The results show that the piles achieved moments very close to the plastic moment capacity ($M_p = 129$ kip-ft), which is far below the connection capacity, calculated according to Marcakis and Mitchell (1980), of approximately 398 kip-ft, for an 18-in embedment.

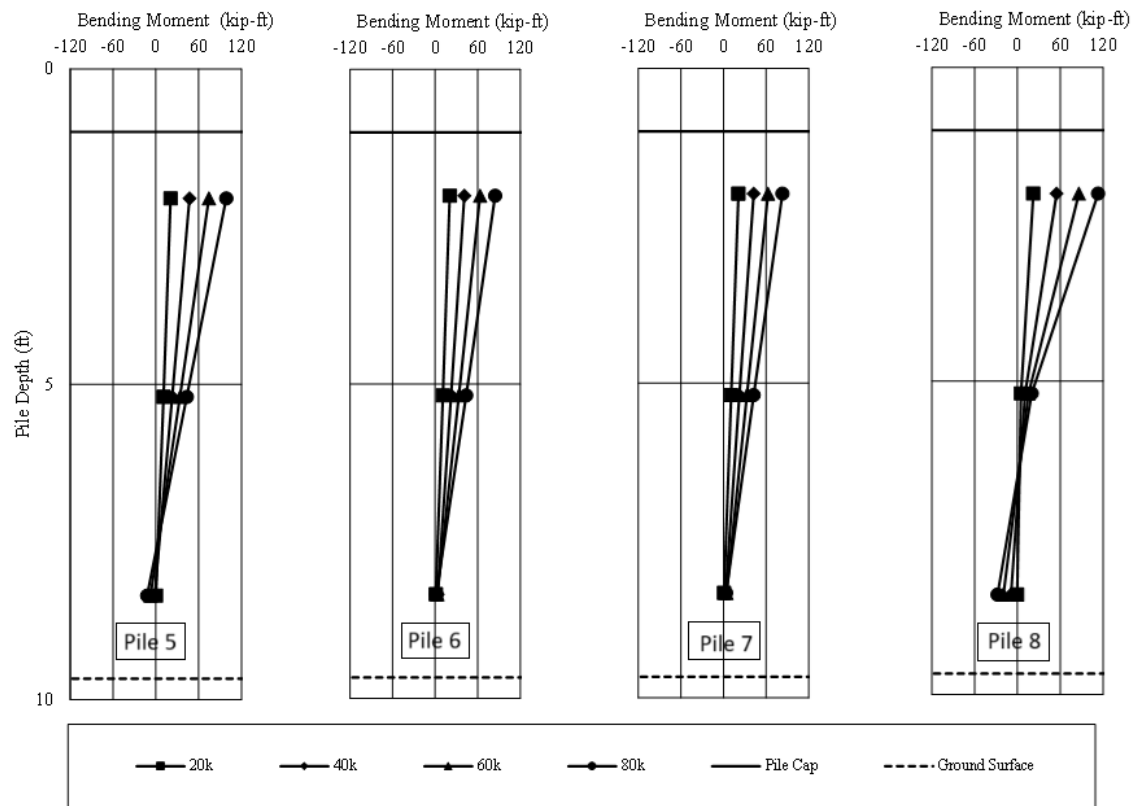


Figure 70. AUNGES vertical bent test, Bending moment diagrams (Marshall et al, 2017).

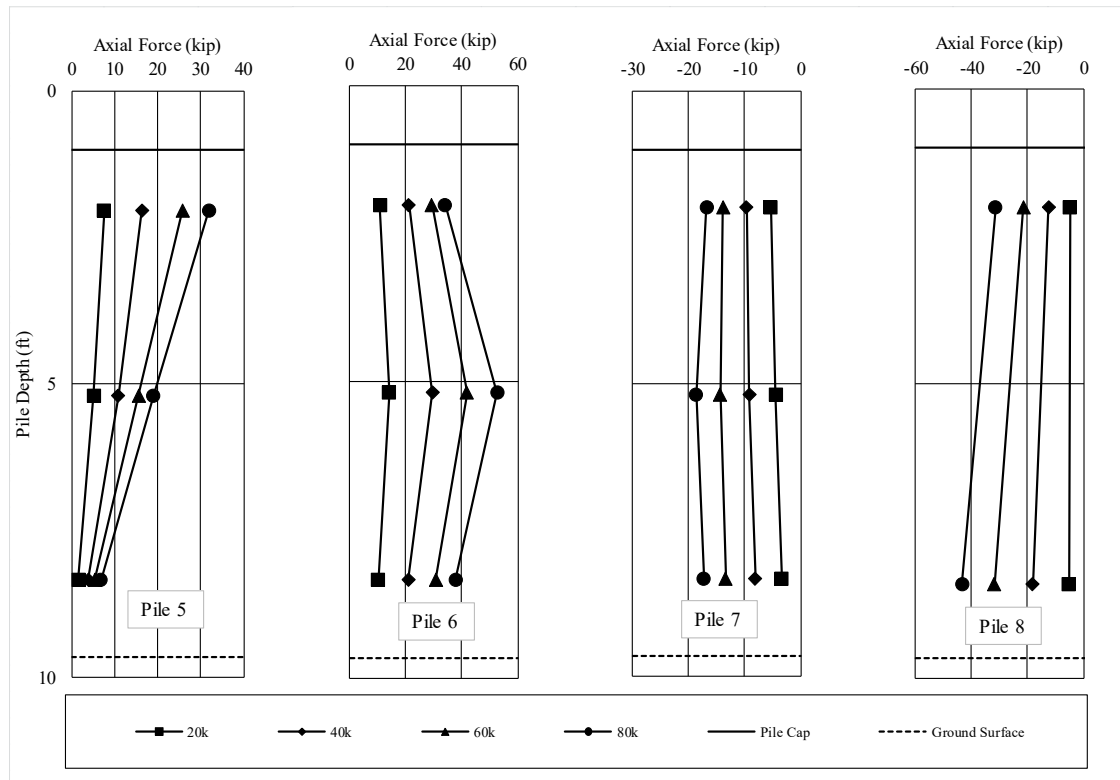


Figure 71. AUNGES vertical bent test, Axial results (Marshall et al, 2017).

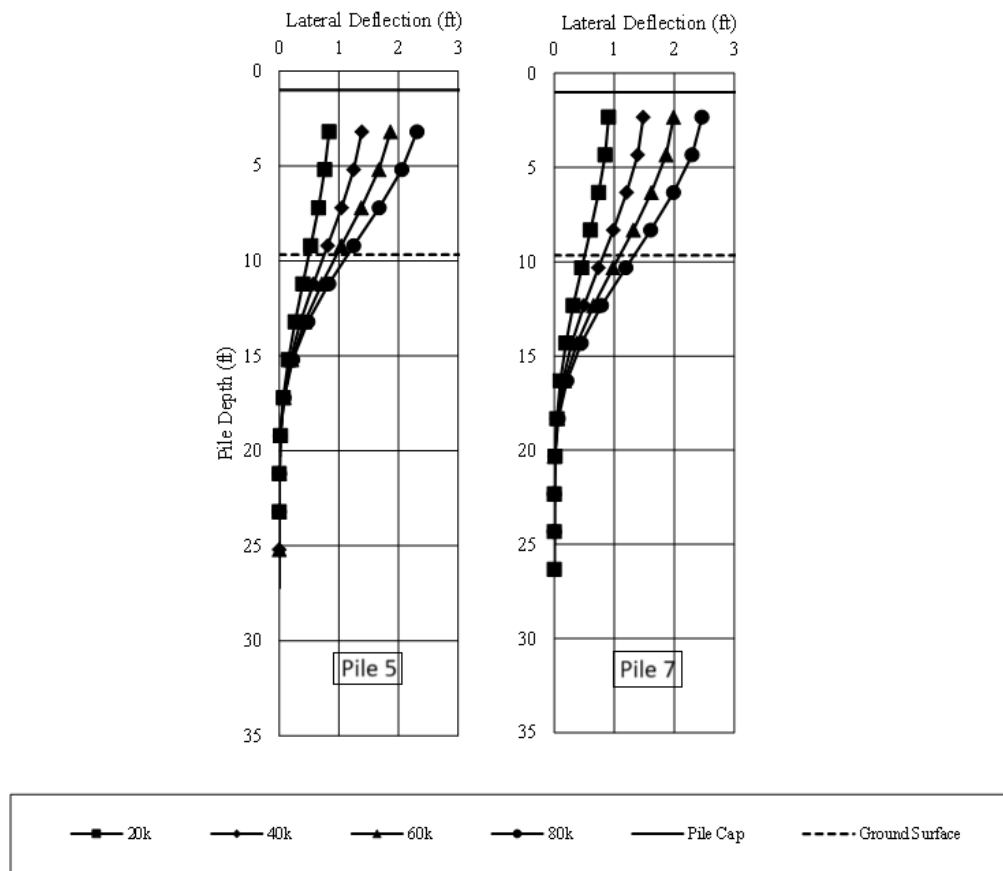


Figure 72. AUNGES vertical bent test, Inclinator results (Marshall et al, 2017).

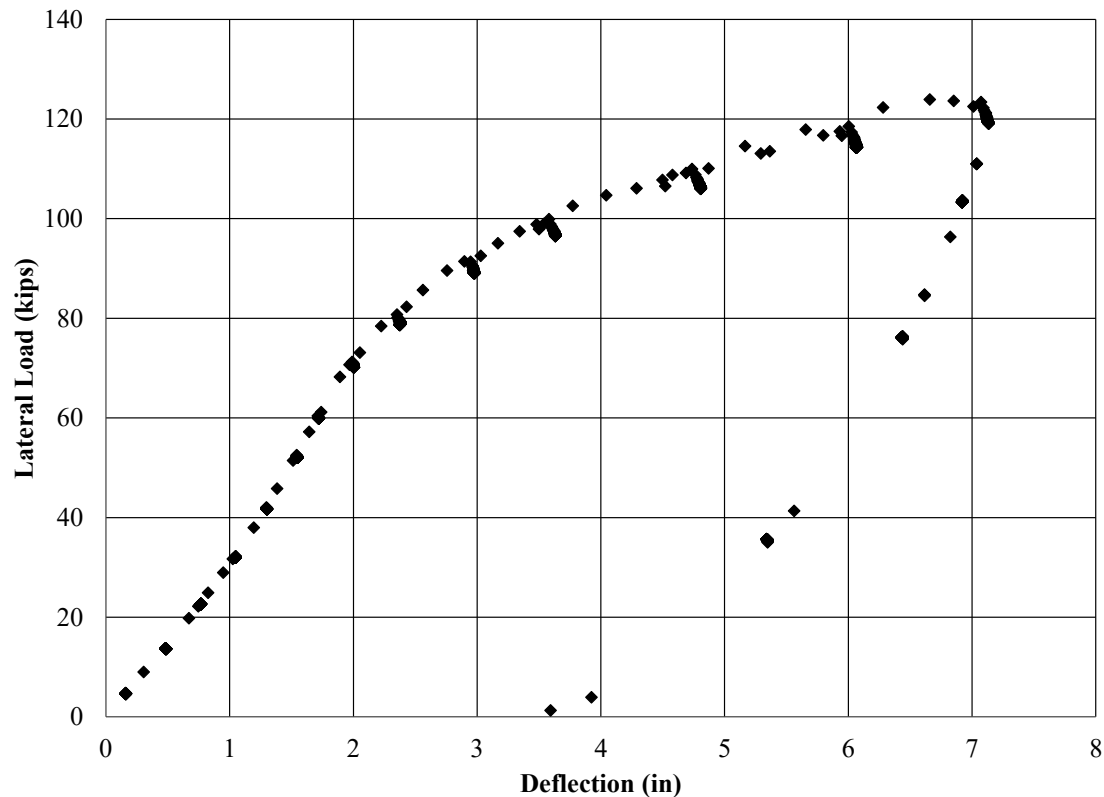


Figure 73. AUNGES vertical bent test, Wire Pot load deflection behavior (Marshall et al, 2017).

In summary, the authors concluded that the vertical bent only experienced yielding of the piles while the battered pile experienced both pile yielding and significant cap cracking above Pile 2. This cracking is due to the tension the pile experienced due to the battered pile in compression wanting to rotate to vertical causing tension in Pile 2 and compression in Pile 3 which is opposite from what is expected and what occurred in the vertical bent. Both the pile bents experienced significant soil deformation and cracking at the ground level. Additionally, it should be noted that the magnitude of the axial forces in the piles are higher in the battered bent.

2.2 Marshall et al. (2021)

Marshall et al. (2021) expanded on the previous research and focused on the distribution of braking forces in steel H-pile bent bridges. The authors performed more

experiments on bridge over Old Town Creek, in Alabama County Road 9, Macon County, Shorter, AL (Figure 74). They investigated how braking forces interacted with the bridge substructures, evaluating static and dynamic braking loads and calibrating models against the experimental data.

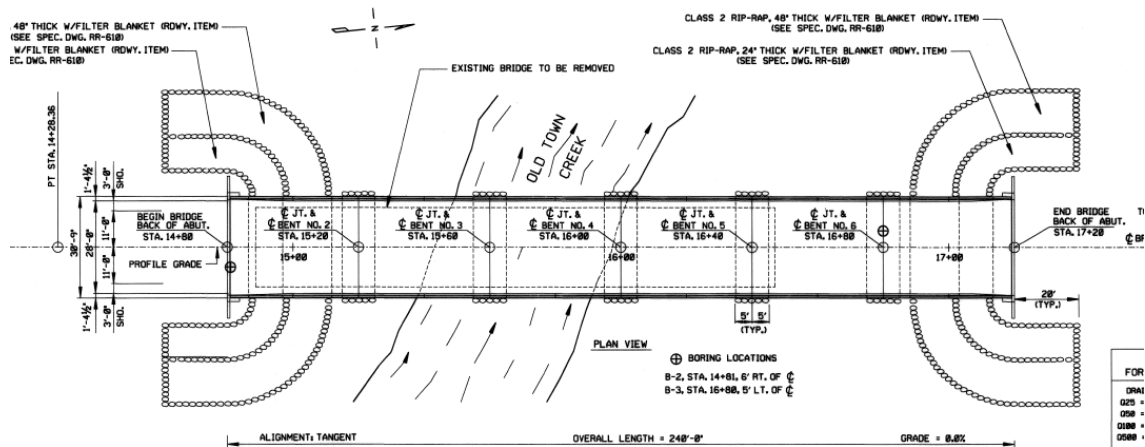


Figure 74. Plan view of bridge over Old Town Creek, Alabama County Road 9, Macon County (Shorter, AL) (Marshall et al, 2021).

In their experimental work, sensors (accelerometers, potentiometers, load cells) were installed on the bridge and connected to a Data Acquisition System (DAQ) to record acceleration, displacement, and force. A tow truck applied static loads, and a load truck conducted dynamic braking tests. Bent displacements and accelerations were recorded and analyzed. The results were used to develop analytical models to supplement test data. Figure 75 shows an example of a sample truck acceleration data.

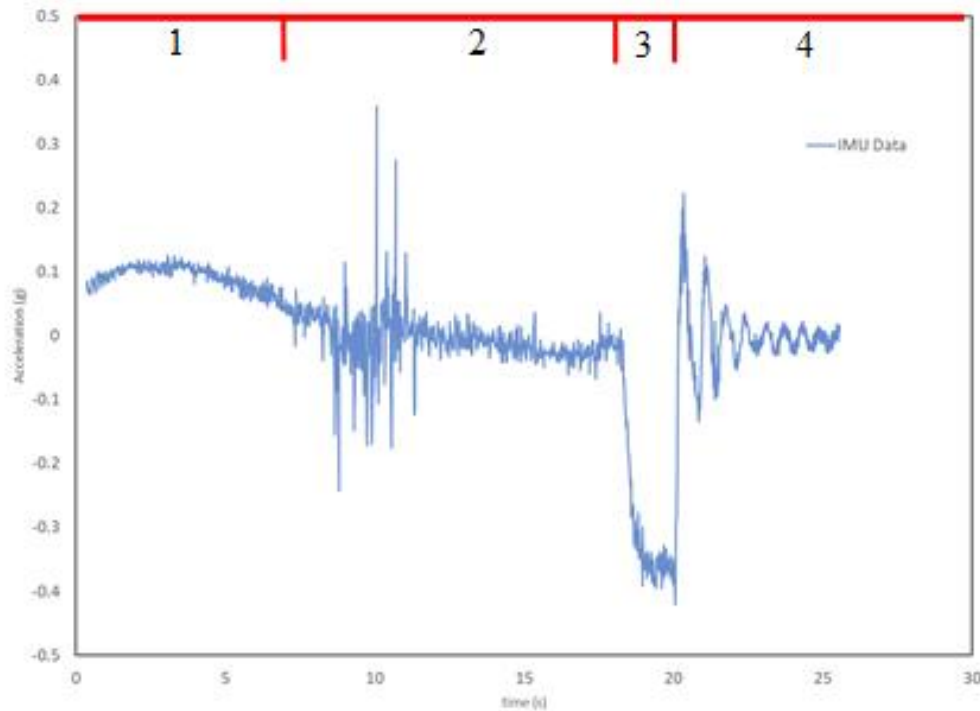


Figure 75. Sample truck acceleration data: 1 is truck initial acceleration to target speed, 2 is truck maintaining constant target velocity, 3 is truck braking, and 4 is truck rocking after initial stop (Marshall et al, 2021).

The static load tests showed greatest deformations at the tallest, least stiff bents (Bent 3, for example, showed 3 mm deflection). The dynamic load tests showed that bridge acceleration was approximately 23% of the truck acceleration, and that braking force was highest near mid-span bents. Figure 76 shows the maximum bent deflections recorded for C5 tests.

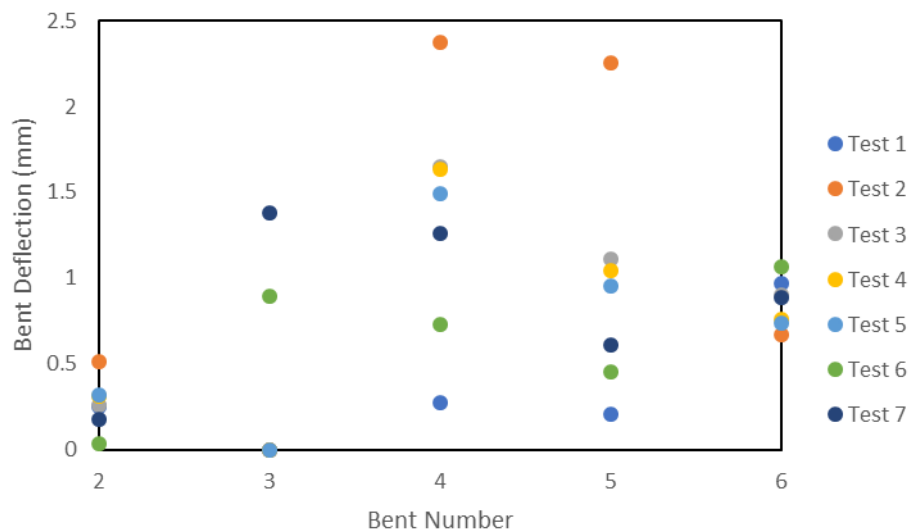


Figure 76. Maximum bent deflections for C5 tests (centerline test on span 5) (Marshall et al, 2021).

The authors then calibrated an analytical model to match the experimental data. Table 7 shows the maximum total and component horizontal shear force for each dynamic test conducted. According to the authors, shear distribution varied, with some bents peaking during free response instead of braking. Additionally, they found that off-center braking increased forces on Abutment 1 & Bent 2. The authors concluded that static analysis is sufficient for estimating total shear forces.

Table 7. Maximum Total and Component Horizontal Shear Force for each Dynamic Test (Marshall et al, 2021).

Braking Position	Max. Static Horizontal Shear (kips)	Max. Dynamic Horizontal Shear (kips)	Max. Comp. Shear (kips): Abutment 1	Max. Comp. Shear (kips): Bent 2	Max. Comp. Shear (kips): Bent 3	Max. Comp. Shear (kips): Bent 4	Max. Comp. Shear (kips): Bent 5	Max. Comp. Shear (kips): Bent 6	Max. Comp. Shear (kips): Abutment 7
Center Span 2	22.5	22.7	7.9	7.5	2.4	2.5*	4.2*	2.3*	4.0*

Right Span 2	26.0	26.5	8.9	8.5	2.7	3.0*	5.2*	3.6*	6.3*
Center Span 3	25.3	26.3	6.6*	4.6	3.0	5.1	4.7	2.7*	4.7*
Right Span 3	21.1	21.2	7.3	6.7	2.2	2.4*	3.9*	2.7*	4.7*
Center Span 5	24.6	25.4	3.7*	2.4*	1.0*	1.7	7.5	6.5	7.6
Right Span 5	20.0	19.7	6.8	6.4	2.0	2.3*	3.4*	2.9*	5.0*

In the last part of the research, the authors performed a limited parametric study using 23 numerical models (Figure 77). They found that abutments absorb ~75% of braking force, while individual bents take up to 35%. Also, stiff columns take more force (0.34 normalized shear), and flexible ones less force (<0.05). According to them, static analysis is effective, but stiffness distribution affects results.

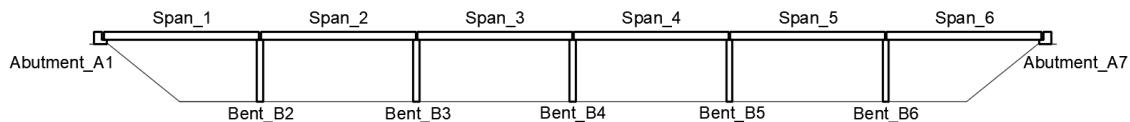


Figure 77. Six-Span Bridge Model used for the parametric study (Marshall et al, 2021).

Chapter 4: Methodology

1. Introduction

This chapter describes the experimental methodology used to investigate the fixity behavior of steel H-pile to concrete bent cap connections relying on plain pile embedment.

2. Experiment Conceptualization

The literature review highlights the study conducted by Marshall et al. (2017), which investigated the response of pile bents subjected to overload conditions. However, this study raised further questions regarding the influence of steel H-Pile embedment into the cap on the lateral load behavior of a pile bent. Building upon this prior work, the current research aims to systematically vary the embedment length of the pile within the cap, evaluate its performance under lateral loading, and analyze the rotational stiffness characteristics of the connection.

Specimens consisting of a concrete cap with an embedded steel H-Pile, as illustrated in Figure 78, will be fabricated. An entire pile bent supported by four piles will be considered. However, for the purposes of the experimental program, a representative segment of this pile bent will be extracted, inverted, and subjected to lateral load testing.



Figure 78. Experiment Conceptualization of a single specimen of a concrete bent cap with a steel H-pile.

2.1 Proposed Specimens

The experimental program relied on material donated by Nucor Skyline. Therefore, the experimental program was limited to the amount and type of material that the donor could provide. The experimental program then opted for the construction of eight specimens. Six of them would use steel piles HP 10x42 and two, HP 12x53. These piles are commonly used by the Alabama Department of Transportation for short-span bridge construction.

Table 8 shows the initial details of the eight specimens.

Table 8. Test specimens for the experimental program

Test Specimen	Pile Type	Load Direction	Embedment (in)	
			0.75 x Pile depth	
W-10-7.5	HP 10x42	Weak axis		7.50

W-10-5			0.50 x Pile depth	5.00
W-10-7.5R			0.75 x Pile depth	7.50
W-10-10			1.00 x Pile depth	10.00
W-10-15			1.50 x Pile depth	15.00
S-10-10			1.00 x Pile depth	10.00
W-12-6	HP 12x53	Weak axis	0.50 x Pile depth	6.00
W-12-12			1.00 x Pile depth	12.00

As observed in table 8, each specimen was named following a standard logic as in “W-10-15”.

The first term of the name “W” or “S” indicates the pile axis that the pile is being bent about. For “W” specimens, the pile is being laterally loaded and bent about its weak axis. For “S” specimens, the pile is laterally loaded and bent about its strong axis.

The second term is either “10” or “12”. It simply indicates whether the specimen uses an HP 10x42 or HP 12x53 pile.

The third term (5, 6, 7.5, 10, 12 or 15) indicates how many inches the pile is embedded into the concrete cap.

Therefore, specimen W-10-15 indicates that it is an HP10x42 pile, embedded 15 in into the cap and bent about its weak axis.

One note about specimens W-10-7.5 and W-10-7.5R. W-10-7.5 was planned to be the first specimen tested. In experimental research, mistakes can occur, or changes can arise during the program. Therefore, W-10-7.5R was created as an exact duplicate of W-10-7.5 to correct potential errors and to ensure replication of results in the dataset.

2.2 Estimated Capacity of Specimens

Each specimen was planned to have a lateral load applied to the pile at a point 7 ft from the cap surface. This point was selected for two reasons: first, it generally corresponds to the average location along the pile length where an inflection point occurred in the moment profiles of Marshall et al. (2017). Therefore, the intent to replicate Marshall et al. (2017) is maintained. Second, the experiments primarily focus on generating moment demand at the connection. With a 7 ft lever arm, the required load to achieve a given connection moment is much lower than with a 3 ft lever arm, for example. Additionally, using this lever arm allows the same desired ratio of flexure to shear in the pile at the connection.

Table 9 shows the expected load necessary to fail each specimen. The maximum loads were calculated taking the minimum value between Marcakis and Mitchell's (1980) theory or the pile structural moment.

Table 9. Expected failure load for each test specimen.

Specimen	Le/d	Expected Failure mode	Expected Horizontal Failure Load (kips)
W-10-15	1.50	Steel yielding	15.0
W-10-10	1.00	Steel yielding	15.0
W-10-7.5	0.75	Concrete cracking/crushing	11.0
W-10-7.5R	0.75	Concrete cracking/crushing	11.0
W-10-5	0.50	Concrete cracking/crushing	5.0
W-12-12	1.00	Steel yielding	21.0
W-12-6	0.50	Concrete cracking/crushing	7.0

S-10-10	1.00	Concrete cracking/crushing	19
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3. Experimental Specimens Design

The design of the test specimens was guided by two primary objectives: generating controlled moment demand at the pile-to-cap connection across a range of embedment depths, and ensuring that specimen geometry was representative of field conditions encountered in short-span bridge pile bents used by ALDOT. The concrete cap dimensions, pile section, reinforcement layout, and embedment depths were established by replicating Marshall et al.'s (2017) pile bent sections, including reinforcement.

Therefore, the selected bent cap section is a reinforced concrete cap section measuring 36 in x 36 in (3 ft x 3 ft) and 6 ft long. Each specimen has a single steel H-pile embedded to a prescribed depth. Axial, shear and moment demands were calculated and the specimen was designed according to AASHTO LRFD Bridge Design Specification (2020) and ACI 318 (2022) to resist such demands. No special connection details were provided other than pure pile embedment. Figure 79 shows a schematic of the specimen geometry.

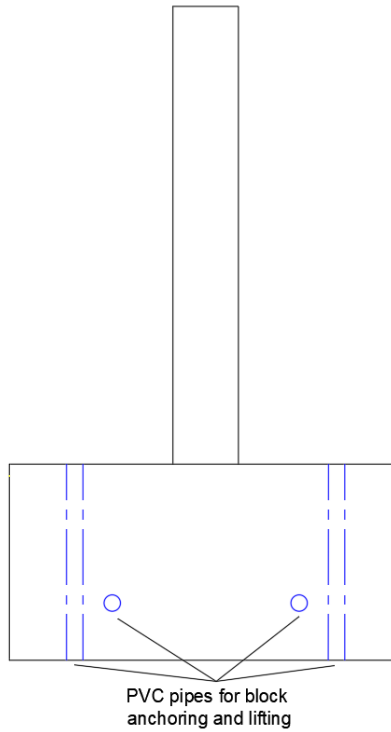
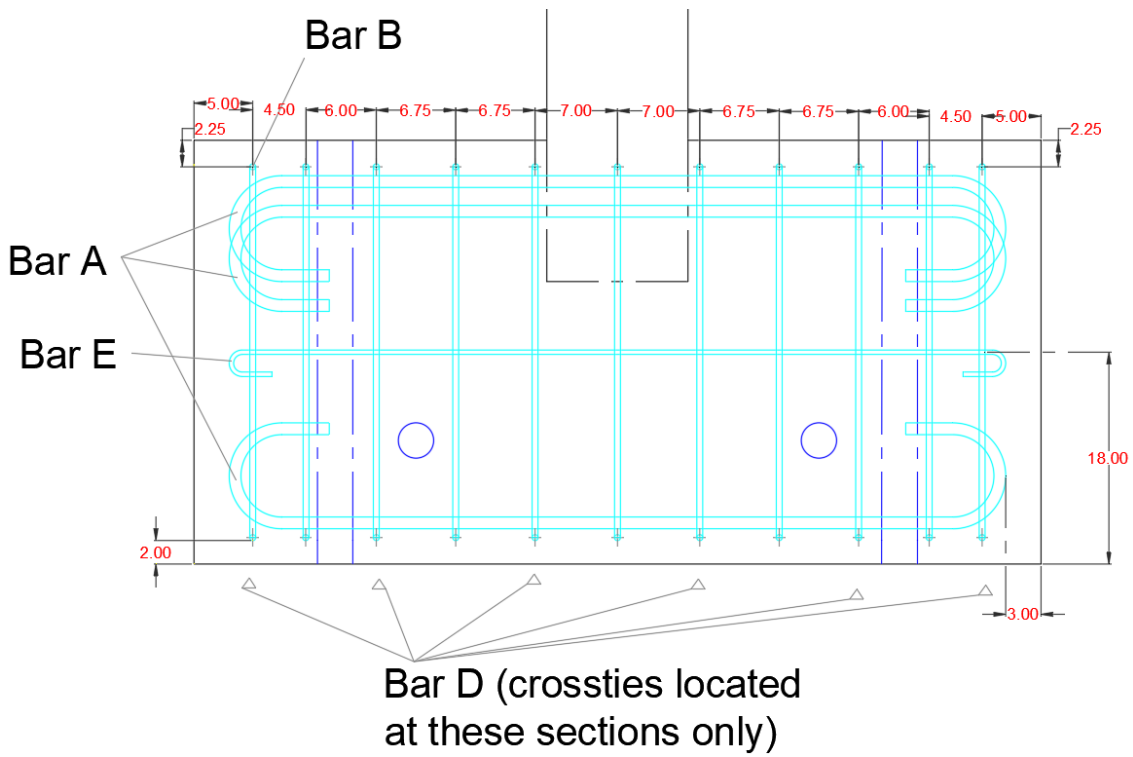


Figure 79. Schematic of Specimen Geometry

3.1 Specimen Details

The concrete cap final reinforcement design can be seen in Figure 80.

Side view:



Cross-section view:

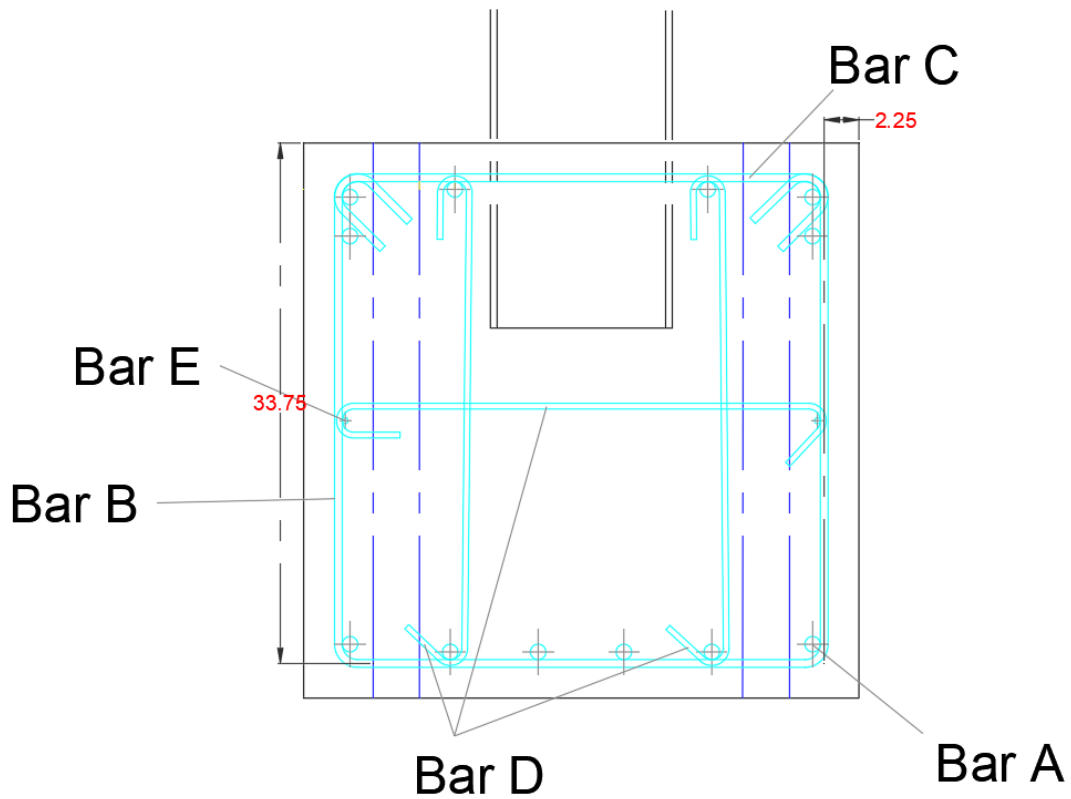


Figure 80. Concrete cap final reinforcement design: Side view (top) and cross-section view (bottom).

As seen in figure 80, the final reinforcement design is comprised of:

- Bar A: 12 - #8 longitudinal rebars, distributed on top and bottom.
- Bar B: 11 - #4 transverse rebars, in a U-shape
- Bar C: 10 - #4 transverse rebars, closing the loop for bar B, except at the midsection.
- Bar D: 18 - #3 cross-ties, placed in six different cross-sections.
- Bar E: 2 - #3 skin rebars, placed on the cap sides.

The reinforcement was designed to leave room in the center so that the steel H-pile can be inserted into the cap, at the desired embedment depth, as seen in figure 81.

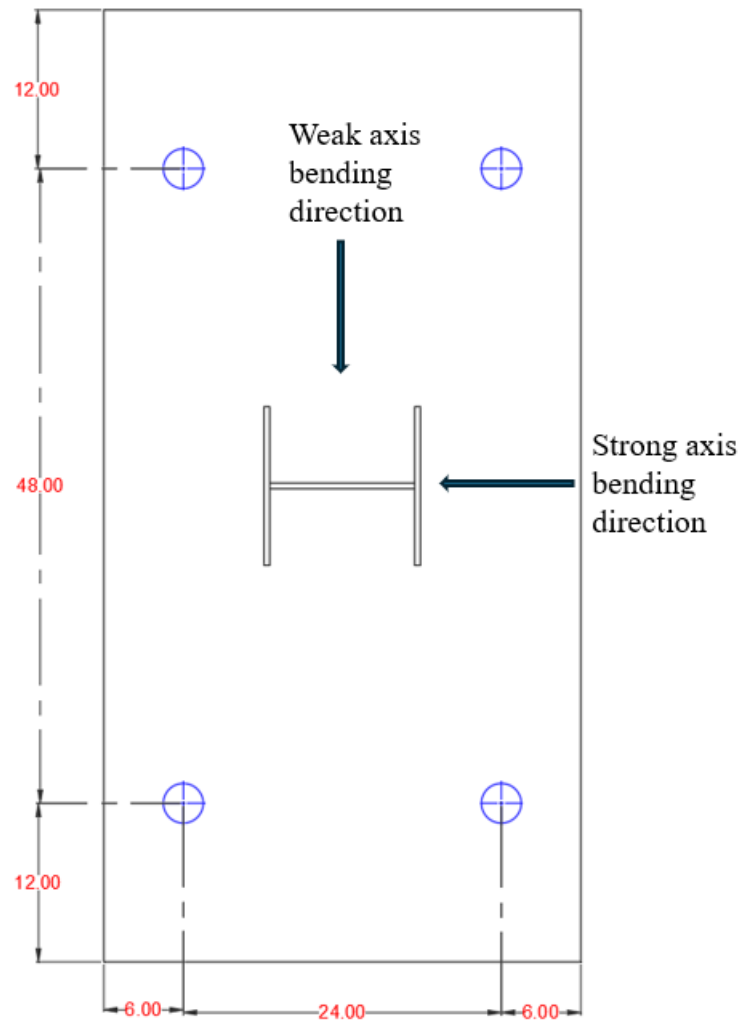


Figure 81. Plan view of the specimen (dimensions in inches).

3.2 Material Properties

The following subsections describe the geometric and material properties of the test specimens, including the steel H-pile cross-sectional dimensions, steel material properties, concrete mix design, and measured fresh and hardened concrete properties.

3.2.1 Reinforcing Steel

The design requirements for mild reinforcing steel follow ASTM A615 Grade 60. The yield strength of steel (f_y) is 60 ksi. The estimated modulus of elasticity (E) is 29,000 ksi.

3.2.2 Steel H Piles

The specimens were fabricated using two standard wide-flange H-pile sections (HP 10x42 and HP 12x53), manufactured by Nucor Skyline Steel according to ASTM A572, A709 and A6.

Table 10 summarizes the cross-sectional properties of both pile sections and figure 82 shows the steel H-pile schematic cross-section.

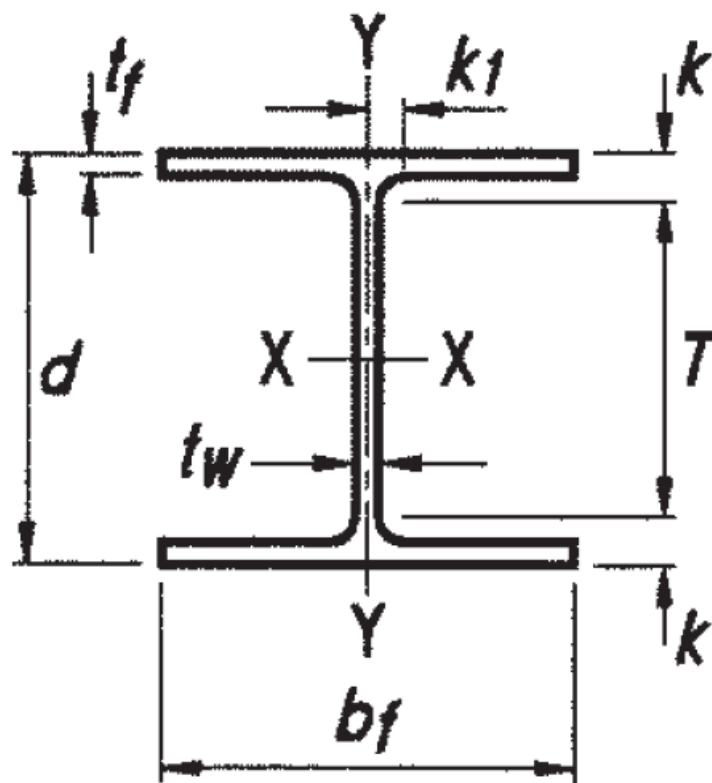


Figure 82. HP section schematic (AISC Steel Manual, 2023)

Table 10. HP section dimensions, properties and calculated Moment Capacities.

Property	HP 10×42	HP 12×53
Geometric Properties		
Cross-sectional area, A (in ²)	12.4	15.5

Nominal depth, d (in)	9.7	11.8
Flange width, b_f (in)	10.1	12
Flange thickness, t_f (in)	0.42	0.435
Web thickness, t_w (in)	0.415	0.435
Compact section criterion - flange, $b_f/2t_f$	12	13.8
Compact section criterion - web, h/t_w	18.9	22.3
Strong-Axis (X-X) Section Properties		
Moment of inertia, I_x (in ⁴)	210	393
Elastic section modulus, S_x (in ³)	43.4	66.7
Radius of gyration, r_x (in)	4.13	5.03
Plastic section modulus, Z_x (in ³)	48.3	74
Weak-Axis (Y-Y)		
Moment of inertia, I_y (in ⁴)	71.7	127
Elastic section modulus, S_y (in ³)	14.2	21.1
Radius of gyration, r_y (in)	2.41	2.86
Plastic section modulus, Z_y (in ³)	21.8	32.2
Torsional Properties		
Torsional constant, J (in ⁴)	0.813	1.12
Warping constant, C_w (in ⁶)	1,540	4,080
Material Properties		
Modulus of elasticity, E (ksi)	29,000	29,000
Yield strength, F_y (ksi)	59	54.5
Tensile strength, F_u (ksi)	80.5	71.75
Calculated Capacities - Strong Axis (X-X)		
Plastic moment, $M_p = Z_x \cdot F_y$ (kip-ft)	237	336
Nominal moment (FLB controls), M_n (kip-ft)	214	292
Yield moment, $M_y = S_x \cdot F_y$ (kip-ft)	213	302
Calculated Capacities - Weak Axis (Y-Y)		
Plastic moment, $M_p = Z_y \cdot F_y$ (kip-ft)	107	146
Nominal moment (FLB controls), M_n (kip-ft)	92	118

Yield moment, $M_y = S_y \cdot F_y$ (kip-ft)	69	95
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3.2.3 Concrete

Normally, the ALDOT Standard Specifications for Highway Construction (2022) specify the type of concrete used bent caps. For this project, however, the concrete target strength was replicated from the work of Marshall et al. (2017).

3.2.3.1 Concrete Mix

Normal-weight ready-mix concrete was used for all specimens, with a target design compressive strength of 5,000 psi (34.5 MPa) at 28 days. All specimens were cast using concrete from the same ready-mix supplier throughout the program, using the Class-B concrete minimum specifications from ALDOT Standard Specifications for Highway Construction (2022), seen in table 11, as a baseline, with adjusted slump for workability around the reinforcement cage and pile.

Table 11. ALDOT Standard Specifications for Highway Construction (2022) for concrete mixtures.

Concrete Class	Class A	Class B	Class C	Class D
Minimum 28-Day Compressive Strength (psi) {MPa}	3,000 {21}	4,000 {28}	3,000 {21}	3,000 {21}
Maximum Water/Cementitious Materials Ratio	0.50	0.45	0.55	0.45
Range of Total Air Content (%)	2.5 - 6.0	2.5 - 6.0	2.5 - 6.0	2.5 - 6.0
Slump (in) {mm}	3.0 {75}	3.5 {90}	3.0 {75}	7.0 {180}
Maximum 28-Day Drying Shrinkage (%)	--	0.04	--	--

Largest Nominal Maximum Aggregate Size (in) {mm}	1.0 {25}	1.0 {25}	1.0 {25}	1.0 {25}
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3.2.3.2 Concrete Properties

Table 12 below summarizes the concrete information and properties measured after pouring. Fresh concrete properties were measured at the time of each pour in accordance with standard ASTM methods: slump (ASTM C143), air content by pressure method (ASTM C231), and concrete temperature. Measurements were taken from the first truckload delivered for each pour.

Table 12. Measured Fresh Concrete properties.

Specimen	Le/d	Concrete pouring date	Slump test (in)	Air test (%)	Temp. at time of pouring (deg F)
W-10-7.5	0.75	10/15/2025	5.250	4.300	84.0
W-10-10	1.00	10/15/2025	5.250	4.300	84.0
W-10-15	1.50	10/22/2025	6.500	4.000	79.0
S-10-10	1.00	10/22/2025	6.500	4.000	79.0
W-10-5	0.50	10/30/2025	7.000	4.000	70.0
W-12-6	0.50	10/30/2025	7.000	4.000	70.0
W-10-7.5R	0.75	11/6/2025	4.625	5.200	76.0
W-12-12	1.00	11/6/2025	4.625	5.200	76.0

The concrete mix was designed to achieve a minimum target compressive strength of 5,000 psi at 28 days using Type I/II Portland cement, crushed limestone coarse aggregate with a maximum nominal size of 3/4 in, natural sand fine aggregate, and water. No supplementary cementitious materials or chemical admixtures were used. Figure 83 shows the measured compressive concrete strengths taken from cylinder compressive tests done according to ASTM C39. The collected concrete cylinders were cured by

leaving the cylinders alongside the specimens. Each datapoint is an average of at least two cylinders.

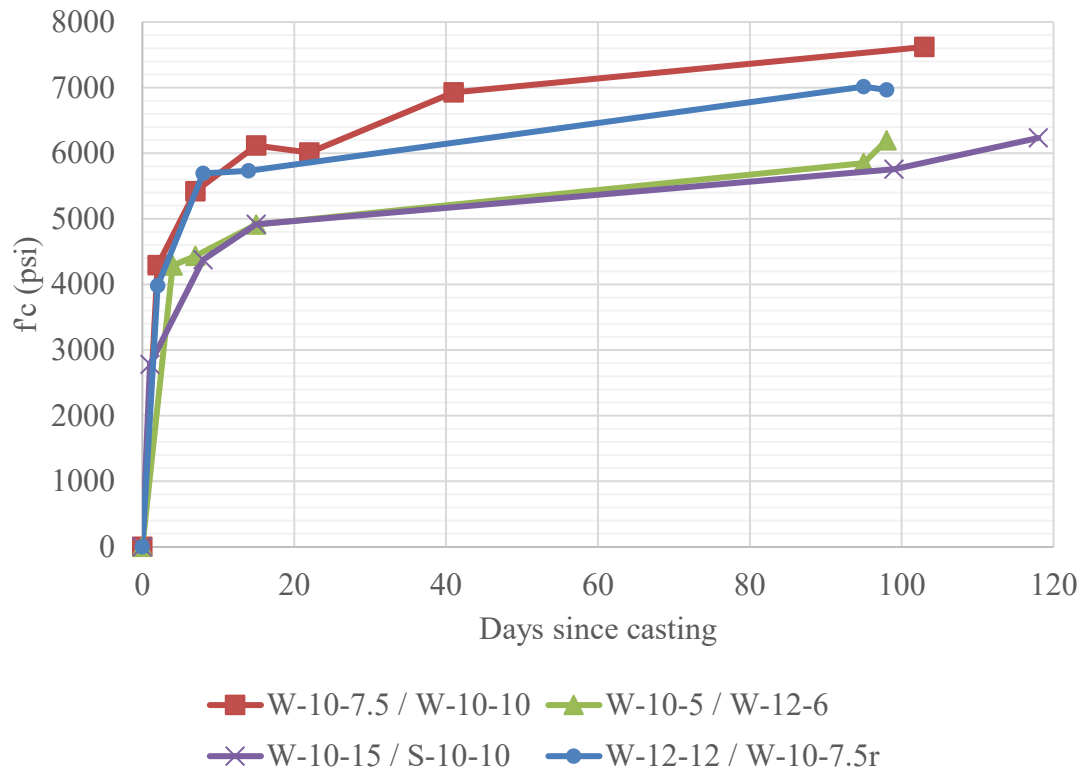


Figure 83. Results of concrete compressive strength tests.

4. Experimental Specimens Fabrication

All eight test specimens were fabricated in the Advanced Structural Engineering Laboratory (ASEL) at Auburn University. Fabrication was performed in four separate pours, with two specimens constructed per pour: Specimens W-10-7.5 and W-10-10 in the first pour, Specimens W-10-15 and S-10-10 in the second pour, Specimens W-10-5 and W-12-6 in the third pour, and Specimens W-10-7.5R and W-12-12 in the fourth pour. The fabrication procedure for each pair followed four sequential steps: (1) assembly of the reinforcement cage, (2) assembly of formwork, (3) insertion of reinforcement cage inside the formwork, (4) construction of support frame and steel H-pile preparation and (5) concrete placement and curing.

(1) Assembly of reinforcement cage:

The reinforcement cages were manufactured according to the design seen in section 3.1. Figure 84 shows all the reinforcement cages ready for placement.



Figure 84. Reinforcement cages ready for placement.

(2) Assembly of formwork:

The formwork was built with plywood and braces in order to accommodate the reinforcing steel inside and the concrete during pouring. Figure 85 shows the formwork ready for cage placement.



Figure 85. Formwork ready for Reinforcement cage placement.

(3) Insertion of reinforcement cage:

Figure 86 shows the formwork with the reinforcement cage ready for pouring. Also, it is possible to observe the PVC pipes installed to make room for the vertical anchors, as well as the horizontal pipes for future lifting of the specimens with a crane and straps.



Figure 86. Formwork and reinforcing steel cage ready for pouring.

(4) Support frame:

Figure 87 shows the frame built to support the steel H-pile so that the correct pile embedment is achieved. Before concrete casting, strain gauges were installed on the pile flanges at the prescribed locations. After gauge installation and protective coating, the pile was inserted into the reinforcement cage with the embedded end positioned at the desired depth. The pile was centered within the cap cross-section and held in alignment using temporary wood bracing screwed to the formwork walls. Pile embedment depth and alignment were verified by direct measurement before concrete placement.



Figure 87. Support frame for steel H-piles. Specimen ready for casting.

(5) Concrete pouring:

Figure 88 shows the pouring of concrete into the specimens, with subsequent vibration.

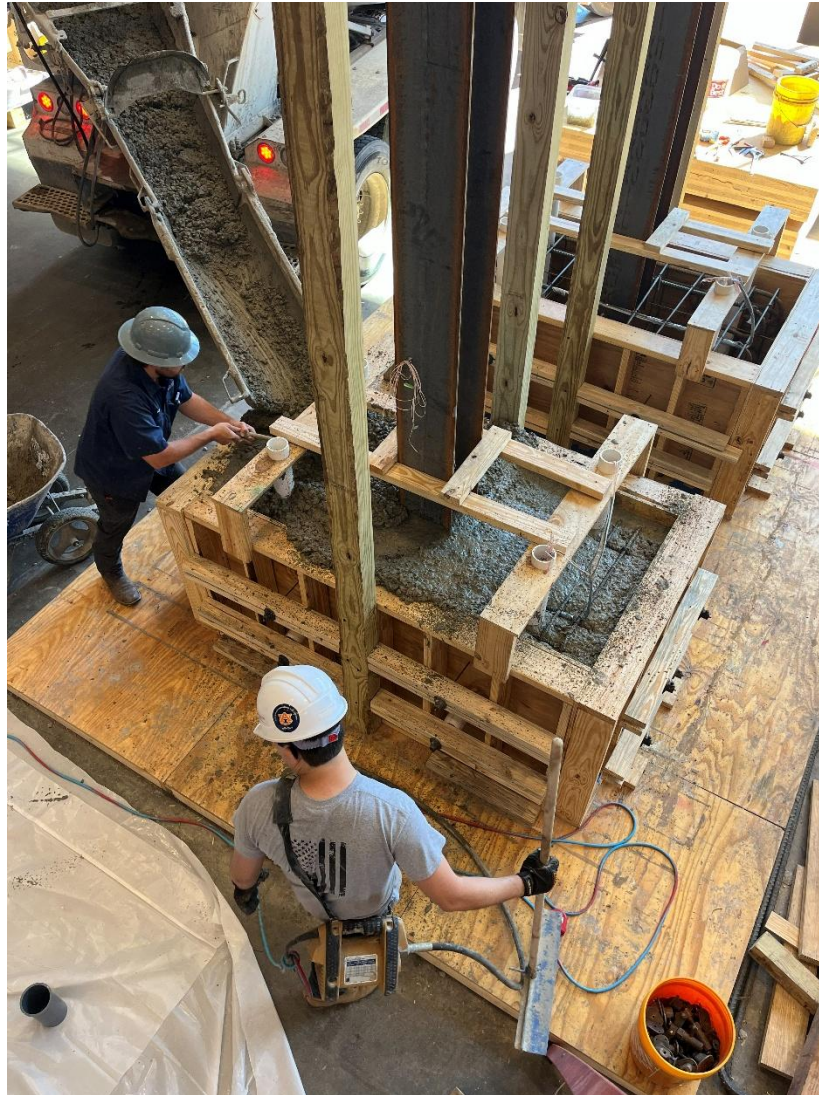


Figure 88. Concrete pouring in specimens.

During placement, slump tests and air content tests were performed, and concrete cylinders were collected. After the concrete achieved the initial set, the specimens were covered with wet burlap and construction plastic to rewet and prevent drying. Figure 89 shows the final specimen after the forms were removed, after 7 days.



Figure 89. Two specimens after form removal and two more specimens ready for casting.

5. Instrumentation

Each test specimen was instrumented with a comprehensive array of sensors to measure the applied load, lateral displacements at multiple heights along the pile, pile rotation at multiple elevations, relative slip between the pile flanges and the concrete cap surface, and strains in the steel pile, reinforcing steel, and concrete cap surface. All sensors were connected to a Campbell Scientific CR6 data acquisition system, which recorded all measurements continuously throughout each load test. The following subsections detail each instrument type, its layout and installation procedure.

5.1 Strain Gauges

Two TML (Tokyo Measuring Instruments Laboratory) strain gauge types were used in this study, each selected for compatibility with its respective material. For steel

instrumentation, the WFLA-6-11-3LDBTB-F was employed, a single-axis foil gauge from the WF waterproof series, with a 6 mm gauge length and 120 Ω nominal resistance. Its entire gauge body and leadwire junction are encapsulated in a transparent epoxy coating, providing moisture and water resistance without requiring additional overcoating against water. The gauge is temperature-compensated for mild steel ($11 \times 10^{-6}/^{\circ}\text{C}$), operates between 0 and $+80^{\circ}\text{C}$, and is connected via a pre-attached 3-meter, 3-wire paralleled vinyl leadwire (LDBTB-F), which eliminates thermal output errors from leadwire resistance changes.

For concrete surface instrumentation, the PL-90-11-3LJCT-F was used, a single-axis polyester wire gauge from the P series specifically designed for concrete, mortar, and rock. Its 90 mm gauge length is well suited to heterogeneous materials like concrete, averaging strain over a representative area that spans multiple aggregate particles, in accordance with TML guidelines recommending gauge lengths at least three times the maximum aggregate diameter. The gauge has 120 Ω resistance, operates between -20 and $+80^{\circ}\text{C}$, and is temperature-compensated for concrete ($11 \times 10^{-6}/^{\circ}\text{C}$), with a transparent polyester backing that facilitates accurate positioning during bonding.

Both gauges are connected in a quarter-bridge 3-wire configuration to compensate for leadwire temperature effects. To interface these gauges with the Campbell Scientific CR6 datalogger, a 4WFBS120 Terminal Input Module (TIM) was used for each gauge. Because each gauge constitutes only one arm of a Wheatstone bridge, the 4WFBS120 supplies the three remaining passive arms (two precision 1 k Ω backplane resistors and a 120 Ω completion resistor matched to the gauge's nominal resistance) allowing the CR6 to execute a full-bridge voltage measurement and convert the output to microstrain.

Figure 90, for example, shows the steel strain gauges glued to one of the tested piles.

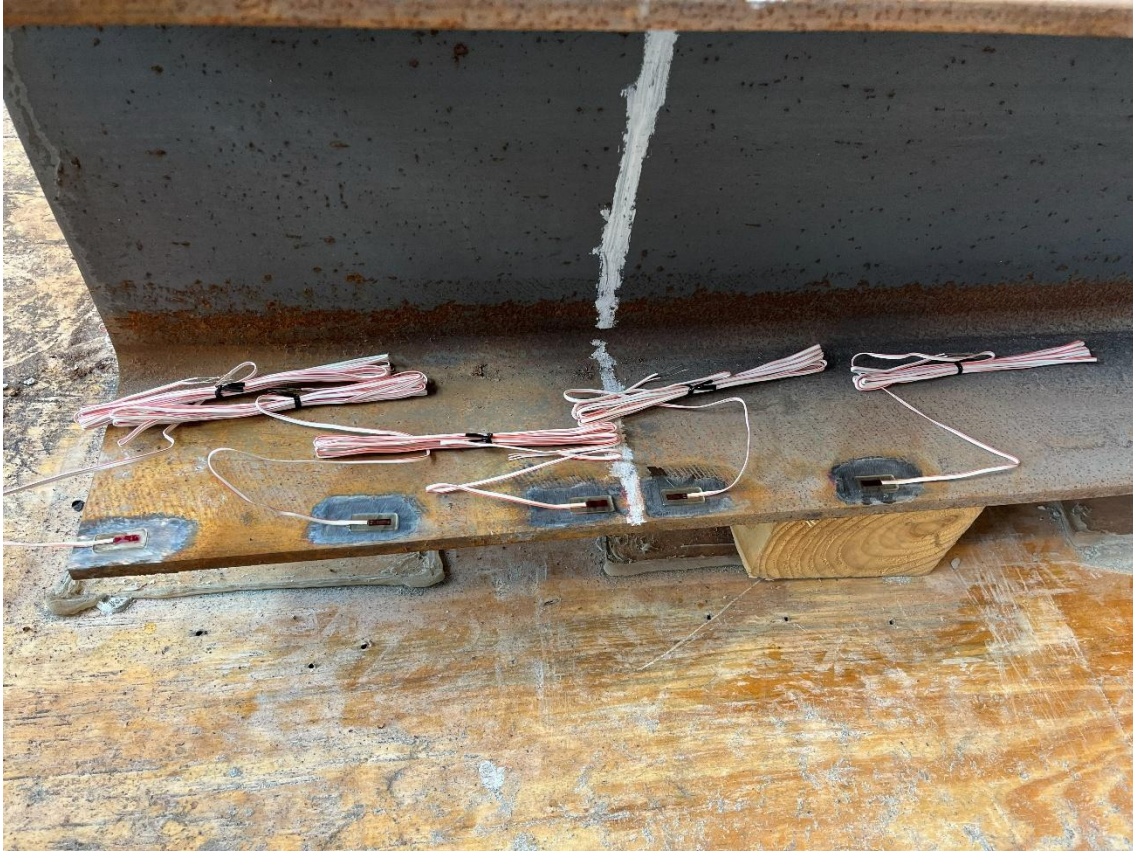


Figure 90. Strain gauges glued to steel H-pile at specific locations.

5.1.1 Steel H Pile Strain Gauges

At the steel H-pile, the strain gauges main purpose is to measure strain and, therefore, moment demand at specific points along the pile's embedded length. For all weak axis bending specimens, the strain gauges were glued in pairs at flanges A and B, A being the trailing flange and B the leading flange, considering the load direction. For the strong axis bending specimens, the strain gauges were glued in pairs at flanges A and D, A being the trailing flange, and D the leading flange. Figure 91 shows the steel H-pile and the adopted nomenclature for each flange with respect to the load direction.

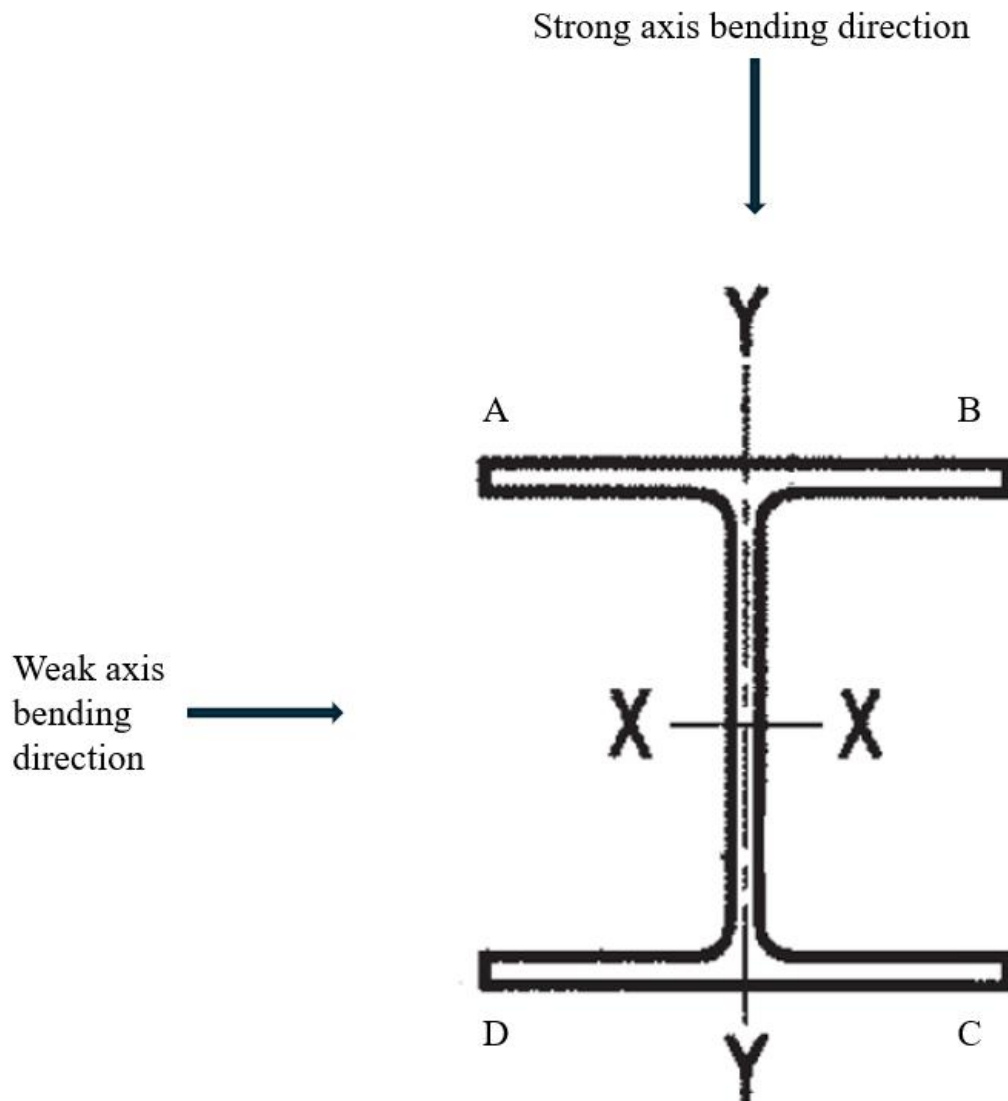
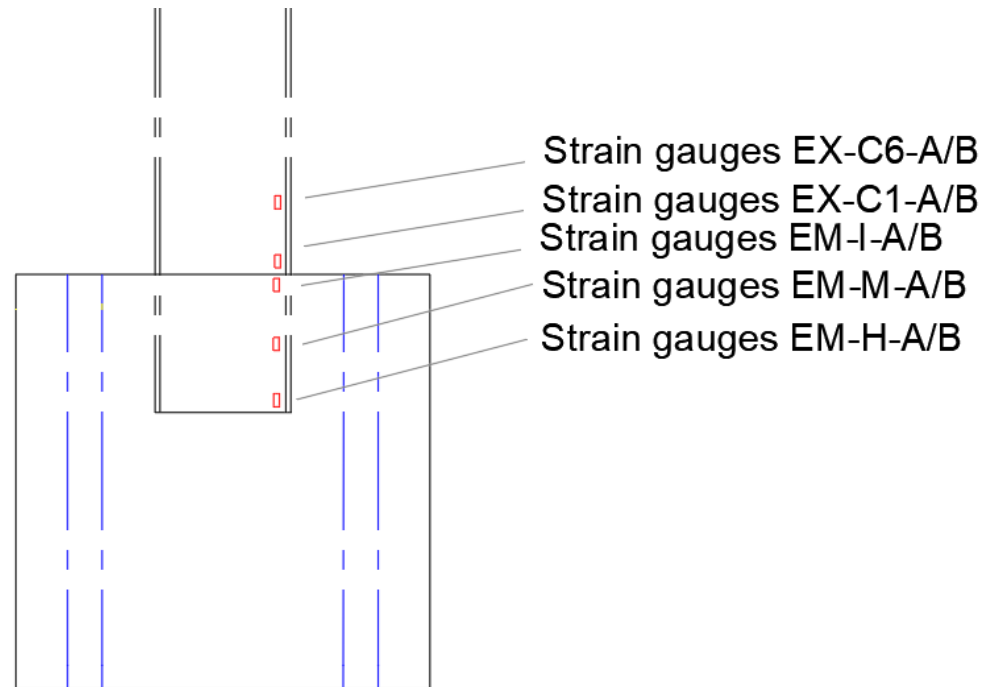


Figure 91. Steel H-Pile flange names.

Figure 92 shows a schematic with the elevations at which each strain gauge pair was glued.



For embedments of 5, 6 and 7.5:
 There are no gauges at EM-M level

Figure 92. Schematic of Strain Gauge elevations in a Steel H-Pile.

The strain gauge names indicate the point and elevation where they were glued to.

“EX” or “EM” indicate whether the gauge is exposed (outside the concrete) or embedded (inside the concrete).

“C6” indicates that the gauge was exposed and glued 6 inches away from the cap.

“C1” indicates that the gauge was exposed and glued 1 inch away from the cap.

“I” stands for interface, indicating that the gauge was glued at 1 in embedded inside the interface.

“M” stands for mid-length, indicating that the gauge was glued at exactly mid-length between the interface and the pile head. If a pile was embedded 12 in into the cap, the EM-M gauge is located at 6 in embedded inside the cap.

“H” stands for head, indicating that the gauge was glued at 1 in away from the pile head.

For shallow embedments, such as 5 in, 6 in or 7.5 in, no EM-M gauge was used.

Finally, the last term of the name “A/B/C/D” indicates at which flange the gauge was glued to.

In all cases, the gauges were glued on the inside of the flanges and 0.5 in from the edges.

Prior to bonding the gauges to the steel, the surface was initially prepared to receive the gauge. The area was first ground to bright metal, then dry-abraded with 220-grit sandpaper followed by wet abrasion using CSM-3 degreaser and MCA-1 Conditioner A (Micro-Measurements, red tip). A final wet abrasion pass was performed with 320-grit sandpaper and Conditioner A, after which the surface was scrubbed with Conditioner A until contaminants were fully removed. Each abrasion step was followed by a single-motion dry gauze wipe to clear residue. The surface was then neutralized with Neutralizer 5A (blue tip) and wiped dry.

After preparing the surface, a thin uniform film of CN adhesive was spread on the gauge back, the gauge was lowered onto the prepared area, covered with a polyethylene sheet, and held under steady thumb pressure for 30 to 60 seconds.

Since the gauges were already protected by an epoxy resin, the gauges were coated only with SB tape (Butyl rubber) for extra moisture and water-proofing. Finally, a final layer of silver tape was applied.

The lead wires from the embedded gauges were routed along one of the reinforcing bars, protected by silver tape, and extended above the cap surface level to allow connection to the data acquisition system after casting.

5.1.2 Reinforcing Steel Strain Gauges

The strain gauges bonded to the reinforcing steel went through the same process as the gauges used on the steel H-piles. Only two gauges were used, per specimen, as seen in figure 93. R1 and R2 were always bonded on the rebar closest to flange A/B, R1 being closer to A and R2 being closer to B. Therefore, R1 was always located on the trailing side of the load direction for weak-axis bending tests, and R2, on the leading side.

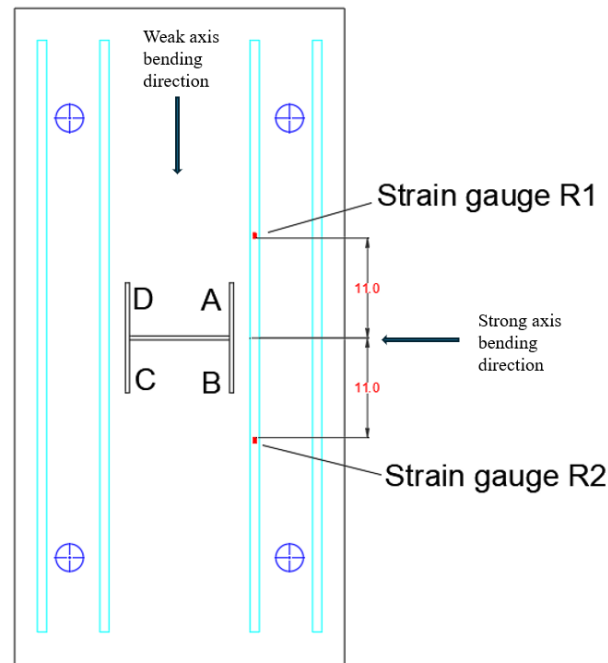


Figure 93. Schematic of Reinforcing Steel Strain Gauges.

5.1.3 Concrete Strain Gauges

After specimen was fully cured, two concrete surface gauges were installed on the top surface of the concrete cap, 6 inches away from pile web, on both sides of the pile, following the weak-axis bending load direction. The gauges were named either “CONC AD” or “CONC BC” indicating which flanges they are close to.

For strong-axis bending, the gauges were positioned 1 in away from the flanges, also following the strong-axis bending load direction. The gauges were named either “CONC AB” or “CONC CD” indicating which flanges they are close to.

Figures 94 and 95 show, respectively, a schematic of the concrete strain gauges and a picture after its installation.

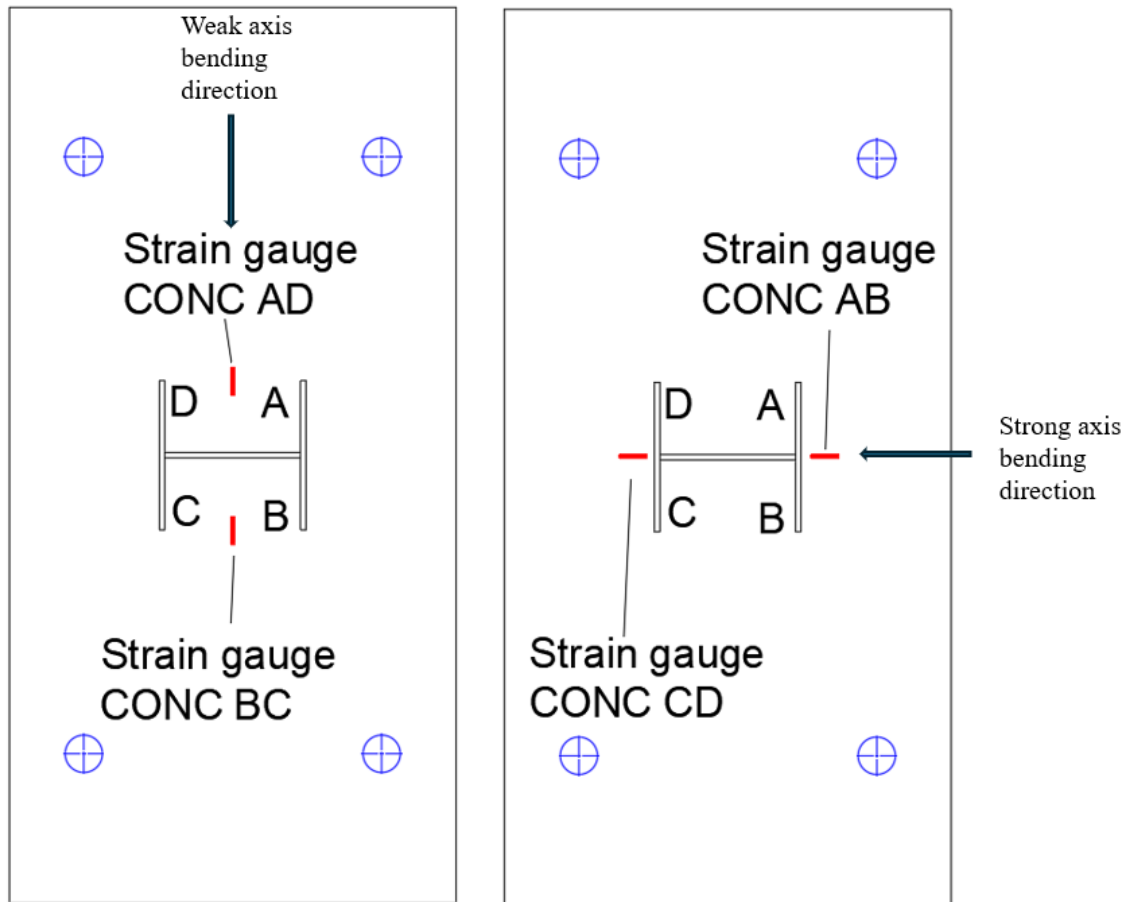


Figure 94. Schematic of Concrete surface gauges.

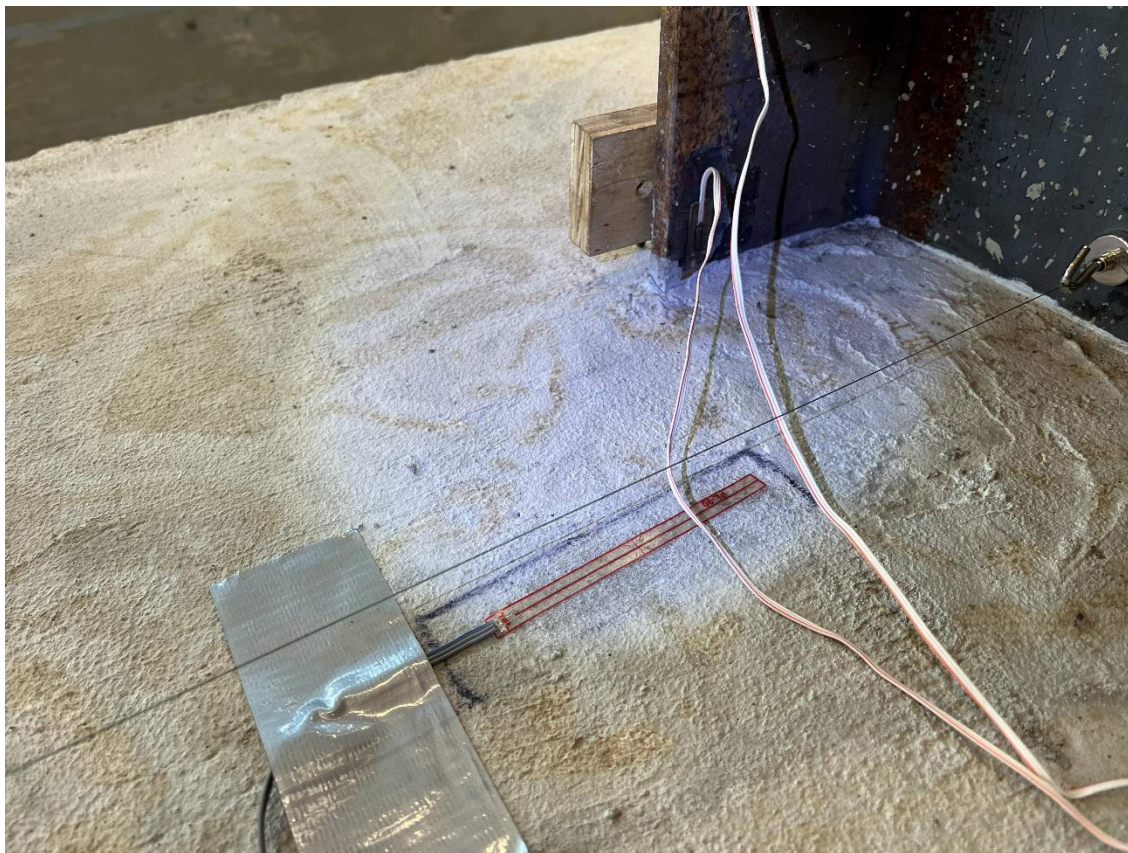


Figure 95. Picture of a concrete strain gauge, after installation.

For surface preparation for the concrete gauges, a similar protocol as the steel H-piles was used. The day prior to bonding, the target area was cleaned with a wet cloth to remove gross surface contamination and allowed to dry overnight. The surface was then abraded with a wire brush. MCA-1 Conditioner A was scrubbed into the surface with gauze, after which the area was dried with a warm air blower. Neutralizer 5A was then applied and scrubbed in, followed by another drying pass with the warm blower. With the surface prepared, the gauge bonded with CN-E, which was spread as a thin uniform film on the gauge back. The gauge was then placed on the surface, covered with a polyethylene sheet, and loaded with a weight for 30-60 seconds before pressure was carefully released.

Since the concrete gauges were exposed to air in a laboratory-controlled environment, no coating was applied other than a plastic cover film and duct tape to prevent accidental contact.

5.2 String Potentiometers

Four Micro-Epsilon wireSENSOR WDS-100-P60-CR-P draw-wire displacement sensors (string potentiometers) were used. These are industrial draw-wire transducers in which a polyamide-coated stainless steel wire ($\varnothing$ 0.45 mm) is wound onto a spring-loaded drum coupled to a multi-turn potentiometer, converting linear wire extension into a proportional resistance change. The sensors have either a 100 mm or 1000 mm measuring range with a potentiometer sensitivity of 9.401 mV/V/mm, a maximum nonlinearity deviation of $\pm 0.5\%$ FSO (± 0.5 mm), and quasi-infinite resolution. The excitation voltage was 10.0 V, and the aluminum housing carries an IP65 protection rating, rated for operation between -20 °C and $+80$ °C.

They were mounted on a rigid reference frame erected independently of the specimen and loading system to measure the lateral displacement of the exposed pile at four heights above the cap surface: 1 in (POT C1), 6 in (POT C6), 42 in (POT C42), and 84 in (POT C84). The 84-in location corresponds to the load application point. The sensors were oriented horizontally in the plane of loading with their wire extensions attached to the pile web by a small magnet at each measurement height. The potentiometer measurement range was selected to accommodate the expected maximum deflections at each location. Figure 96 shows an example of a string potentiometer.



Figure 96. String potentiometers (Micro-Epsilon, 2021).

5.3 Spring Potentiometers (Slip meters)

Pile displacements at the connection interface were measured using Sensata BEI Sensors 9600 Series spring-return linear position sensors (model [9610/9615]). These devices are resistive potentiometers with a spring-loaded plunger that extends to the actuated position and returns to the reference position upon unloading, making them suited for monotonic and cyclic displacement measurements without requiring a fixed connection to the moving surface.

The [9610/9615] unit provides an electrical travel range of [25.4/38.1] mm, a total DC resistance of [3.4/5.1] k Ω ($\pm 25\%$), and a standard linearity of $\pm 2\%$ over the active electrical travel. Output voltage was read by the datalogger. Sensor displacement was independently calibrated prior to testing to verify linearity and zero-position accuracy.

Six slip meters were used. Four were installed each on each flange, with the sensor stroke in contact with the concrete surface - SLIP A, SLIP B, SLIP C and SLIP D. These sensors detected whether the flanges were separating from the concrete on the trailing side (SLIPS A and D) or bearing into the concrete on the leading side (SLIPS B and C) as the pile rotated.

Two additional slip meters were also mounted horizontally at the ends of the concrete cap to monitor for any longitudinal translation or rocking of the cap. SLIP AD was placed on the trailing edge of the cap, and SLIP BC, on the leading edge.

Figures 97 and 98 show two examples of, respectively, SLIP B and SLIP BC mounting positions.



Figure 97. SLIP B and SLIP A mounting positions



Figure 98. SLIP BC mounting position.

5.4 Inclination

Two Rieker inclinometers and two Geokon tiltmeters were used to measure rotational movement of the cap and pile during the load tests. The Geokon tiltmeters were named TILT-CAP and TILT-C6. The Rieker inclinometers were named TILT-HEAD and TILT-C1. Figure 99 shows the position of all tiltmeters and inclinometers.

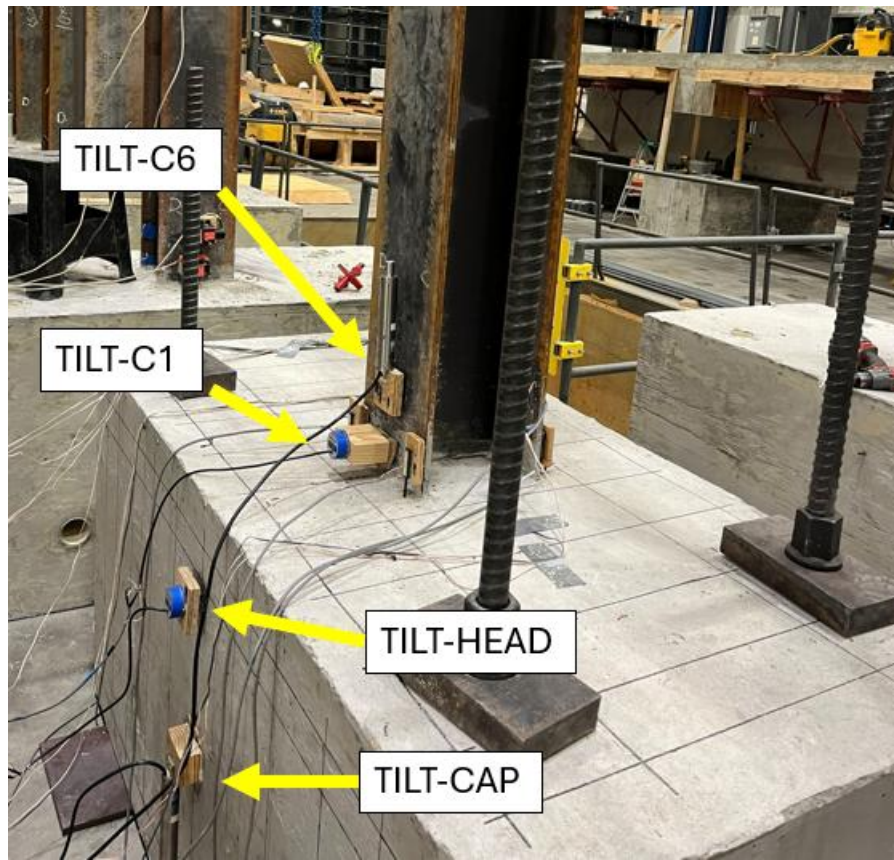


Figure 99. Positions of Tiltmeters and Inclinometers.

5.4.1 Geokon Tiltmeters

Two Geokon Model 6160 MEMS biaxial tiltmeters were deployed, one attached to the pile, 6 in away from the cap (TILT-C6) and one to the side of the pile cap at mid-height (TILT-CAP). They enable direct measurement of the differential rotation between these two structural elements, from which the degree of rotational fixity at the pile-to-cap connection can be inferred. The MEMS (Micro-Electro-Mechanical Systems) transduction principle was selected for its robustness to shock loading, good sensitivity, and compatibility with automated continuous data acquisition, which allowed the sensors to be logged throughout the loading protocol without manual intervention.

The operating principle relies on a solid-state MEMS accelerometer mounted on a printed circuit board inside a stainless steel housing. The sensor output is a differential analog voltage proportional to the sine of the angle of inclination, with a full-scale output of ± 4 V over a $\pm 15^\circ$ range. Resolution is ± 4 arc seconds (± 0.02 mm/m), with an accuracy

of ± 0.05 mm/m (± 10 arc seconds) established under laboratory conditions using a second-order polynomial calibration. The sensor was powered at 12 V DC nominal (9-15 V range).

5.4.2 Rieker Tiltmeters

Also called Seika NG2 electronic inclinometer, this sensor is a liquid-capacitive, gravity-referenced sensor with a $\pm 10^\circ$ measuring range and a resolution of less than 0.001° . The sensor produces a non-amplified analog DC voltage output with a sensitivity of approximately $10 \text{ mV}/^\circ$ and a linearity deviation below 0.1% of the full measuring range.

Two inclinometer installations were used. The first (TILT-C1) was mounted directly on the pile, 1 in above the top face of the cap, providing a direct measure of the pile inclination at that elevation.

The second installation was designed to capture the embedded pile head rotation, independently of the cap geometry. Prior to concrete casting, a steel rod was welded to the pile head with its axis oriented horizontally, following the pile web direction, and extending laterally beyond the cap boundary. It was protected along its embedded length by a vinyl sleeve to prevent bond with the surrounding concrete. A steel plate was attached to the exposed tip of the rod, and the NG2 sensor was attached to this plate using a permanent magnet. Under lateral loading, any rotation of the pile head is transmitted through the embedded rod to the sensor at its tip, registering as a change in inclination. The magnetic attachment allowed the sensor to be removed and repositioned between test specimens while maintaining a consistent measurement reference through the fixed rod-plate assembly.

Figure 100 shows a schematic of the steel rod with the TILT-HEAD sensor attached to it.

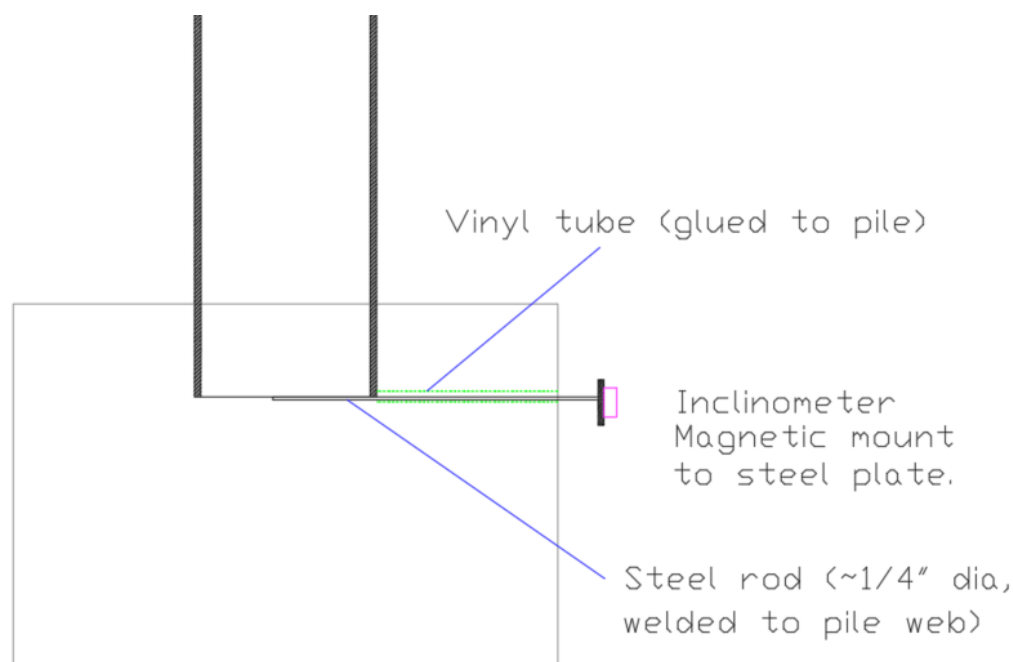


Figure 100. Schematic cross-sectional view of a specimen with a steel rod welded to the pile web, and inclinometer attachment.

5.5 Geokon Load Cells

Two GEOKON Model 4900-200-3 vibrating wire load cells (serial numbers 2438882 and 2438883) were used to measure the lateral load applied to the pile during each test. Figure 101 shows the load cells positioned for the load test.

The Model 4900 operates on the vibrating wire principle. The load cell body is a high-strength steel cylinder in which three vibrating wire strain gauges are embedded around the circumference. As a compressive load is applied to the cylinder, the cylinder shortens but (due to Poisson's effect) expands radially. This causes increase of tension on the embedded wires, and therefore the resonant frequency of each wire. An electromagnet plucks each wire and the resulting frequency signal is transmitted to the data acquisition system. The three gauge readings are averaged to account for eccentric loading effects. Because the raw output is frequency, it is immune to cable resistance variations and compatible with long cable runs without signal degradation.

Each load cell has a rated capacity of 200 kips. Factory-reported accuracy is $\pm 0.5\%$ full-scale ($\pm 1,000$ lb), resolution is 0.025% full-scale (± 50 lb), and the frequency output

range is 1,200-2,800 Hz. An embedded thermistor allows temperature correction if needed.

The load cells were paired with one hydraulic jack each. It is noted that the 200-kip capacity significantly exceeds the expected failure loads of most specimens (Table 9), which range from 5 to 21 kips. At the maximum expected test load of 21 kips, the load cells operate at approximately 10% of full-scale range. In this regime, the absolute accuracy of $\pm 1,000$ lb (± 1 kip) represents a relative uncertainty of approximately $\pm 5\%$ of the applied load for the largest specimens. However, for the lowest-capacity specimen (W-10-5, expected failure at 5 kips), the 1-kip accuracy represents $\pm 20\%$ of the expected failure load). This limitation is inherent to the available instrumentation and is acknowledged when interpreting load measurements at low load levels. The fact that pressure gauges were used in conjunction with the hydraulic jacks and pumps partially mitigates this concern, as the two readings can be cross-checked throughout each test.



Figure 101. Load cells positioned behind each hydraulic jack during the load test.

5.6 Campbell Scientific CR6 Data Acquisition System

All sensor data were acquired using a Campbell Scientific CR6 datalogger programmed in CRBasic at a 15-second scan interval. The CR6 served as the central hub of the instrumentation system, interfacing with 16 resistive strain gauges, 2 Geokon MEMS tiltmeters, 4 string potentiometers, 6 slip potentiometers, 2 Rieker inclinometers, and 2 Geokon vibrating-wire load cells. Given the number of sensors relative to the CR6's native terminal count, three Campbell Scientific AM16/32B relay multiplexers were deployed to expand channel capacity.

Two multiplexers (SN 23546 and SN 23505) were controlled directly by the CR6 via control port pairs C1/C2 and C3/C4, respectively. The first (SN 23546) routed signals from the 16 strain gauge channels to terminals U1, U2, and U3 for full-bridge measurement. The second (SN 23505) sequentially connected 14 channels - MEMS tiltmeters (channels 1-2), string potentiometers (channels 3-6), slip potentiometers (channels 7-12), and Rieker inclinometers (channels 13-14) - to terminals U5 and U7 for half-bridge or single-ended voltage measurements. Both multiplexers were operated using PortSet() and PulsePort() instructions with a 150 ms enable delay and powered from the CR6's 12 V output terminal.

The vibrating-wire load cell channels required dedicated signal conditioning and were handled separately. A Campbell Scientific AVW200 2-channel vibrating-wire analyzer module was connected to the CR6 via RS-232 on terminals U9/U10 (38,400 baud) and powered through the SW12_2 switched output. A third AM16/32B multiplexer (SN 23507) was wired directly to the AVW200, routing the coil and thermistor leads of six vibrating-wire gauges (three per load cell) through its COM terminals. The AVW200 was configured with a swept excitation range of 450 to 6,000 Hz at 5 V peak-to-peak, and all six channels were acquired sequentially in a single "AVW200()" instruction call with 60 Hz thermistor noise rejection enabled. In addition to resonant frequency, the module returned four diagnostic values per gauge (response amplitude, signal-to-noise ratio, noise frequency, and decay ratio), which were logged throughout testing to monitor signal integrity. Thermistor resistance was converted to temperature in CRBasic.

All channels across the three multiplexers were recorded in a single data table at 15-second intervals. The CRBasic program used for monitoring in this research can be found in Appendix X.

6. Experimental Test Setup

This section describes the physical test setup used for the load tests, including the positioning of the specimen, the reaction frame, the hydraulic loading system, and the load protocol. The configuration was designed to apply a controlled, quasi-statically increasing lateral load at a 7-ft cantilever arm on the exposed pile while maintaining the concrete cap in a fixed position, simulating the moment demand imposed on the connection by lateral loads on a pile bent.

6.1 Test Setup

Each specimen was oriented horizontally on the laboratory strong floor with the concrete cap at the bottom, resting on steel bearing plates, and the steel pile extending vertically upward. This orientation is the physical inversion of the field condition, in which the pile is below the cap, but it produces the same force and moment demands at the connection. The concrete cap was anchored to the strong floor to restrain it against horizontal translation.

Lateral load was applied to the pile at a point 7 ft (84 in) above the cap surface using two hydraulic jacks bearing through a steel load spreader cylinder on both flanges of the pile. An independent reference frame, separate from the specimen and reaction frame, was erected adjacent to the specimen to serve as the fixed reference for the string potentiometer mountings.

The reaction frame was a steel assembly fabricated in-house and anchored to the laboratory structural strong-floor using high-strength threaded rods. The primary load-carrying member was a wide-flange I-beam, oriented to bend about its strong axis. The frame also included a diagonal bracing. It was verified to resist twice the highest expected load in this research program.

An important note regarding the reaction frame: the frame used for the initial attempt of specimen W-10-7.5 on November 24, 2025 was found to be inadequate, and

the test was interrupted at approximately 9 kips. An upgraded, heavier reaction frame was installed before the second attempt on January 22, 2026. All specimens from the second attempt of W-10-7.5 onward were tested with the “upgraded frame”, which remained in service without modification for the remainder of the experimental program.

Figure 102 shows the overall test setup with the specimen, reaction frame and loading system.



Figure 102. Overall test setup.

Lateral load was applied to the pile using two Powerteam RH6010 hollow-ram hydraulic jacks, operating independently. Each jack has a rated push capacity of 60 tons (65.7 tons at 10,000 psi) with a 10-inch stroke and effective push area of 13.14 in². The RH series through-hole design (with a 2.13-inch center hole) allowed a reaction DYWIDAG bar to pass through each cylinder, enabling the load to be reacted against the reaction frame. Each jack was operated from its own hydraulic pump, allowing

independent load control at each loading point. Load at each jack was monitored by the Geokon 4900 vibrating-wire load cells described in the preceding section. Additionally, the applied pressure was independently measured with SSI MG-9V 3000-psi pressure gauges (0.25% accuracy) mounted between the hoses and the pumps.

The hydraulic jacks were supported by a hollow square tube that transfers the load onto the reaction frame. The DYWIDAG bars extend to the piles and connect to a thick steel cylinder via shackles. The cylinder transfers the load to the pile at the desired load application point, bending it.

6.2 Load Protocol

Static lateral load tests were conducted in accordance with ASTM D3966/D3966M-22 (Standard Test Methods for Deep Foundation Elements Under Static Lateral Load). Load application point was at 7 ft away from the cap. The design load for each specimen was taken as half the theoretical lateral capacity reported in Table 9.

Each test began with Procedure A, in which the applied lateral load was increased stepwise from 25% to 200% of the design load following the schedule in Table 13. At each increment, the load was held for the prescribed duration before the next increment was applied. At 200% of the design load, the load was held for 60 minutes. If no failure occurred by the end of this hold, loading continued as described in Table 14, for procedure B. Otherwise, the load was removed in four unloading steps.

Table 13. Procedure A, Standard Loading Schedule (ASTM D3966/D3966M-22)

Load Increment (% of Design Load)	Hold Duration (min)
0	-
25	10
50	10

Load Increment (% of Design Load)	Hold Duration (min)
75	15
100	20
125	20
150	20
170	20
180	20
190	20
200	60
Unloading	
150	10
100	10
50	10
0	-

When a specimen had not failed upon completion of the 200% hold under Procedure A, loading was continued beyond 200% in increments of 10% of the design load, following the duration schedule of ASTM D3966/D3966M-22 Procedure B. Unlike the strict sequential interpretation of the standard, which prescribes a complete unloading cycle after Procedure A before initiating Procedure B, the transition here was made without intermediate unloading. This modification was deliberate: it preserved a continuous load-deformation history and eliminated any recovery that an intermediate

unloading cycle would introduce. Each increment beyond 200% was held for 15 minutes. The sequence continued until the failure was achieved.

Table 14. Procedure B, Excess Loading Schedule (ASTM D3966/D3966M-22)

Load Increment (% of Design Load)	Hold Duration (min)
210	15
220	15
230	15
240	15
250	15
(+10% increments)	15
Failure load	60

In all load tests, failure was defined as the condition in which the pile head exhibited continuous progressive lateral displacement under constant pressurization of the hydraulic jacks, without concurrent increase in the measured load.

To distinguish genuine failure from transient fluctuations in the hydraulic system, it was necessary to confirm that failure had occurred. The following operational rule was applied: upon observing progressive displacement under jack pressurization without an increase in load, the load stage was held for 15 minutes. Then, a second attempt was made. Only after three consecutive unsuccessful attempts to increase the load was failure declared. The load was then removed in four equal decremental steps.

This three-attempt confirmation rule is not prescribed by ASTM D3966/D3966M-22. It was adopted to ensure conservative and repeatable failure across all tests.

Chapter 5:

Results of Experimental Phase

1. Results of Laboratory Load Tests

In this section, the results from the load test program is presented, including load vs displacement curves, rotation results and strain gauge deductions.

1.1. Specimen W-10-15

Specimen W-10-15 was tested on 1/30/2026. The load test required 3 tries because the first two tries were terminated early due to load imbalance between the two hydraulic jacks, which introduced unintended torsion into the specimen. The third attempt resolved this issue by operating each jack on an independent hydraulic pump, achieving balanced load application throughout. Figure 103 shows the load vs deflection curves.

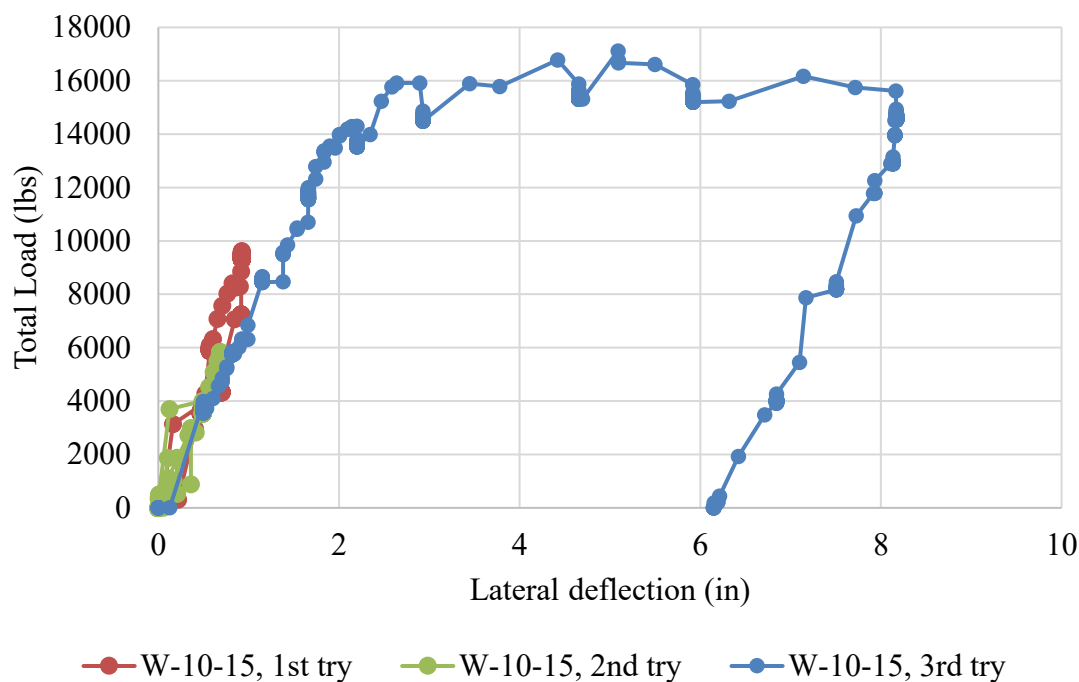


Figure 103. Specimen W-10-15 horizontal load vs lateral deflection curves.

At the start of the valid test, one pre-existing crack was noted: a hairline-width opening along the steel-concrete interface on the trailing side of the pile (opposite to the direction of applied load), carried over from the preliminary attempts. No additional cracking was present at zero load.

Through Stage 4 (peak total load approximately 11.9 kips, pile head deflection of 1.65 inches), no new cracks were observed in the concrete cap. The steel-concrete interface gap on the trailing side continued to widen gradually, though it remained narrower at comparable load levels than observed in shallower embedded specimens.

At Stage 5 (peak total load approximately 13.7 kips, pile head deflection 2 in), two distinct crack systems appeared for the first time. One consisted of diagonal cracks radiating outward from the flange tips at the top surface of the cap, as seen in Figure 104. The second consisted of hairline cracks on the vertical side faces of the cap, localized near the top of the cap on the trailing side, flanking the zone opposite to the load direction. These two systems were spatially distinct and did not appear to be initially connected.



Figure 104. Specimen W-10-15 crack formation after target loads of 14.5 kips and 17.5 kips (applied loads of 13.8 kips and 15.8 kips, respectively) (load direction: from right to left).

At Stage 6 (peak total load approximately 14.8 kips, pile head deflection 2.50 in), additional cracks appeared on the vertical side faces of the cap. The applied load plateaued at approximately 15-16 kips total despite continued pressurization.

At Stage 7 (peak total load approximately 15.8-16.2 kips, pile head deflection 4.50 inches), further cracking developed on the vertical side faces of the cap. Cracking remained strictly confined to the trailing side of the cap, no cracking was observed on the leading side other than the diagonal cracks from the flanges. Compression flange buckling was observed visually on flanges B and C, accompanied by mill scale loss, consistent with local flange yielding on the compression side. The specimen was considered to have reached its failure condition at this stage. Figure 105 shows the buckling in flanges B and C and mill scale loss.

At Stage 9 (peak total load dropped to approximately 14.8 kips, pile head deflection 8.10 in), additional hairline cracks appeared on the vertical side faces of the cap, with the crack pattern having propagated modestly downward from its original near-top initiation

elevation. Mill scale loss was also observed on the tension flanges (A and D), indicating yielding had extended to the tension side of the pile cross-section. The specimen was unloaded following this stage.



Figure 105. Specimen W-10-15 after final load stage (approx. 16 kips). Observed buckling in flanges B and C and mill scale loss.

Post-test crack width measurements, taken using a crack width ruler at the final load stage, confirmed that all cracks in the concrete cap were very thin and measured

0.03 mm or less, indicating that despite pile yielding and flange buckling, concrete damage remained superficial throughout the test.

The expected maximum horizontal load for the test was 13.1 kips, the point at which the pile's theoretical flange local buckling moment (92 kip-ft) is achieved. As observed, the load test curve remained elastic until approximately the expected maximum load. Then, the curve started to transition into the plastic phase, plateauing at approximately 16 kips, 1.22 times the expected maximum. At this point, the load-generated moment in the pile is slightly above its theoretical full-plastic moment of 107.18 kip-ft.

The specimen suffered an elastic deflection of approximately 1.83 inches at 13.1 kips, then transitioned to the full plastic portion, when deflection reached 2.64 inches at 16 kips. The load test continued until a maximum lateral deflection of 8.16 inches was reached, with the 16-kip load plateau maintained.

The inclinometers and tiltmeters generated rotation measurements displayed in Figure 106.

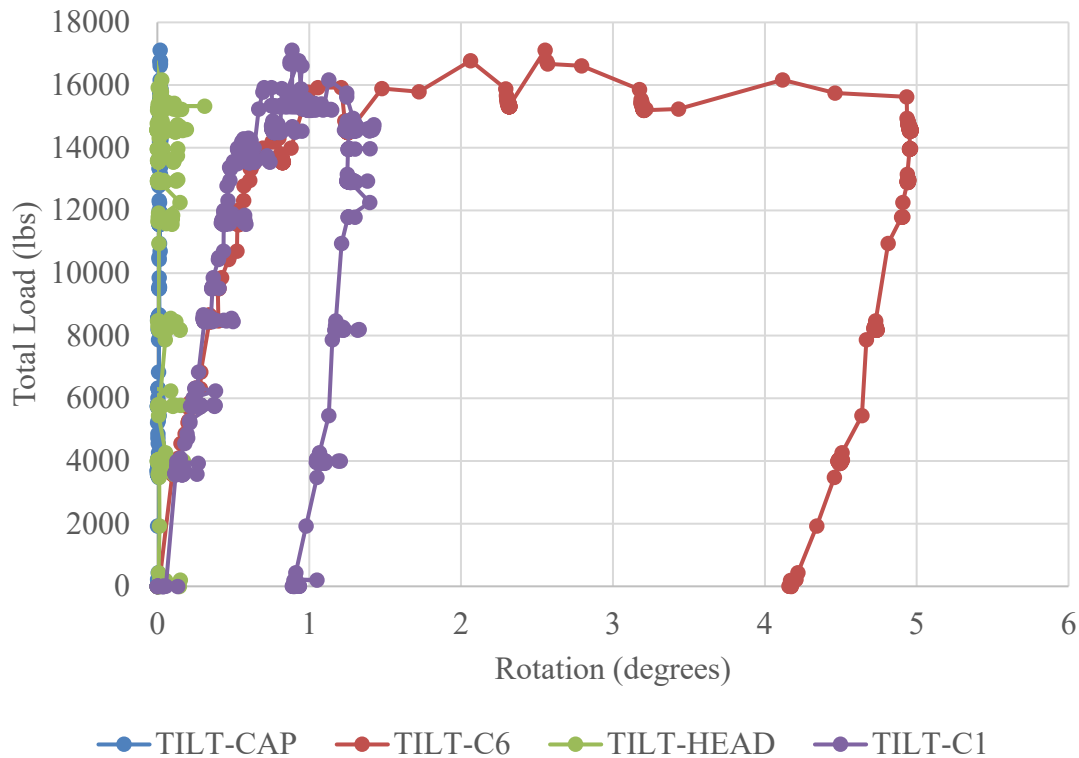


Figure 106. Specimen W-10-15, rotation measurements (positive rotation follows the load direction).

TILT-CAP showed negligible rotation throughout the entire loading sequence, confirming that the concrete cap remained stationary and providing a baseline sanity check for the instrumentation.

TILT-HEAD, measuring rotation at the embedded pile head inside the concrete, also registered negligible rotation at all load stages. Scatter did not exceed approximately 0.1 degrees even at peak load. This indicates that at the depth of the embedded pile head, no measurable rotation occurred during the test.

In the elastic loading range, up to approximately 13 kips, TILT-C1 and TILT-C6 tracked closely together, consistent with the pile rotating approximately as a rigid body over that portion of its length. Both instruments recorded rotation in the direction of applied load, reaching on the order of 0.5-0.6 degrees at that load level.

At approximately 13 kips, TILT-C1 and TILT-C6 began to diverge, indicating the onset of local flange buckling. At approximately 16 kips, divergence became pronounced,

consistent with full local buckling of the compression flanges. This is corroborated by the visual observation of flange buckling on flanges B and C at approximately 2 to 3 inches above the cap surface (between the two instrument locations). Above the buckle, TILT-C6 continued to accumulate rotation freely, ultimately reaching nearly 5 degrees. Below the buckle, TILT-C1 was partially decoupled from the deformation above, and recorded a maximum rotation of approximately 1.0 to 1.2 degrees.

Upon unloading, both instruments retained residual rotation, reflecting permanent plastic deformation associated with flange yielding and buckling. TILT-C6 retained approximately 4 to 5 degrees of residual rotation, while TILT-C1 retained approximately 1 degree. The larger residual at TILT-C6 is consistent with its position above the buckle region, where most of the inelastic deformation was concentrated.

Spring potentiometers at each flange (SLIP A, SLIP B, SLIP C, and SLIP D) measured how much the flanges moved relative to the cap surface, as well as two spring potentiometers at each end of the concrete cap (SLIP AD and SLIP BC) measured if the cap moved longitudinally. SLIP C malfunctioned during the test and its data were not used. The results are seen in Figure 107.

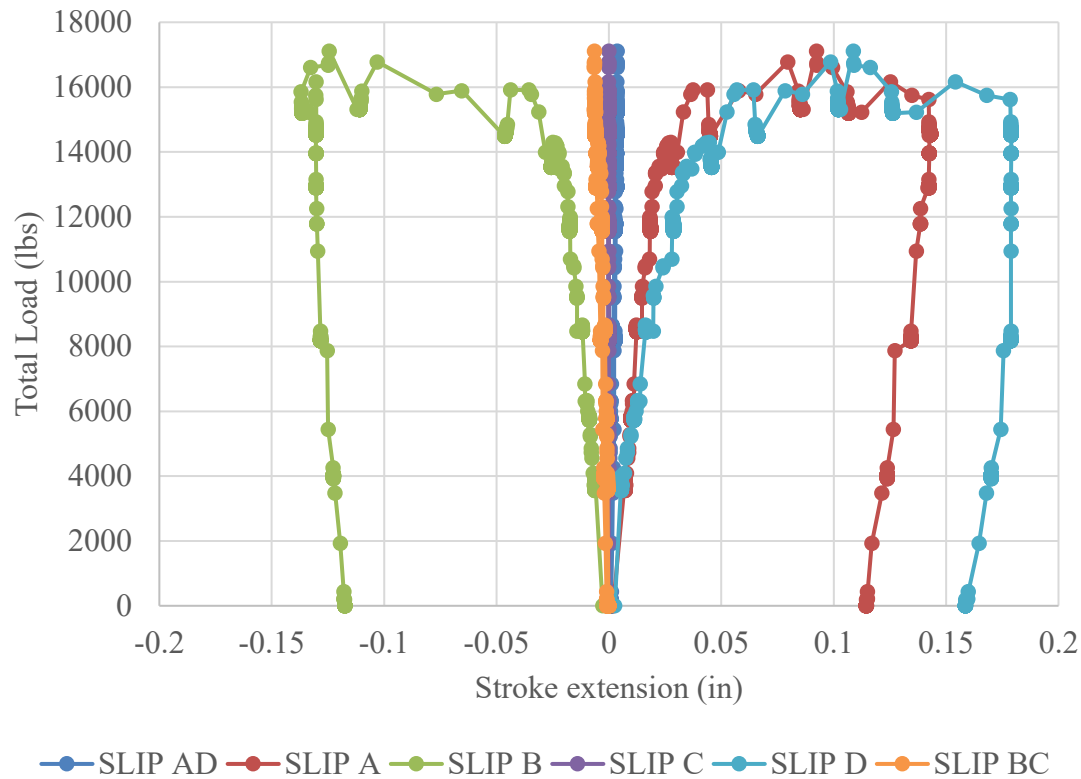


Figure 107. Specimen W-10-15, spring potentiometer measurements.

SLIP AD and SLIP BC recorded negligible displacement throughout the entire loading sequence, confirming that the concrete cap remained stationary and did not translate or rock on its support at any load stage.

On the tension side, flanges A and D were pulled away from the concrete cap surface as the pile rotated under lateral load, producing positive stroke extension. On the compression side, flange B was progressively inserted into the concrete cap surface, producing negative stroke extension. This behavior is consistent with pile rotation about a point near the cap surface.

A notable result is the close symmetry between SLIP A and SLIP B throughout the test. These two instruments measure opposite flanges parallel to the load direction (flange A on the tension side and flange B on the compression side). Their magnitudes tracked nearly identically and in opposite directions across all load stages, indicating that the pile

rotated cleanly about a point near the cap surface with negligible net translation at that flange pair.

In the elastic loading range, up to approximately 13 kips, all functioning sensors showed a stiff, approximately linear load-displacement response. SLIP A and SLIP D tracked closely together in this range, consistent with symmetric behavior on the tension side. This stiffness transition is consistent with the onset of local flange buckling identified at the same load level in the tiltmeter data.

Beyond approximately 13-14 kips, the load-displacement curves flattened, with stroke extension continuing under little additional load increase, indicating yielding and loss of stiffness. In this inelastic range, SLIP A and SLIP D began to diverge, with SLIP A reaching approximately 0.15 inches and SLIP D reaching approximately 0.18 inches at peak load. This asymmetry, although not large, is attributed to differential concrete cracking and localized spalling at the cap surface on the tension side, which affected the two flange measurements unequally rather than reflecting any fundamental asymmetry in the pile section behavior.

The strain gauge measurements were collected and reduced in order to obtain and visualize different behaviors in the pile, rebars, and concrete surface. The strain measurements are presented in Appendix A. The calculated stresses, moments, and axial loads are presented in this section. In processing, strain measurements that were larger than the steel yielding point or larger than the estimated strain at 70% of the concrete compressive strength were simply eliminated from analysis.

For this specimen, strain gauges EM-I-A, EM-I-B, EM-M-B, and EM-H-A malfunctioned. Therefore, analysis originating from these gauges is limited. Additionally, when appropriate for these gauges, the results were manually replaced with their symmetric pairs' data for analysis.

Figures 108 and 109 show the total stresses deduced from the strain gauges in the steel reinforcement cage and the concrete cap surface.

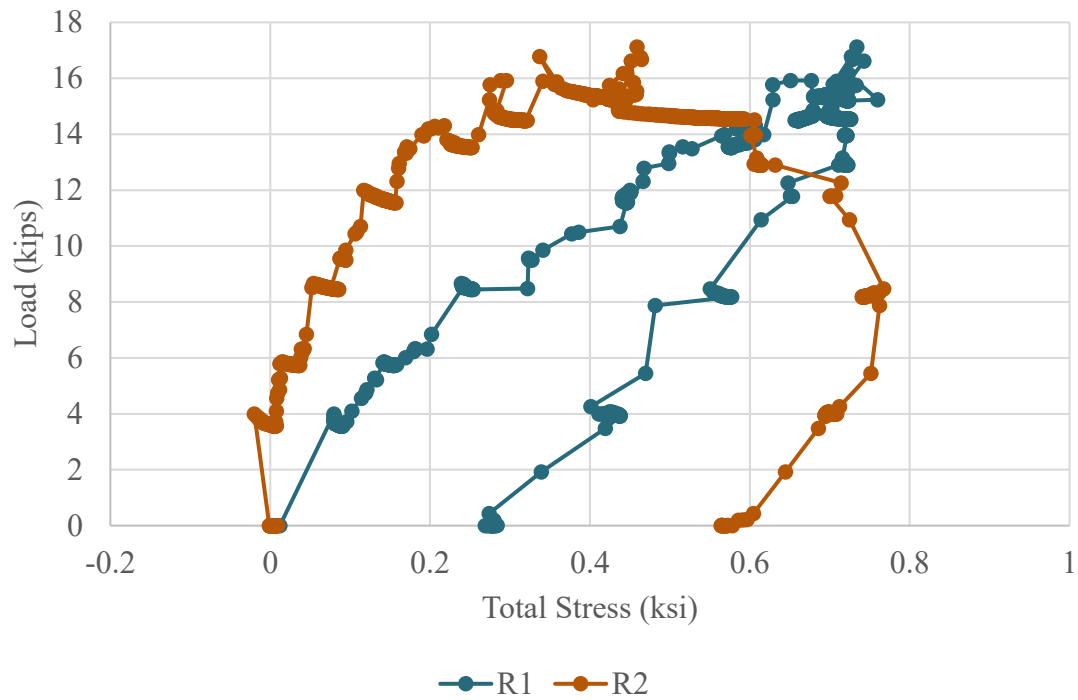


Figure 108. Specimen W-10-15, total stress measured by strain gauges R1 and R2 on steel reinforcement cage.

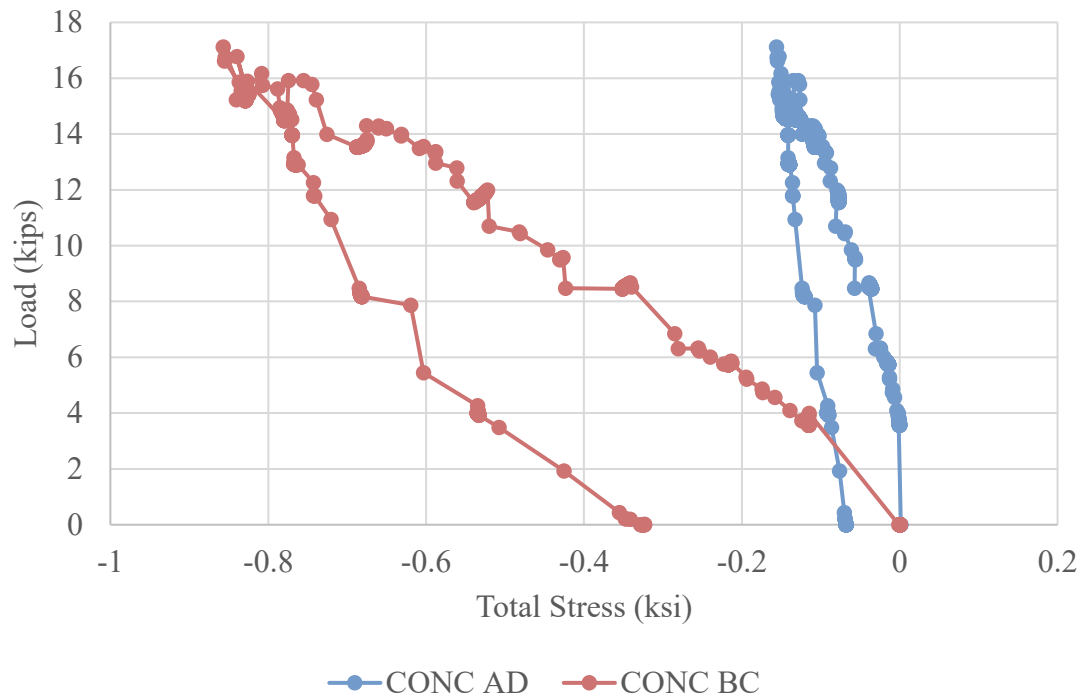


Figure 109. Specimen W-10-15, strain gauges CONC AD and CONC BC on the concrete cap surface.

As observed, the gauges in the steel reinforcement cage showed very little stress throughout the entire load test, with both gauges indicating tension. The concrete cap surface on the trailing and leading sides both showed compression behavior due to pile movement. CONC BC on the leading side was compressed to approximately 900 psi, and CONC AD (trailing side) showed smaller compression of approximately 150 psi, due to the internal rotation of the embedded pile. These results help confirm previous observations that the 15-in embedment generates greater fixity in the system, leading to steel pile structural failure rather than concrete failure.

Regarding strain gauges on the pile itself, plots containing the total stresses can be found in Appendix A. In figures 110, 111 and 112, the calculated bending moments and axial loads are seen for, respectively, the exposed gauges and the embedded gauges.

Bending moment results clearly show how flange A is behaving in tension and flange B, in compression. EX-C6-A/B and EX-C1-A/B presented similar results of bending moment, the small difference was only due to their physical placement 5 in apart. Bending

moment increased almost linearly until horizontal load close to 13.5 kips, when they achieved approximately 75 kip-ft and 71 kip-ft, for EX-C1-A/B and EX-C6-A/B, respectively. This value falls within the range where the steel is transitioning from elastic to plastic, yielding having started already.

EM-M-A/B and EM-H-A/B were able to register moment values at the maximum horizontal load of 16 kips: 76.15 kip-ft and 22.15 kip-ft, respectively. It is clear that, after the moment profile peaks near the steel/concrete interface, it decreases with embedment depth. Figure 113 shows the moment profile along the pile length, along with a comparison to the expected theoretical moment based on the applied horizontal load. The strain gauge measurements matched the expected moments well at the lower load stages. As the horizontal load increases, the steel pile begins to reach moment levels near or beyond the onset of yielding, at which point the measured moment no longer matches the expected values.

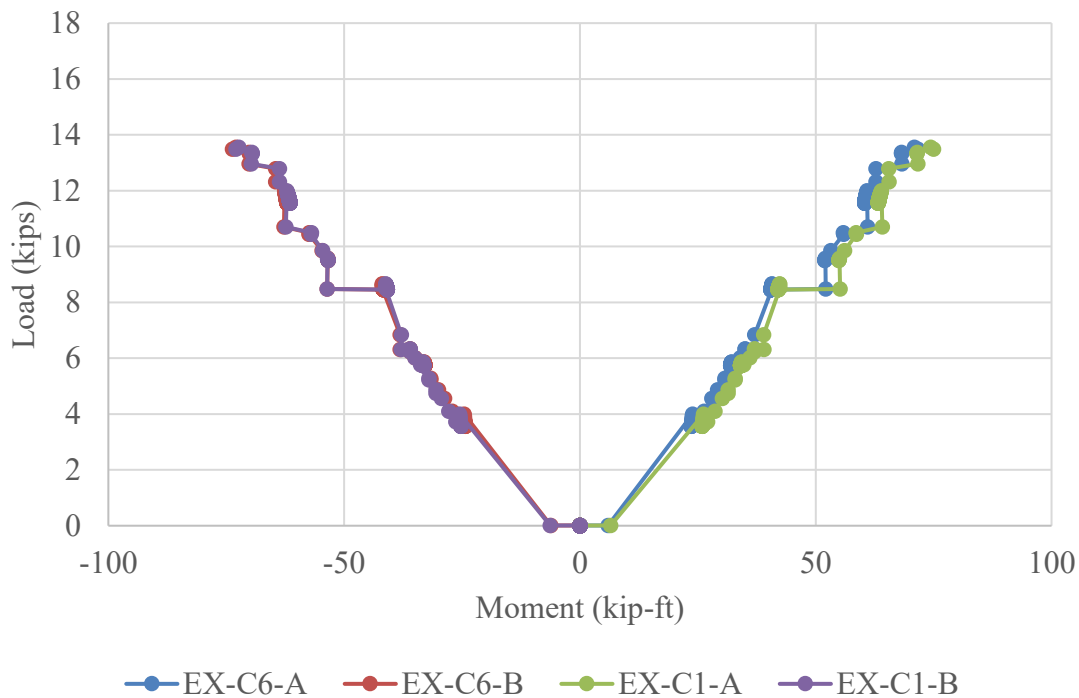


Figure 110. Specimen W-10-15, calculated bending moment for EX strain gauges.

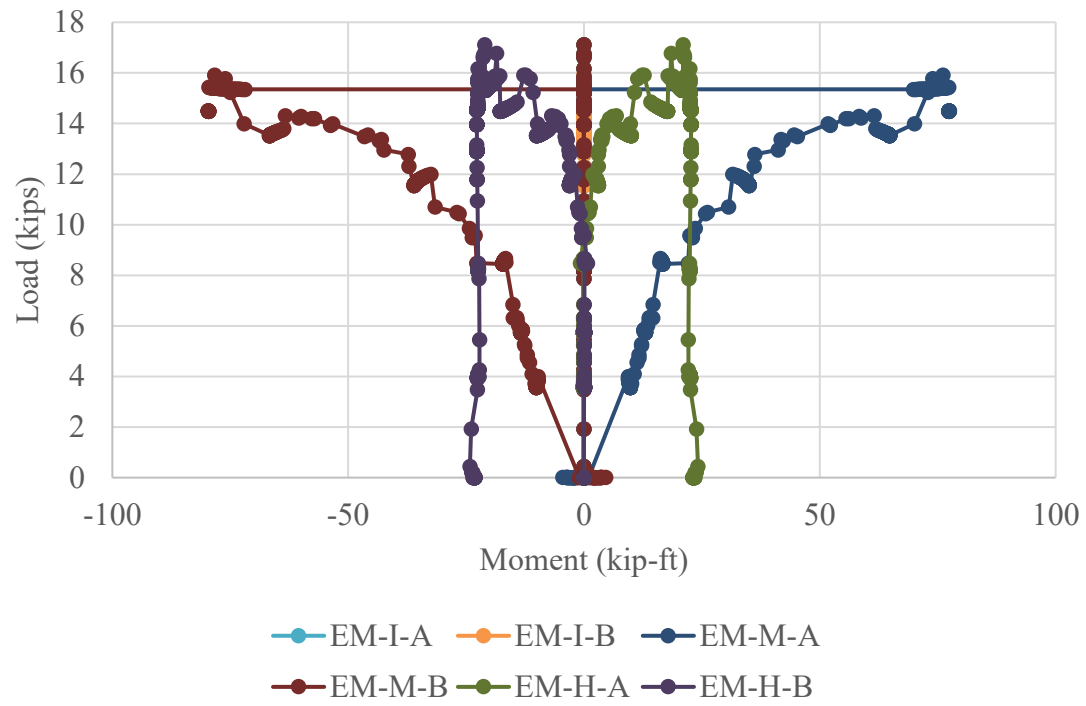


Figure 111. Specimen W-10-15, calculated bending moment for EM strain gauges.

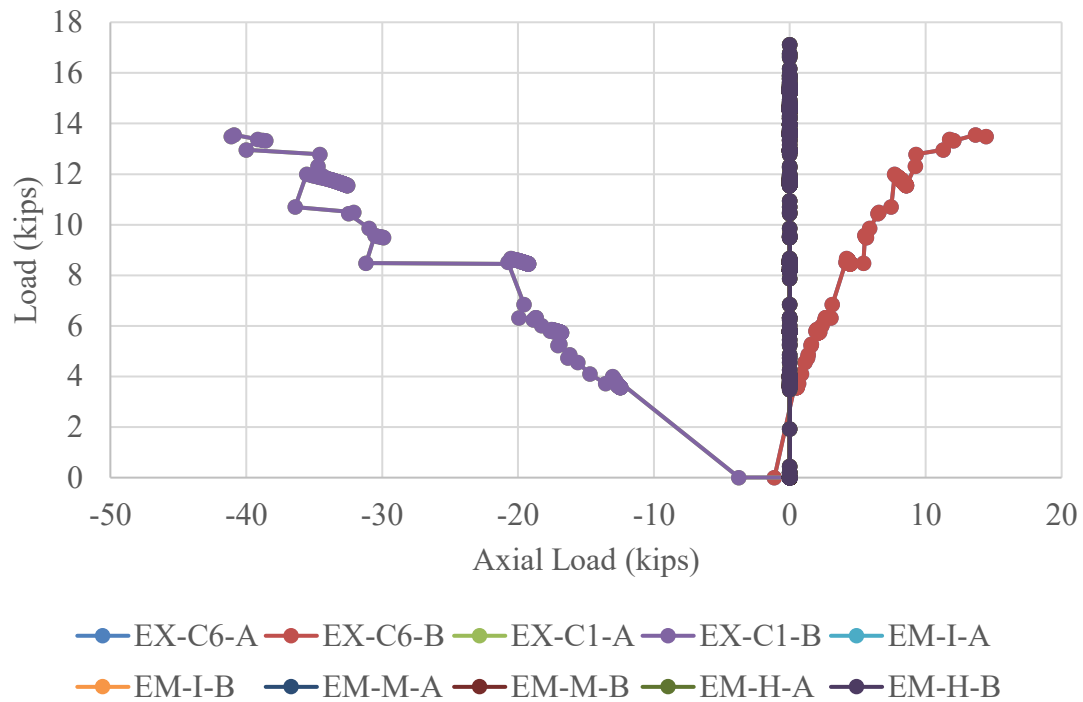


Figure 112. Specimen W-10-15 axial loads calculated from strain gauges on the steel pile.

Axial loads were measured only at the C6 and C1 levels. At C6, the values were reasonable and consistent with the load test. At C1, the axial load was higher, suggesting a possible strain gauge malfunction beyond a certain point in the load test.

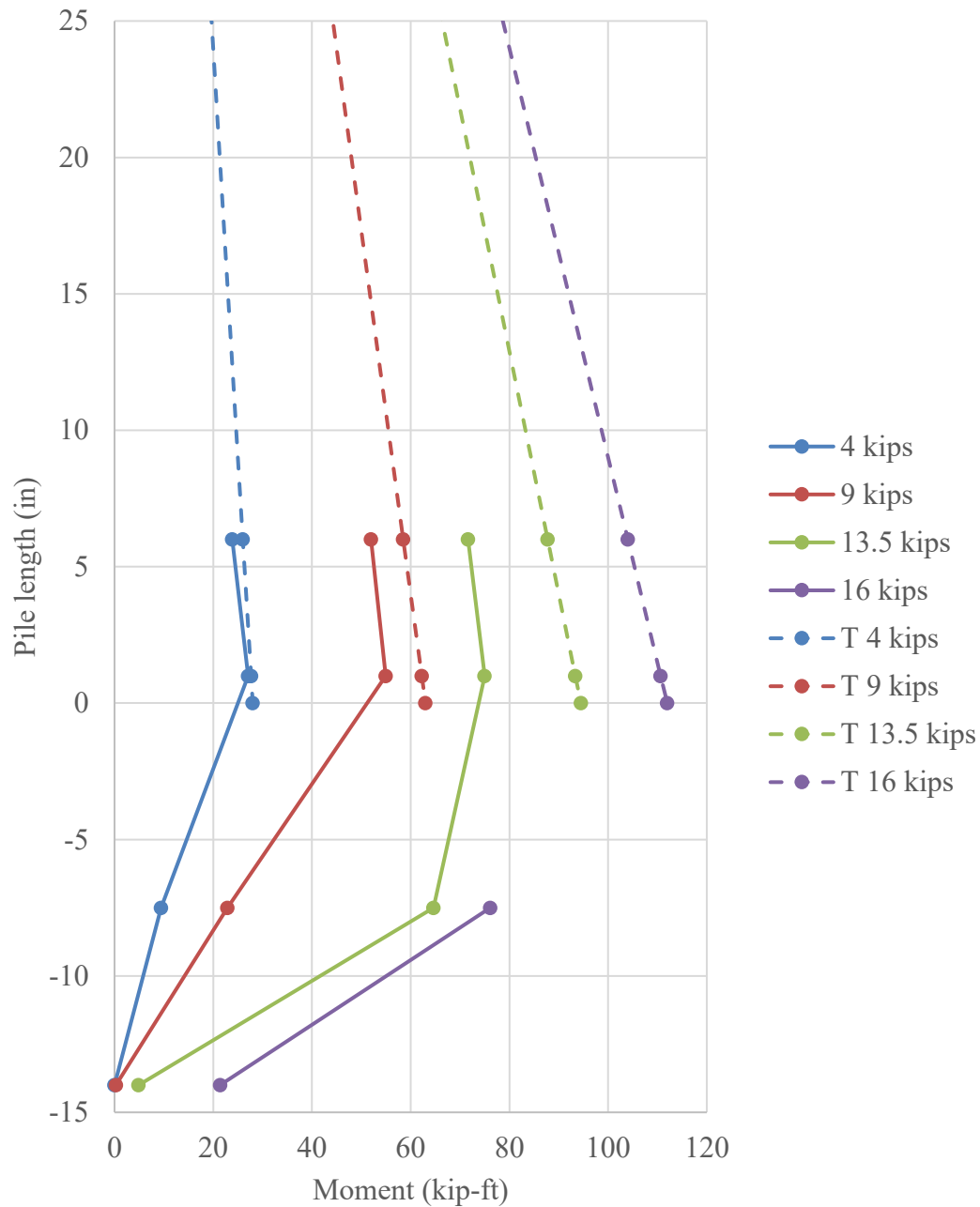


Figure 113. Specimen W-10-15 measured moments (solid lines) and theoretical moments (dashed lines) for each load stage. Zero y-axis marks the cap surface: positive is the exposed pile length, and negative is the embedded pile length.

1.2. Specimen W-10-10

Specimen W-10-10 was tested on 1/27/2026. Figure 114 shows the load vs. deflection curves.

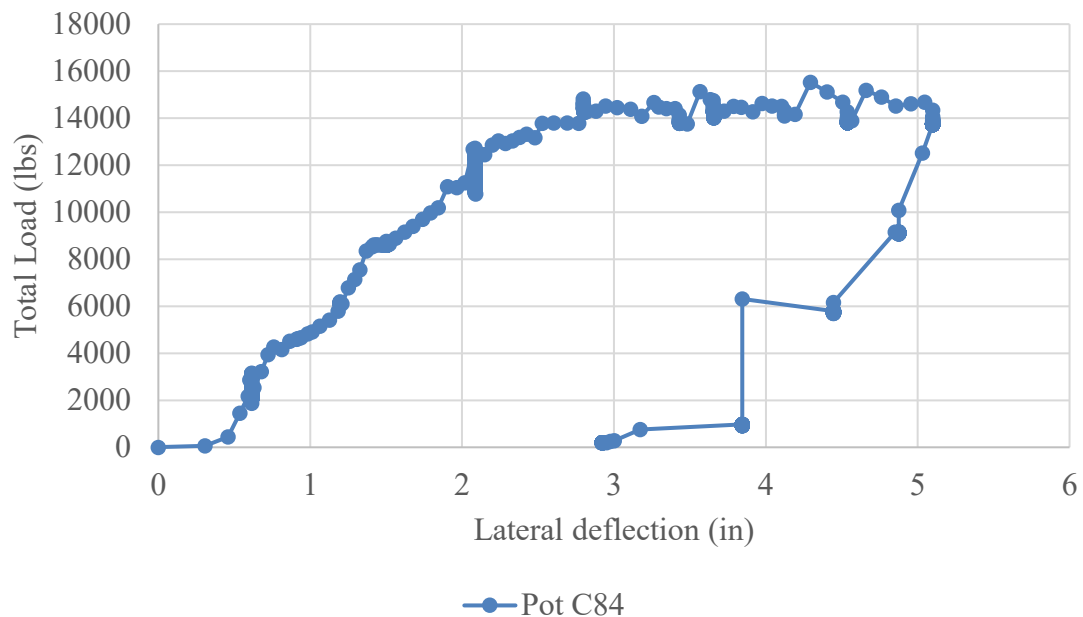


Figure 114. Specimen W-10-10 horizontal load vs lateral deflection curve.

Through Stage 4 (peak total load approximately 12.6 kips, pile head deflection of 2.06 inches), no new cracks were observed in the concrete cap. The steel-concrete interface gap on the trailing side widened progressively with each load stage, and by Stage 3 the pile was visibly battering under lateral load, consistent with the growing separation at the connection interface.

At Stage 5 (peak total load approximately 14.6 kips, pile head deflection 2.78 inches), two distinct crack systems appeared for the first time. One consisted of diagonal cracks radiating outward from the flange tips at the top surface of the cap. The second consisted of cracks on the vertical side faces of the cap, localized near the top of the cap on the trailing side. These two systems were spatially distinct, consistent with the crack pattern observed in specimen W-10-15 at the same load stage.

At Stage 6 (peak total load approximately 15.0 kips, pile head deflection 3.66 inches), the applied load plateaued and would not increase beyond approximately 15 kips despite continued pressurization. Visual observation confirmed the onset of flange buckling on flange C at this stage. Additional cracking continued on the vertical side faces of the cap.

At Stage 7 (peak total load approximately 15.1 kips, pile head deflection 4.59 inches), flanges B and C were clearly buckling, and one new crack appeared on the vertical side face of the cap. The load remained unable to exceed 15 kips. The specimen was considered to have reached its failure condition at this stage.

At Stage 8 (peak total load approximately 14.3 kips, pile head deflection 5.19 inches), flanges B and C continued to exhibit pronounced buckling. One additional minor crack appeared on the vertical side face of the cap. The specimen was unloaded following this stage. Figure 115 shows the crack formation observed during the load test, and Figure 116 shows the flange buckling condition after the test concluded.



Figure 115. Specimen W-10-10 crack formation after target loads of 14.5 kips and 17.5 kips (applied loads of 14.6 kips and 15.1 kips, respectively) (load direction: from right to left).

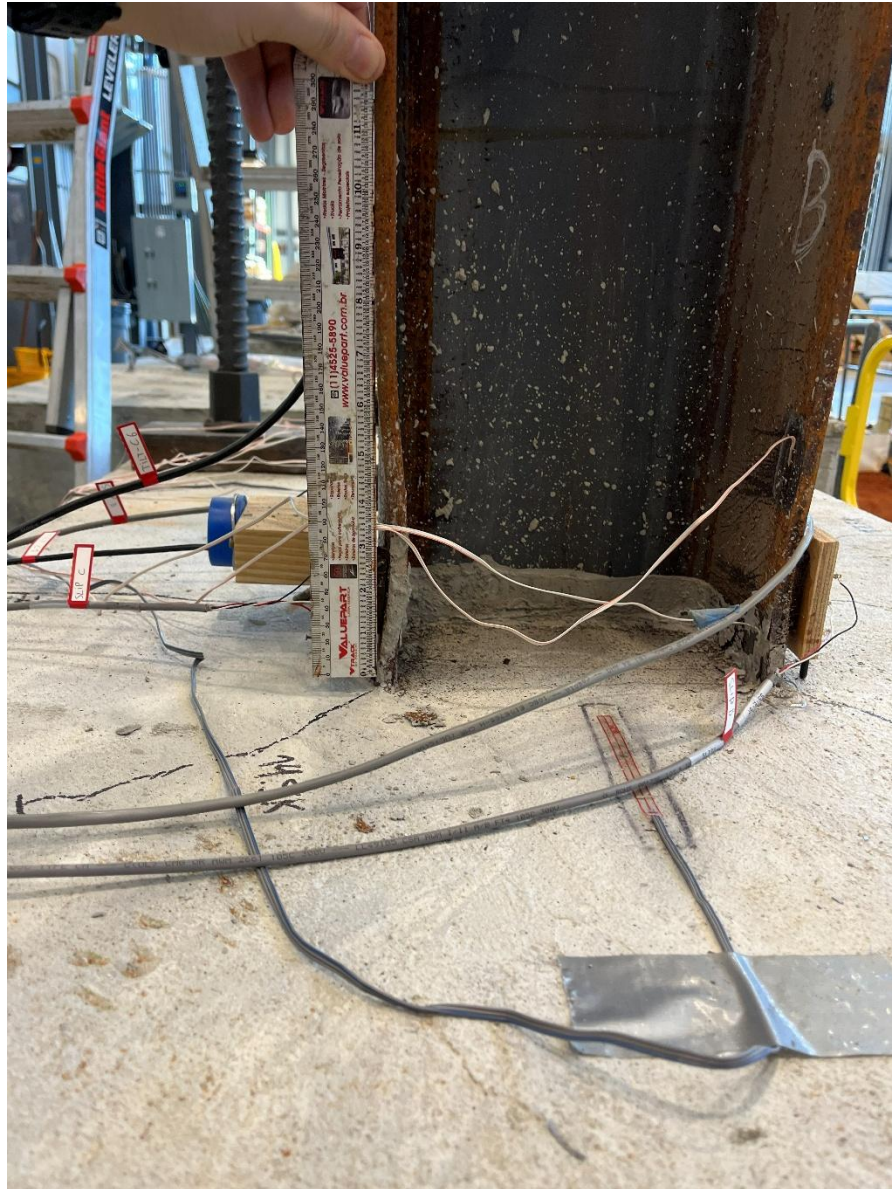


Figure 116. Specimen W-10-10 after final load stage (approx. 16 kips). Observed buckling in flanges B and C and mill scale loss.

Post-test crack width measurements confirmed that all cracks in the concrete cap measured 0.08 mm or less, indicating that concrete damage remained superficial throughout the test despite pile yielding and flange buckling.

The expected maximum horizontal load for the test was 13.1 kips, the point at which the pile's theoretical flange local buckling moment (92 kip-ft) is achieved. The load test curve remained approximately elastic until this range, then transitioned into a plastic

plateau at approximately 14.6-15.0 kips (1.11 to 1.14 times the expected maximum). At the maximum load, the theoretical moment generated at the connection interface is approximately 102 kip-ft, above the flange local buckling limit and near the full plastic moment of the pile (107 kip-ft). This confirms that the specimen failed by steel pile structural failure rather than concrete failure.

It is worth noting, however, that the concrete bearing capacity of the connection, estimated using the Marcakis and Mitchell (1980) method, falls in the range of approximately 13 to 19 kips depending on the assumed bearing width. The observed failure load of 14.6-15.0 kips falls within this range, indicating that while steel structural failure controlled in this specimen, the 10-inch embedment places the system near the boundary where the controlling failure mode could transition from steel structural failure to concrete bearing failure. This is consistent with the elevated concrete stress measurements discussed below.

The specimen experienced an elastic deflection of approximately 2.06 inches at 12.6 kips, transitioning to the plastic plateau at approximately 14.6 kips and 2.78 inches of deflection. The load test continued until a maximum lateral deflection of 5.19 inches was reached, with the load plateau maintained near 15 kips.

The inclinometers and tiltmeters generated rotation measurements displayed in Figure 117.

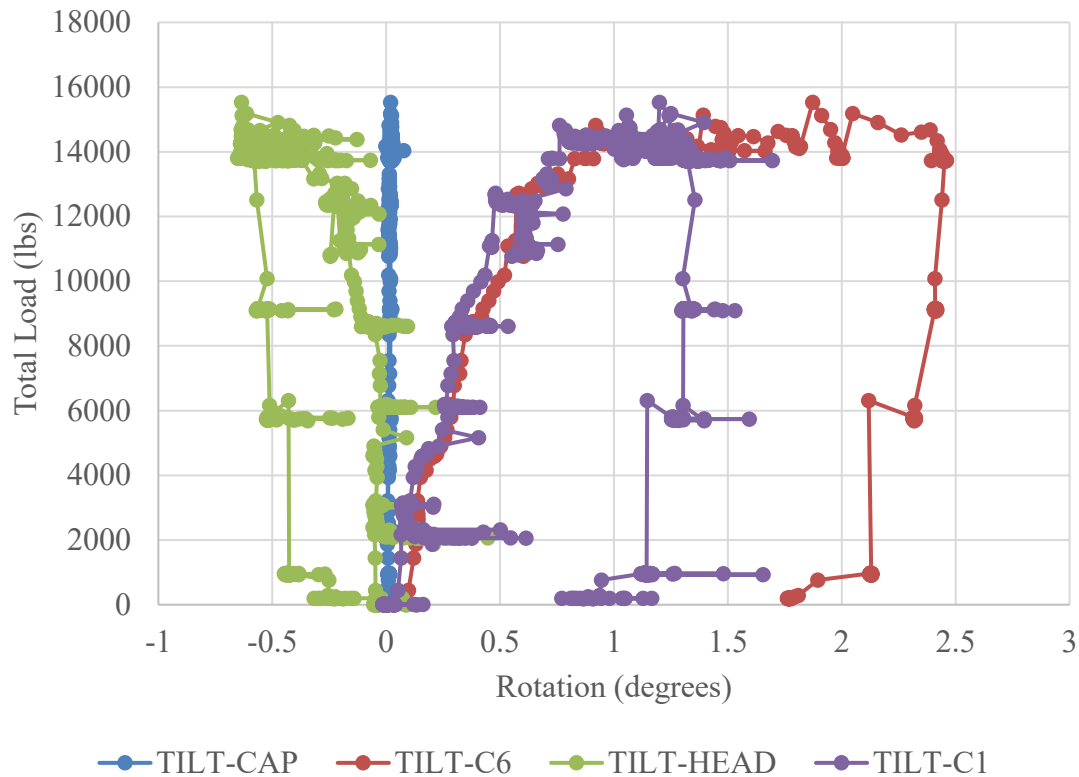


Figure 117. Specimen W-10-10, rotation measurements (positive rotation follows the load direction).

TILT-CAP, again, showed negligible rotation throughout the entire loading sequence, confirming that the concrete cap remained stationary and providing a baseline sanity check for the instrumentation.

TILT-C1 and TILT-C6 behaved similarly throughout the elastic loading range, both recording rotation in the direction of applied load and tracking closely together, consistent with the pile rotating approximately as a rigid body over that portion of its exposed length. Both instruments reached approximately 1 degree of rotation near the failure load of 14.6 kips. At approximately 13-14 kips, TILT-C1 and TILT-C6 began to diverge, with TILT-C6 continuing to accumulate rotation freely as the pile deformed above the buckle zone, ultimately reaching larger values, while TILT-C1, located below the buckle zone, was partially decoupled from the deformation above and showed limited additional rotation. This behavior is consistent with what was observed in specimen W-10-15 and reflects the

localization of inelastic deformation at the buckle zone between the two instrument positions.

TILT-HEAD produced the most counterintuitive rotation result. Rather than rotating in the direction of the applied load, the tiltmeter exhibited a counterclockwise rotation (opposite to the load direction) beginning at approximately 9 kips. This onset coincides with the stage at which the pile was visibly battering and gap formation at the connection interface was well established. The counterclockwise rotation grew progressively with increasing load. Unlike specimen W-10-15, where TILT-HEAD registered negligible rotation throughout the test, the W-10-10 specimen exhibited a measurable and consistent signal at the pile head.

Although interesting, this measurement likely has a very simple physical explanation. The TILT-HEAD sensor is attached to the end of a rod, which in turn is welded to the very base of the web of the pile head. This creates an effect similar to a book balancing on a cylinder. As the load increases and rotates the pile clockwise, the pile head (book) pushes the rod (cylinder), making it rotate in the opposite direction. Therefore, although the sensor rotates counterclockwise, the pile head is, in fact, rotating clockwise (in favor of the load direction). Figure 118 shows a photograph of the demolished specimen confirming the final pile head position, and Figure 119 shows an analogous, exaggerated and simplified explanation of the phenomenon.



Figure 118. Specimen W-10-10. Demolished specimen showing the final position of the pile head inside the embedded zone.

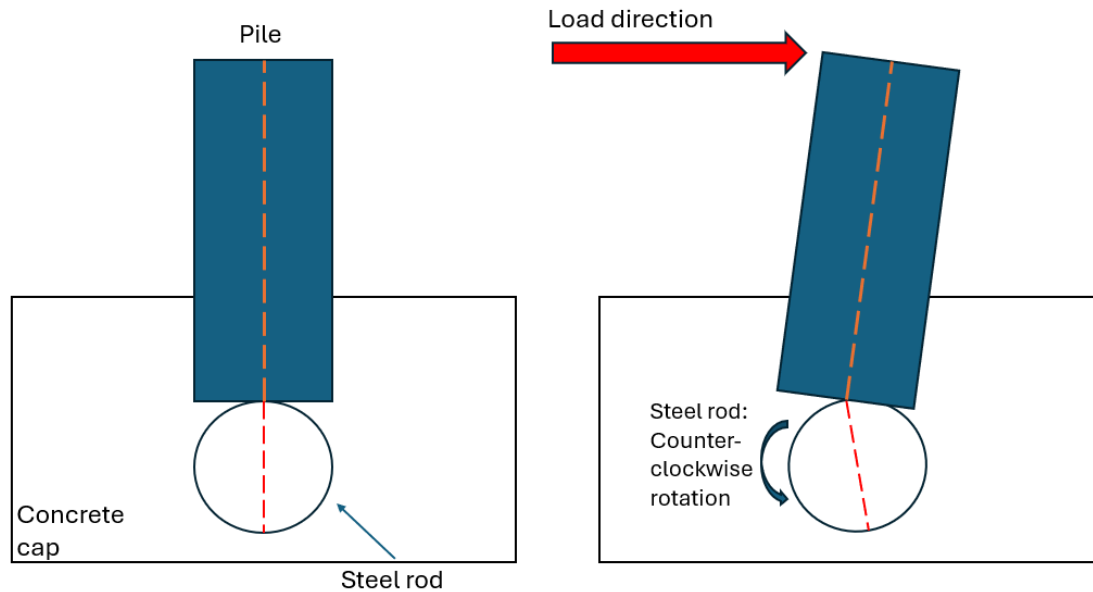


Figure 119. Specimen W-10-10, exaggerated explanation of the TILT-HEAD rotation phenomenon.

Spring potentiometers at each flange (SLIP A, SLIP B, SLIP D) measured relative displacement between the pile flanges and the cap surface. SLIP C malfunctioned during the test and its data was not used. Results are shown in Figure 120.

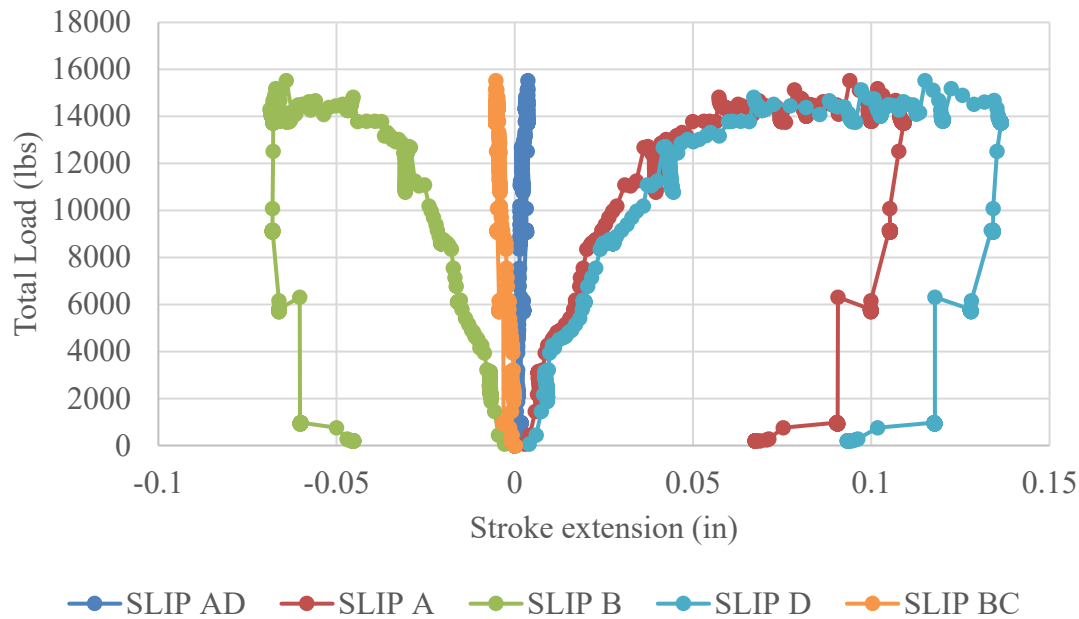


Figure 120. Specimen W-10-10, spring potentiometer measurements.

SLIP AD and SLIP BC recorded negligible displacement throughout the entire loading sequence, confirming that the concrete cap remained stationary and did not translate or rock on its support at any load stage.

On the trailing side, flanges A and D were pulled away from the cap surface under lateral load, producing positive stroke extension of approximately 0.075 inches at the failure load of 14.6 kips. On the leading side, flange B was progressively pushed into the cap surface, producing negative stroke extension of approximately 0.075 inches at the same load. The close symmetry between the tension-side and compression-side measurements indicates that the pile rotated cleanly about a point near the cap surface with negligible net translation at that flange pair, consistent with the behavior observed in specimen W-10-15.

The strain gauge measurements were collected and reduced to obtain bending moments, axial loads, and stress distributions along the pile and in the cap. Strain measurements exceeding the steel yield point or 70% of the estimated concrete compressive strength were eliminated from analysis. For this specimen, strain gauges EX-C6-B, EX-C1-B, EM-I-A, EM-I-B, EM-M-B, and EM-H-B all malfunctioned. Because nearly all B-flange gauges were non-functional, they were replaced by the opposite value

of their symmetric A-flange counterparts for bending moment calculations. As a consequence of this substitution, axial load is assumed to be zero for all gauge pairs in this specimen.

Figures 121 and 122 show the total stresses measured by the strain gauges in the steel reinforcement cage and on the concrete cap surface, respectively.

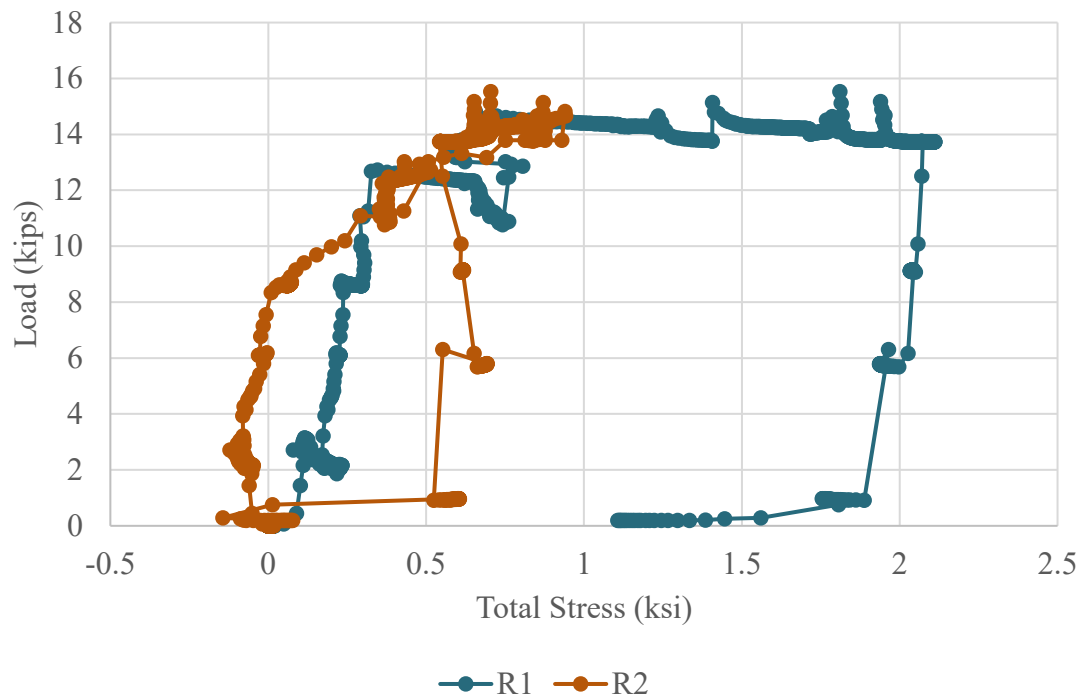


Figure 121. Specimen W-10-10, total stress measured by strain gauges R1 and R2 on steel reinforcement cage.

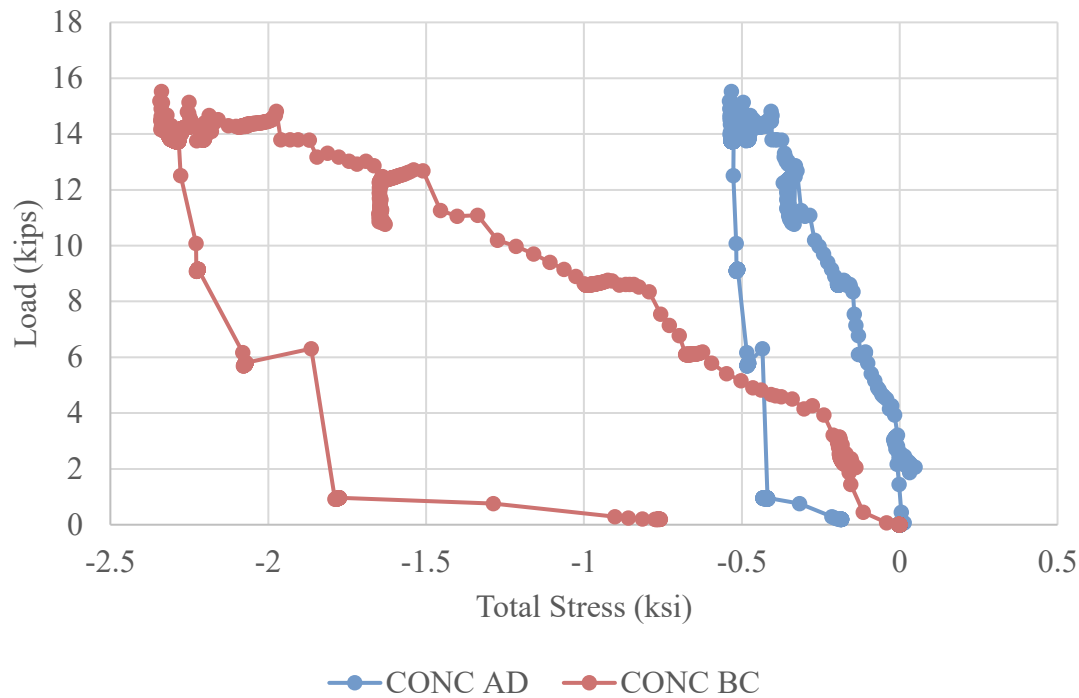


Figure 122. Specimen W-10-10, total stress measured by strain gauges CONC AD and CONC BC on the concrete cap surface.

The reinforcement cage gauges showed very little stress throughout the test. R1, on the trailing side, remained in tension throughout, accumulating approximately 2 ksi at the end of the test. R2, on the leading side, began in slight compression up to approximately 8 kips, then transitioned to tension, reaching a maximum of approximately 0.75 ksi. Both values are small relative to the reinforcement yield stress, consistent with the specimen failing by pile structural failure rather than cap failure.

The concrete surface gauges showed compression on both the leading and trailing faces. CONC BC, on the leading side, reached approximately 2.5 ksi in compression (higher than the 0.9 ksi recorded at the same location in specimen W-10-15). CONC AD, on the trailing side, showed smaller compression values. The substantially elevated concrete stress on the leading face, despite the specimen still failing by steel structural buckling, reflects the reduced embedment depth generating higher bearing demand on the concrete. This supports the interpretation that the 10-inch embedment places the

system near the transition between steel-controlled and concrete-controlled failure, consistent with the Marcakis and Mitchell (1980) capacity range discussed above.

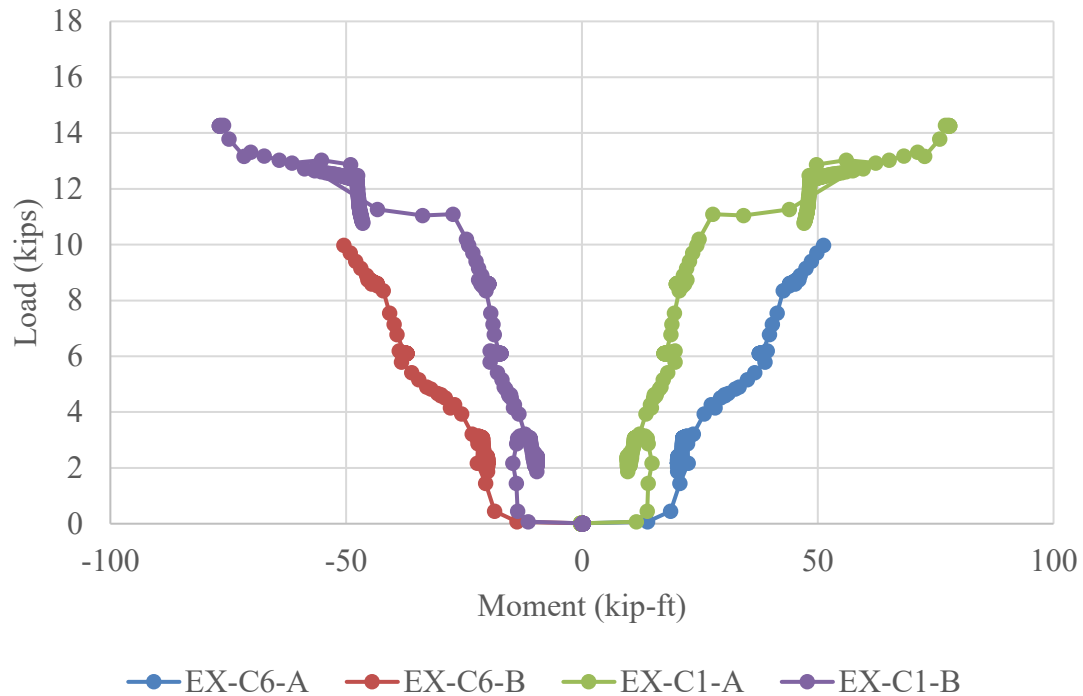


Figure 123. Specimen W-10-10, calculated bending moment for EX strain gauges.

Figure 123 shows the bending moment results for the exposed strain gauges. The EX-C6 and EX-C1 gauge pairs showed anomalous behavior throughout the test, with a stiffer moment response and lower-than-expected moment magnitudes during the elastic phase relative to specimen W-10-15 and relative to theoretical expectations. One possible explanation is that during concrete placement, concrete splashed onto and hardened around the unprotected gauge locations on this specimen (unlike W-10-15, duct tape protection was not applied to these gauges prior to casting). The hardened concrete encasing the gauges may have partially restrained their movement, reducing the measured strains below their true values. As a result, the EX-C6 and EX-C1 moment data are considered unreliable and are not used for quantitative conclusions.

Figure 124 shows the bending moment results for the embedded strain gauges. EM-M-A/B showed reasonable behavior up to approximately 4 kips, reaching

approximately 18 kip-ft at 3 kips and growing as expected through that range. Beyond approximately 4 kips, the gauge output became erratic, reaching implausibly high values, and is considered unreliable past that threshold. EM-H-A/B performed well throughout the entire test, reaching approximately 15 kip-ft at the maximum load of 14.6 kips and returning to near-zero residual upon unloading, consistent with the elastic behavior expected at that embedment depth.

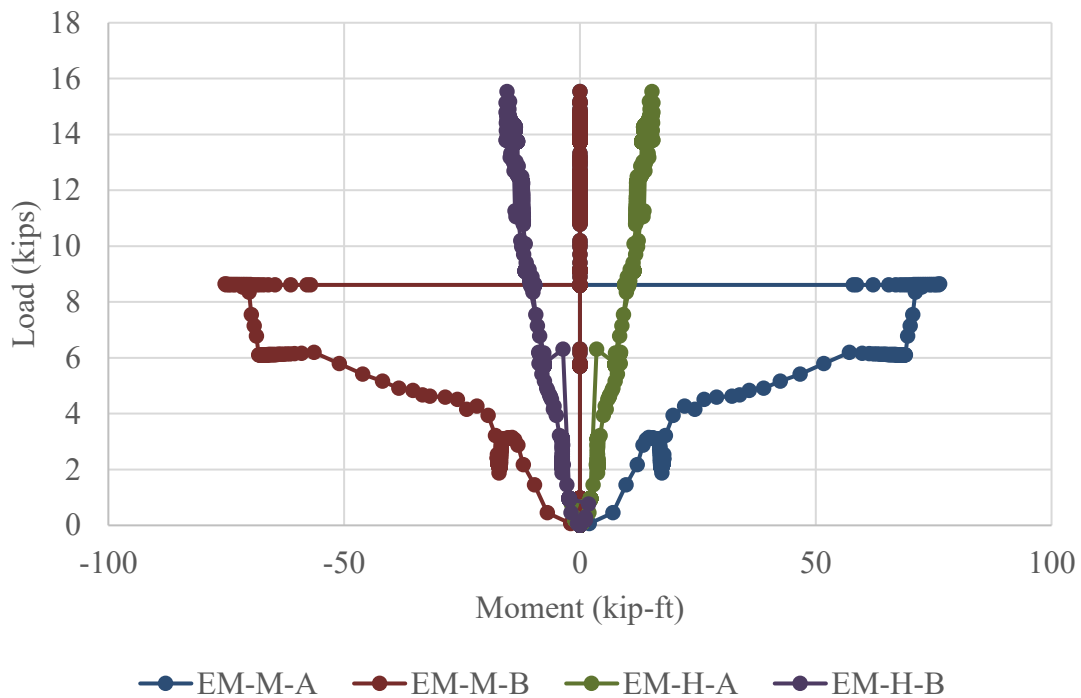


Figure 124. Specimen W-10-10, calculated bending moment for EM strain gauges.

Figure 125 shows the moment profile along the pile length for each load stage, along with a comparison to the theoretical moment based on applied horizontal load. At low load levels, the measured moments at the reliable gauge locations were reasonably consistent with theoretical expectations. At higher load levels, the unreliable EX-C6 and EX-C1 data preclude a complete moment profile comparison. The EM-H-A/B results at the deepest reliable location, however, confirm that moment is being transferred into the embedment zone and decreasing with depth, consistent with the connection providing partial fixity.

The 15 kip-ft moment measured by EM-H-A/B at maximum load represents a 32% reduction compared to the 22 kip-ft recorded at the same location in specimen W-10-15. This reduction is physically consistent with the difference in embedment depth and connection fixity between the two specimens. The 15-inch embedment in W-10-15 generates a more fixed boundary condition at the pile head, resulting in larger moment demands transferred into the embedment zone. The 10-inch embedment in W-10-10 provides less fixity, and accordingly the moment magnitude decreases more rapidly with depth, producing lower values at the EM-H level. This trend is meaningful and supports the interpretation that embedment depth has a measurable effect on the moment distribution within the connection zone.

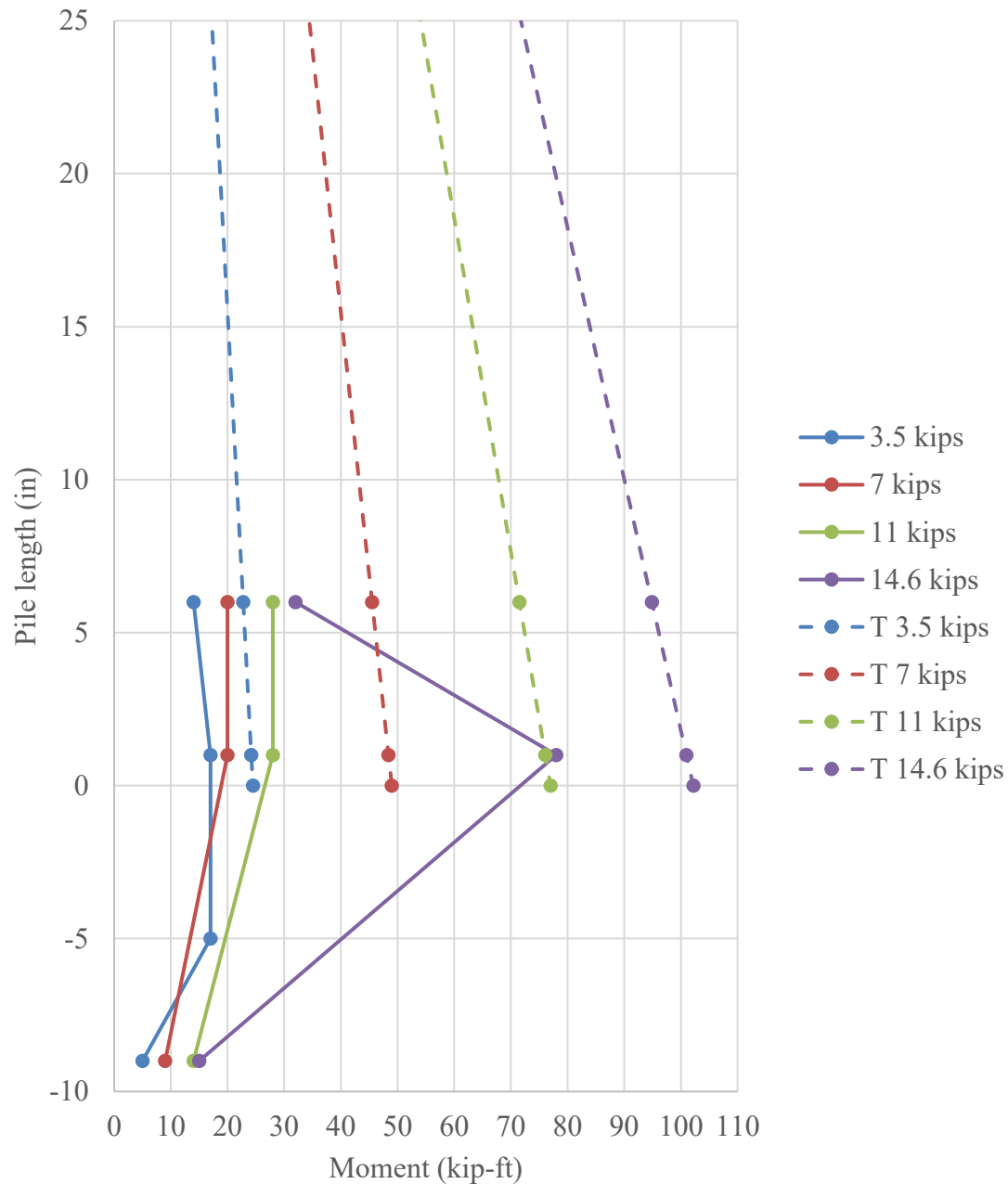


Figure 125. Specimen W-10-10 measured moments (solid lines) and theoretical moments (dashed lines) for each load stage. Zero y-axis marks the cap surface: positive is the exposed pile length, and negative is the embedded pile length.

All other plots for specimen W-10-10 (wire potentiometers, strain measurements, and pile total stress results) are in Appendix A.

1.3. Specimens W-10-7.5 and W-10-7.5R

On 11/24/2025, specimen W-10-7.5 was tested. However, the reaction frame used for the test was deemed inappropriate, and the test was interrupted at approximately 9 kips. On 1/22/2026, a second try with the same specimen W-10-7.5, this time with an appropriate reaction frame, was done. Finally, specimen W-10-7.5R was tested on 2/12/2026. The results for the three tests are seen in Figure 126.

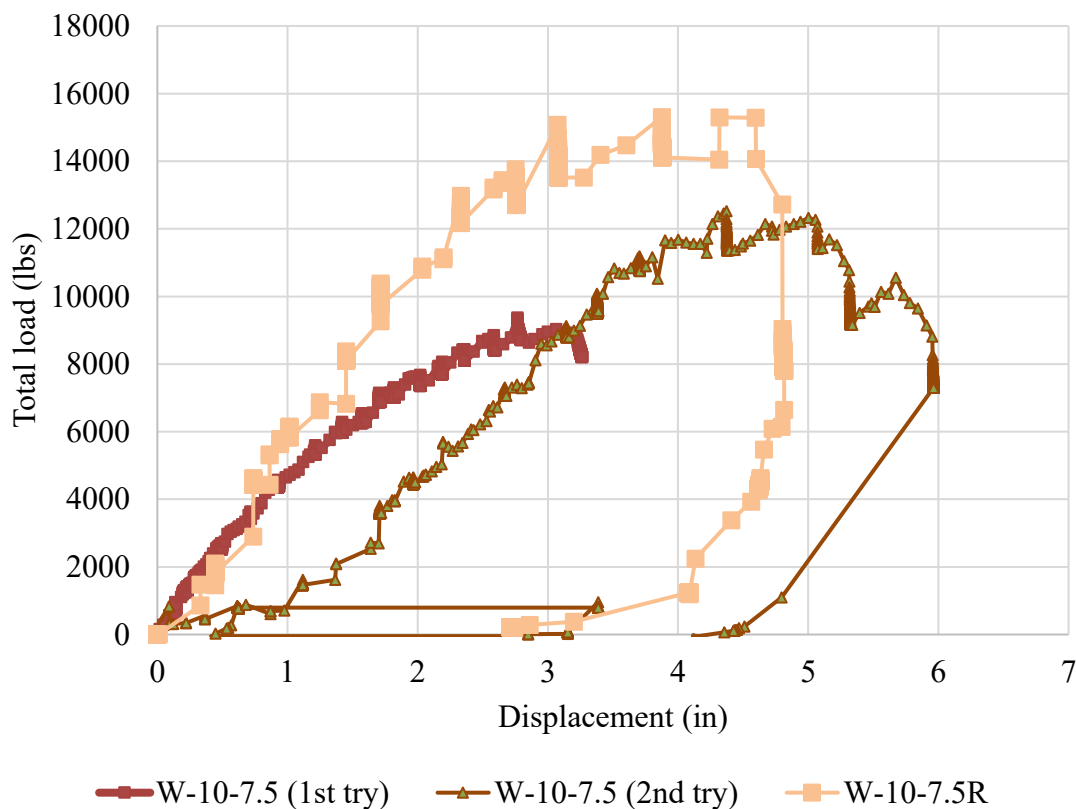


Figure 126. Specimens W-10-7.5 and W-10-7.5R horizontal load vs lateral deflection curves.

As observed, the W-10-7.5 (1st try) showed a steady load increase until approximately 9 kips, when the test had to be interrupted. The second try shows an initial increase characteristic of a previously load specimen. Maximum achieved load for the 2nd try was approximately 11.5-12 kips. Specimen W-10-7.5R, by contrast, reached a

maximum horizontal load of 14 kips, 20% above W-10-7.5 (2nd try). Furthermore, the initial slope of the elastic part of the W-10-7.5R curve (up to approximately 4 kips) matches the W-10-7.5 (1st try) closely, but not the W-10-7.5 (2nd try). This suggests that specimen W-10-7.5 experienced repeated damage, resulting in reduced initial stiffness and lower maximum capacity in the second try. W-10-7.5R, being a fresh specimen tested without prior loading history, provides more reliable dataset for this embedment depth. Therefore, the detailed results from W-10-7.5R will be presented in this section. The complete results for W-10-7.5 (both tries) are provided in Appendix A.

The load vs. deflection curve for W-10-7.5R shows a transition between elastic and plastic behavior beginning at approximately 8 kips, corresponding to a theoretical moment of approximately 56 kip-ft at the steel-concrete interface. This moment level coincides with the onset of yielding based on the pile's elastic section modulus. The specimen reached a maximum horizontal load of approximately 14 kips at a deflection of approximately 3 inches, after which the load plateaued and deflection continued without further load increase. The elastic deflection at approximately 8 kips was approximately 1.44 inches.

The expected maximum horizontal load for this specimen, calculated using the Marcakis and Mitchell (1980) method, falls in the range of approximately 7.4 to 11.1 kips depending on the assumed bearing width. The observed failure load of 14 kips exceeds this range. The full flange local buckling moment of the HP10x42 pile corresponds to an expected horizontal load of approximately 13.1 kips. The observed failure load is therefore slightly above both the upper bound of the Marcakis and Mitchell (1980) range and the load associated with flange local buckling. However, no signs of flange buckling were observed at failure. At Stage 9, mill scale loss was noted on flange C, consistent with the onset of steel yielding at that load level, but the pile did not reach full local buckling. The controlling failure mode was clearly concrete cracking and crushing rather than steel structural failure, as evidenced by the extensive cracking and concrete spalling observed at the cap at failure. Post-test crack width measurements exceeded 1 mm at several locations, in contrast to the 0.03 mm or less recorded in specimens W-10-15 and W-10-10. At Stage 10, concrete spalling was observed on the leading side between flanges B and C, accompanied by a loud bang, consistent with crushing of the concrete

in the bearing zone inside the cap. Cracking was extensive on the trailing side, and the concrete-steel interface gap on the tension side became large. These observations are consistent with a concrete bearing failure mechanism at the connection. Figures 127, 128 and 129 show the damage results in the specimen after the test completion.

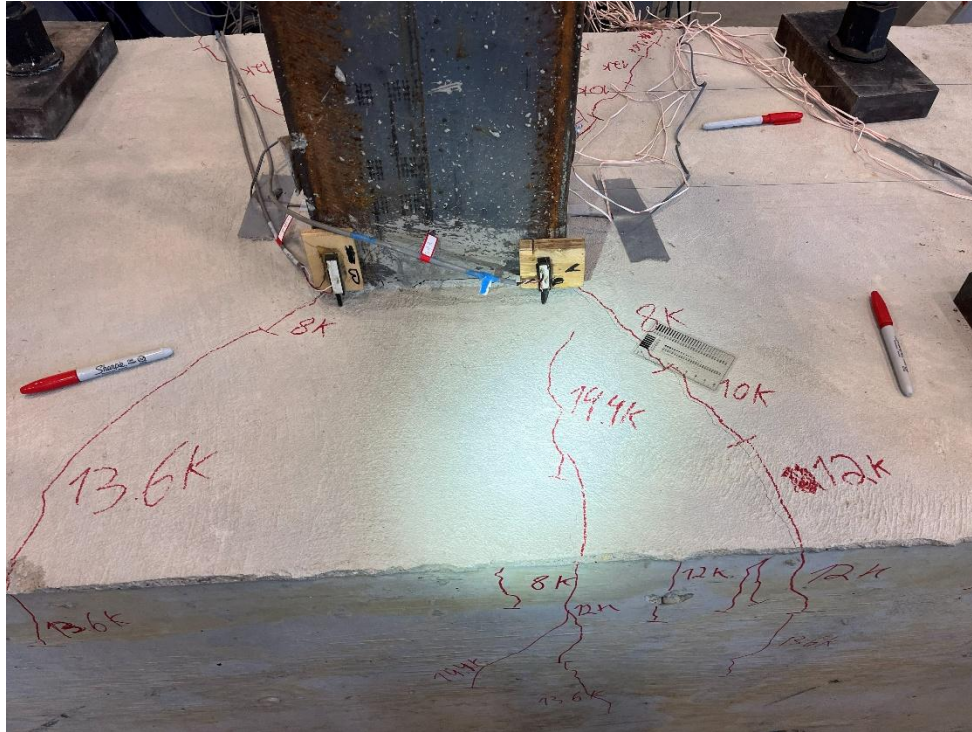


Figure 127. Specimen W-10-7.5R crack formation after target load of 14.4 kips (actual load of 14.6 kips) (load direction: from right to left).

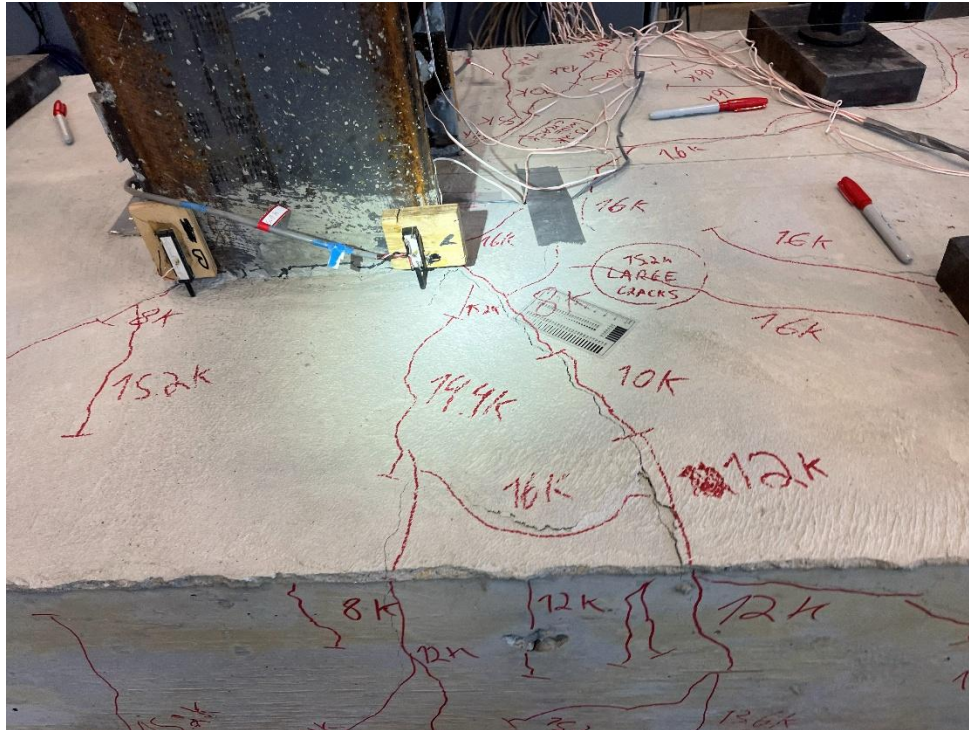


Figure 128. Specimen W-10-7.5R after final load stage (target load 16 kips, actual load 8.7 kips). Observed excessive cracking on the trailing side and cap sides.



Figure 129. Specimen W-10-7.5R after final load stage (target load 16 kips, actual load 8.7 kips). Observed excessive concrete crushing on the leading side.

The inclinometers and tiltmeters generated rotation measurements displayed in Figure 130.

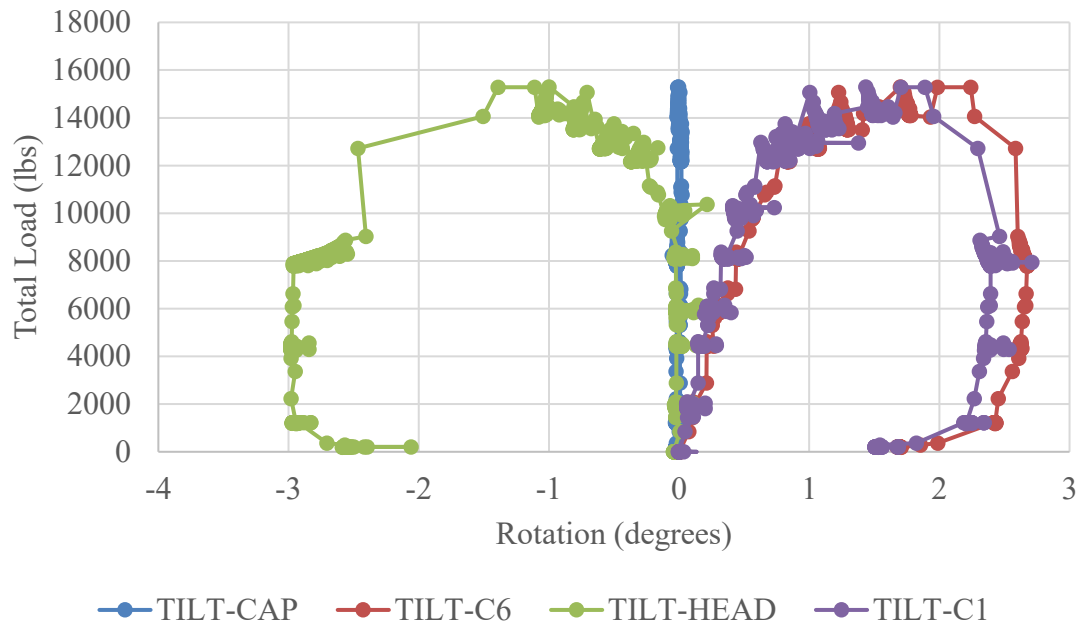


Figure 130. Specimen W-10-7.5R, rotation measurements (positive rotation follows the load direction).

TILT-CAP remained negligible throughout the entire loading sequence, confirming that the concrete cap remained stationary.

TILT-C1 and TILT-C6 tracked each other closely throughout the entire test, including after the failure plateau and during unloading, showing nearly identical behavior at all load stages. Both instruments reached approximately 1.2 degrees of rotation at failure. This behavior is distinctly different from what was observed in specimens W-10-15 and W-10-10, where TILT-C1 and TILT-C6 diverged significantly beyond approximately 13-14 kips due to flange buckling localizing deformation between the two instrument positions. The absence of divergence in W-10-7.5R is consistent with the absence of flange buckling, confirming that the inelastic deformation was not concentrated at a buckle zone between the two instruments. Both instruments retained residual rotation upon unloading, reflecting permanent deformation in the system.

TILT-HEAD again showed counterclockwise rotation (opposite to the load direction) as observed in specimen W-10-10. As explained, this behavior results from the geometry of the sensor attachment: the tiltmeter is fixed to a rod welded to the pile web

at the pile head, and as the pile rotates in the direction of applied load, the rod rotates in the opposite direction, producing an apparent counterclockwise reading at the sensor. The pile head itself is rotating in the direction of the applied load. TILT-HEAD began showing negative rotation at approximately 8-10 kips, coinciding with the onset of pile yielding and the beginning of visible pile head movement. At failure, TILT-HEAD recorded approximately -1 degree of rotation, comparable in magnitude to TILT-C1 at the same load stage. This similarity in magnitude between TILT-HEAD and TILT-C1 indicates that the embedded portion of the pile exhibited very little relative bending, indicating that the embedded pile was rotating largely as a rigid body rather than bending significantly within the cap. This differs from the W-10-10 behavior, where TILT-HEAD recorded approximately -0.5 degrees at failure, a smaller magnitude than W-10-7.5R. The larger TILT-HEAD magnitude in W-10-7.5R is attributed to the shorter embedment depth, which provides less rotational restraint at the pile head and allows greater rigid-body rotation of the embedded segment.

Spring potentiometer results are shown in Figure 131.

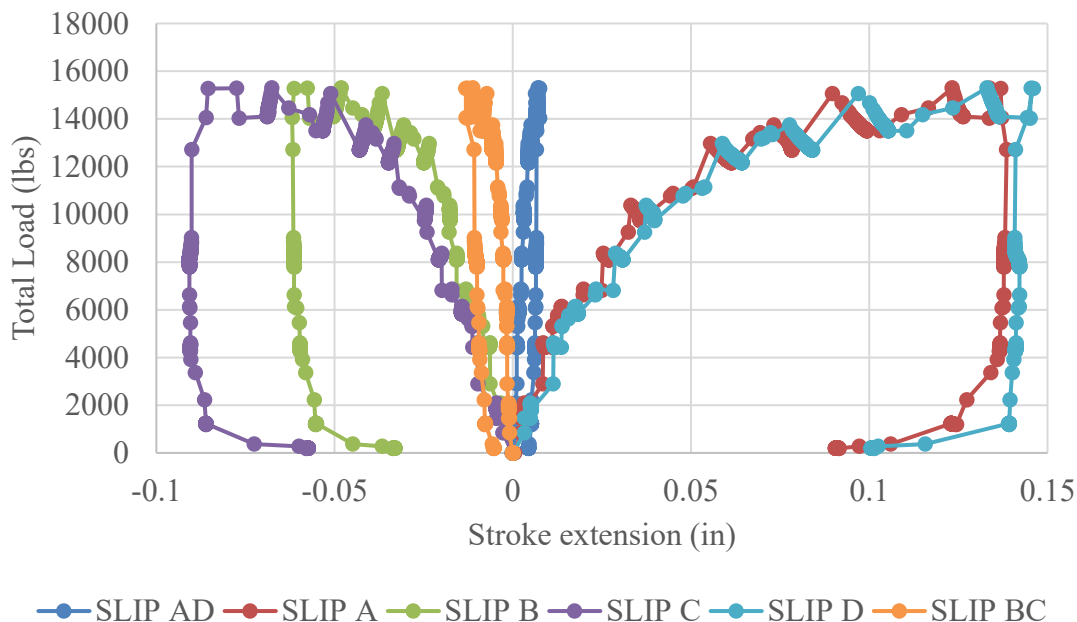


Figure 131. Specimen W-10-7.5R, spring potentiometer measurements.

SLIP AD and SLIP BC recorded negligible displacement throughout the entire loading sequence, confirming that the concrete cap remained stationary and did not translate or rock on its support at any load stage.

On the trailing side, flanges A and D behaved very similarly, reaching a maximum stroke extension of approximately 0.1 inches at failure, as the tension flanges were pulled away from the cap surface. On the leading side, flanges B and C also behaved similarly, reaching a maximum stroke compression of approximately 0.05 inches at failure, as the compression flanges were “pushed into the cap”. The asymmetry between the tension-side and compression-side magnitudes - 0.1 inches versus 0.05 inches - indicates that the trailing flanges separated from the concrete substantially more than the leading flanges were inserted into it. This is consistent with the observed failure mode: on the trailing side, concrete cracking and gap opening allowed the flanges to pull away more freely, while on the leading side, concrete crushing limited further compression penetration.

The strain gauge measurements were collected and reduced to obtain bending moments, axial loads, and stress distributions along the pile and in the cap. Strain measurements exceeding the steel yield point or 70% of the estimated concrete compressive strength were eliminated from analysis. For this specimen, strain gauges EX-C6-A, EX-C1-A, EX-C1-B, EM-I-B, and EM-H-B malfunctioned, and EM-M-A and EM-M-B were not installed. Malfunctioning gauges were replaced by the opposite value of their symmetric counterparts where appropriate. EX-C1-B, while malfunctioning, is shown for reference. Its anomalous behavior is attributed to the same concrete splash mechanism hypothesized for specimen W-10-10, where unprotected gauges near the cap surface were encased by hardened concrete during casting, partially restraining their movement and reducing measured strains below their true values.

Figures 132 and 133 show the total stresses measured by the strain gauges in the steel reinforcement cage and on the concrete cap surface, respectively.

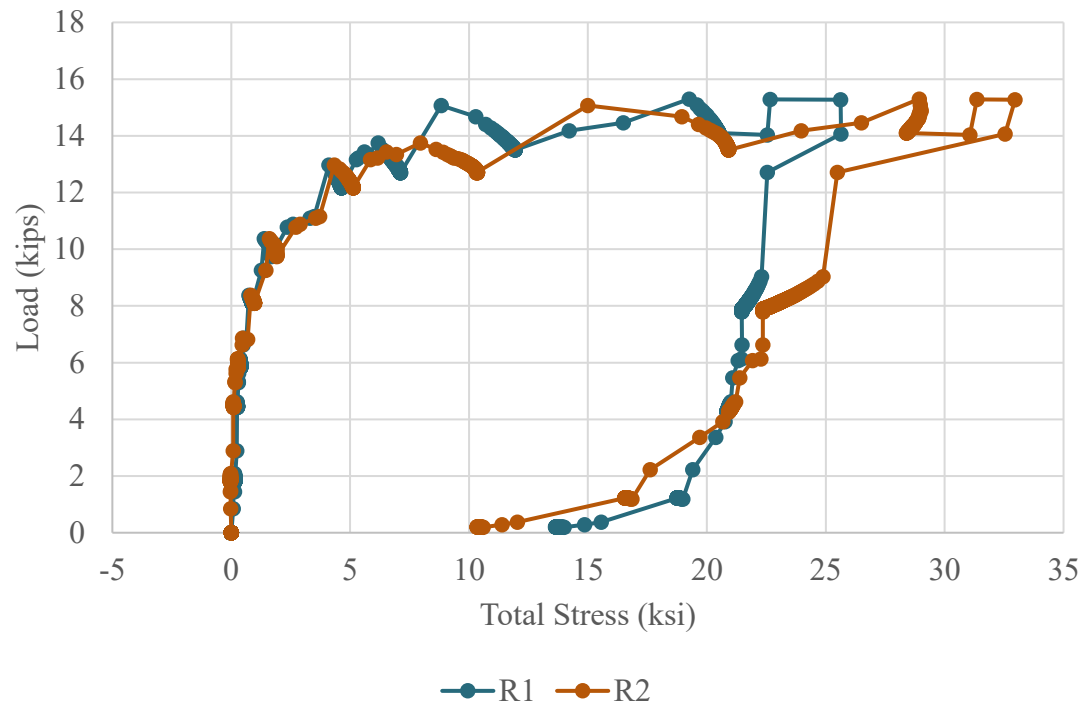


Figure 132. Specimen W-10-7.5R, total stress measured by strain gauges R1 and R2 on steel reinforcement cage.

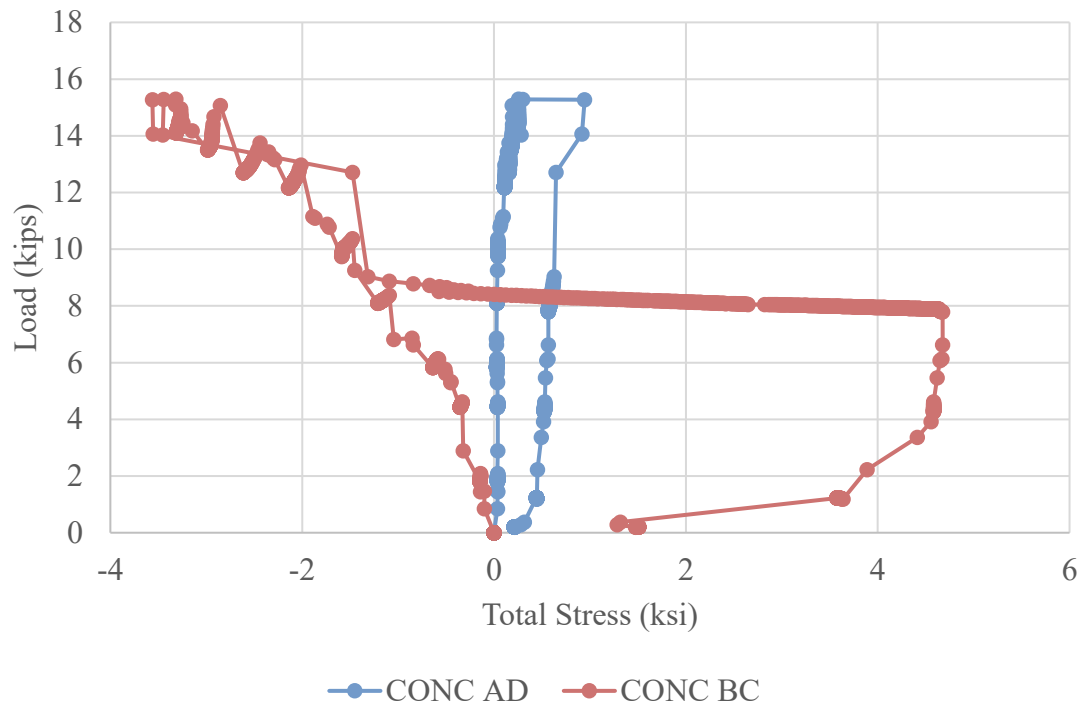


Figure 133. Specimen W-10-7.5R, total stress measured by strain gauges CONC AD and CONC BC on the concrete cap surface.

The reinforcement cage gauges showed notably higher stresses than observed in specimens W-10-15 and W-10-10, consistent with the greater concrete demand at this shorter embedment. Both R1 and R2 indicated tension throughout the test, reaching approximately 15 ksi at the failure plateau. While still below the reinforcement yield stress, these values are substantially higher than the 2 ksi and 0.75 ksi recorded in W-10-10, reflecting the increased load transfer into the cap at reduced embedment depth.

The concrete surface gauges also showed notable behavior. CONC AD on the trailing side exhibited tension throughout the test, reaching approximately 300 psi (close but not exceeding the estimated concrete tensile rupture strength of approximately 559 psi). CONC BC on the leading side behaved in compression up to approximately 3.5 ksi, at which point the specimen reached failure. This gauge subsequently transitioned to apparent tension, likely due to a crack forming through the compressed zone at failure. The 3.5 ksi compression on the leading face substantially exceeds the values recorded in W-10-10 (2.5 ksi) and W-10-15 (0.9 ksi), reinforcing the trend that shorter embedment

depths generate progressively higher concrete bearing stress, eventually transitioning the controlling failure mode from steel structural failure to concrete failure.

Figure 134 shows the bending moment results for the exposed strain gauges. EX-C6-A/B showed moment values consistent with theoretical expectations based on the applied horizontal load throughout the test, providing a reliable record at that instrument level. EX-C1-A/B results are considered unreliable due to the concrete splash malfunction described above and are not used for quantitative conclusions.

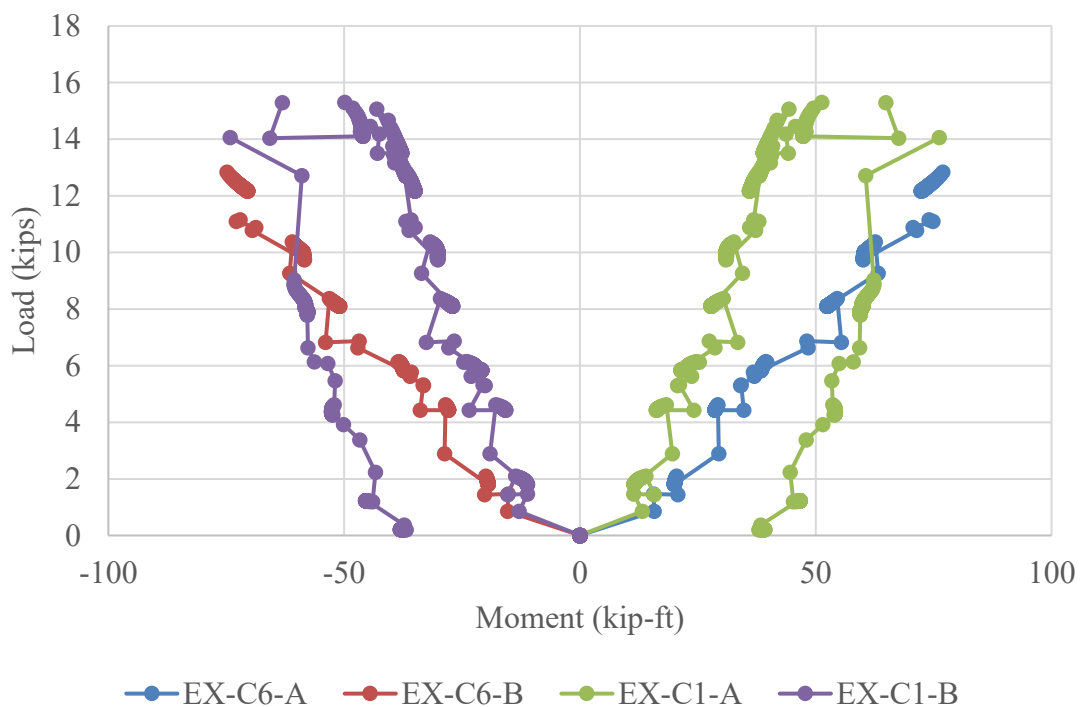


Figure 134. Specimen W-10-7.5R, calculated bending moment for EX strain gauges.

Figure 135 shows the bending moment results for the embedded strain gauges. EM-I-A/B showed good behavior up to approximately 8 kips, with values close to those expected at the steel-concrete interface. Beyond approximately 8 kips, both gauges ceased functioning. This coincides with the load level at which the embedded pile head began to show rotation, and it is speculated that gap opening or concrete cracking at the interface at that stage disrupted the gauge bond, causing loss of signal. EM-H-A/B performed well throughout the entire test, recording approximately 25 kip-ft at the maximum load of 14 kips.

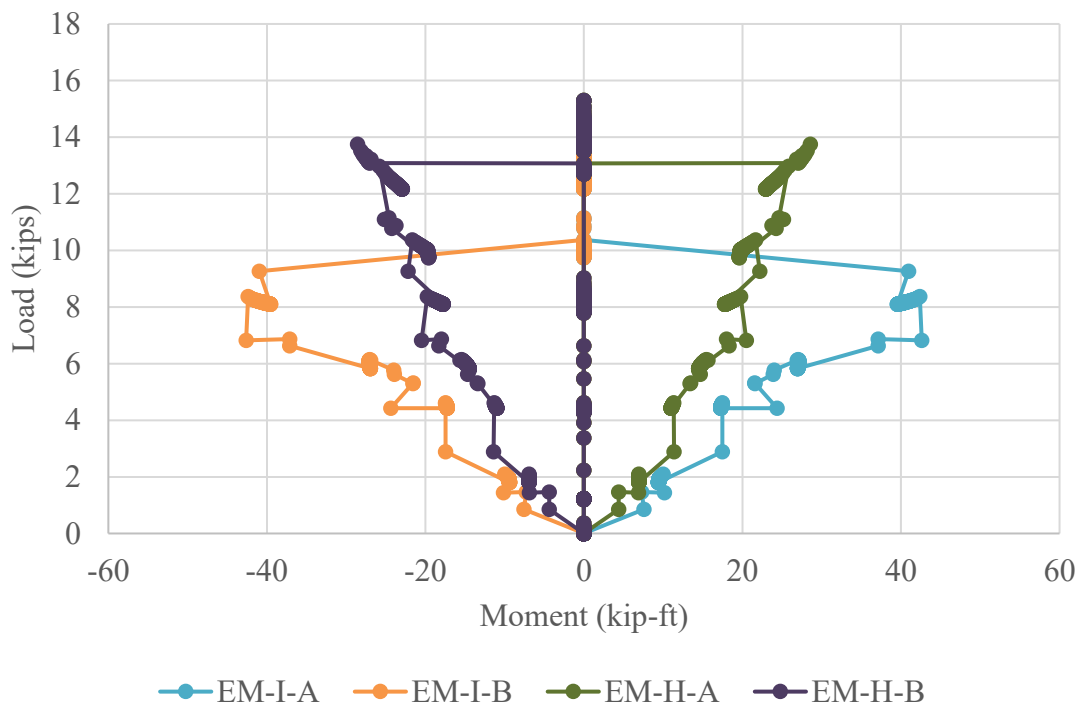


Figure 135. Specimen W-10-7.5R, calculated bending moment for EM strain gauges.

The 25 kip-ft moment recorded by EM-H-A/B at failure is higher than the values measured at the same location in specimens W-10-10 (15 kip-ft) and W-10-15 (22 kip-ft). A monotonically decreasing trend with embedment depth would be expected if all else were equal, but the W-10-7.5R value does not follow this pattern. This result is attributed to specimen-to-specimen variability: minor differences in concrete placement, aggregate distribution, and internal force paths between individual cast specimens can produce

scatter in the moment measurements at a given location, particularly at the pile head where the connection geometry is most sensitive to local conditions.

Figure 136 shows the moment profile along the pile length for each load stage, along with a comparison to the theoretical moment based on the applied horizontal load. At lower load levels (3.5 kips and 7 kips), EX-C6 showed good agreement with the theoretical moment, and the embedded gauges confirmed a decreasing moment with embedment depth. At the 7 kip load stage, the moment at the EM-I level increased from approximately 17 to 37 kip-ft, and at the pile head (EM-H) the moment was approximately 11 to 19 kip-ft. At 10 kips, EX-C6 remained consistent with theoretical expectations, while EM-I had already ceased functioning. EM-H showed approximately 22 kip-ft. Near failure, EX-C6 continued to track theoretical expectations, and EM-H recorded approximately 27 kip-ft. The moment profile confirms that moment is being transferred into the embedment zone and decreasing with depth, consistent with the connection providing partial fixity, despite the not-so-deep embedment depth.

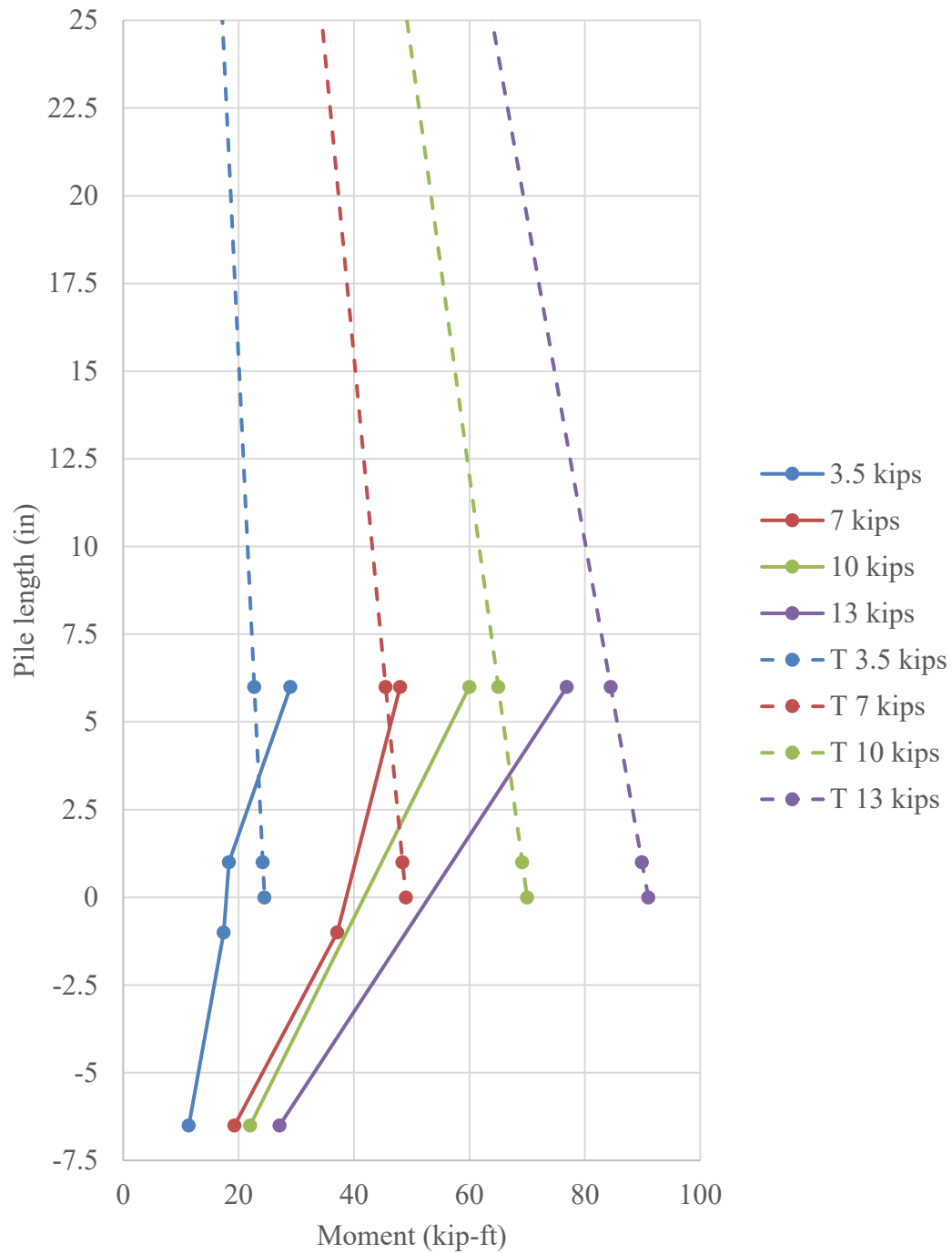


Figure 136. Specimen W-10-7.5R measured moments (solid lines) and theoretical moments (dashed lines) for each load stage. Zero y-axis marks the cap surface: positive is the exposed pile length, and negative is the embedded pile length.

All other plots for specimens W-10-7.5 and W-10-7.5R (wire potentiometers, strain measurements, and pile total stress results) are provided in Appendix A.

1.4. Specimen W-10-5

On 2/03/2026, specimen W-10-5 was tested. Figure 137 shows the load vs deflection curve.

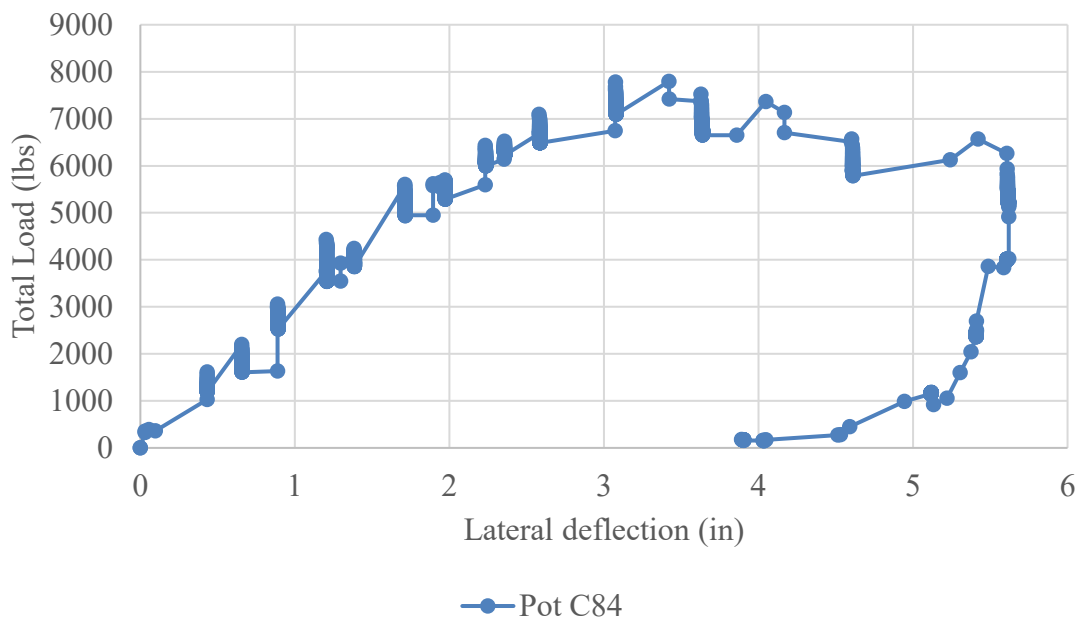


Figure 137. Specimen W-10-5 horizontal load vs lateral deflection curve.

The load vs. deflection curve for W-10-5 showed a steady load increase until approximately 4.5-5 kips, at which point the transition from elastic to plastic behavior began, corresponding to a lateral deflection of approximately 1.5 inches. The specimen reached a maximum horizontal load of approximately 7.5 kips at a deflection of approximately 3.5 inches. Unlike the deeper embedment specimens, where the load plateau was sustained over large deflections, the plateau in W-10-5 was not maintained, it dropped quickly upon continued pressurization of the hydraulic jack, indicating a sudden and brittle failure compared to the more ductile behavior observed in specimens W-10-15, W-10-10, and W-10-7.5R.

The expected maximum horizontal load for this specimen, calculated using the Marcakis and Mitchell (1980) method, falls in the range of approximately 3.34 to 5.04 kips depending on the assumed bearing width. The observed failure load of 7.5 kips exceeds this range by a substantial margin. As noted by other authors in the literature, the Marcakis and Mitchell (1980) equation tends to produce conservative estimates of connection capacity, particularly for shallow embedments, and this conservatism was anticipated across all specimens in this test program. The failure load nonetheless remains below any threshold associated with steel yielding onset or full flange local buckling, and visual observations confirmed that failure occurred by concrete cracking and crushing rather than steel structural failure.

A distinctive characteristic of specimen W-10-5, compared to all deeper embedment specimens, was the very early onset of cracking and gap formation. Diagonal cracks radiating from all four flange tips appeared as early as Stage 2 (approximately 2.1 kips total load), and the steel-concrete interface gap began opening at Stage 1 (approximately 1.5 kips). In contrast, cracking in specimens W-10-15 and W-10-10 did not appear until Stage 5, at loads exceeding 13 kips. This early cracking reflects the substantially reduced ability of the 5-inch embedment to confine and engage the concrete in bearing, with the connection beginning to deteriorate almost from the start of loading.

Through Stage 6 (peak total load approximately 5.6 kips, pile head deflection 1.5 inches), diagonal cracks continued to grow in length and the interface gap widened progressively. No cracking was observed on the vertical side faces of the cap at this stage.

At Stage 7 (peak total load approximately 5.7 kips, pile head deflection 1.75 inches), cracks began appearing on the vertical side faces of the cap, propagating outward from the diagonal crack system. The interface gap continued to widen.

At Stage 9 (peak total load approximately 6.5 kips, pile head deflection 2.125 inches), the diagonal cracks near the flanges had widened to approximately 0.10-0.15 mm. On the leading side, between flanges B and C, a bulge began to form at the cap surface, accompanied by a crack consistent with concrete being displaced vertically. This is interpreted as an early sign of the concrete bearing zone failing internally, the pile bearing load displacing the concrete mass upward rather than being resisted through compression within the cap.

At Stage 11 (peak total load approximately 7.5 kips, pile head deflection 2.8125 inches), a similar bulge became visible on the trailing side between flanges A and D, indicating that vertical displacement of the concrete mass was now occurring on both sides of the pile. The interface gap continued to increase noticeably, and additional cracks appeared along the diagonal crack system on the cap sides.

At Stage 12 (peak total load approximately 7.5 kips, pile head deflection 3.4 inches), the specimen appeared to have reached failure. The bulge on the tension side continued to grow, additional cracks formed on the cap sides, and the interface gap became large.

At Stage 14 (target load 8 kips, actual total load approximately 6.2 kips), complete failure was evident. The interface gap was very large. A longitudinal crack on the cap side exceeded 2 mm in width. The concrete on the leading side had crushed entirely, and the concrete on the trailing side had formed a large bulge, pushed upward by the displaced bearing zone. Post-test crack width measurements confirmed cracks exceeding 1-2 mm at several locations, consistent with the concrete bearing failure mode and in contrast to the 0.03 mm or less recorded in specimens W-10-15 and W-10-10.

Figure 138 shows the crack formation observed during the load test, and Figure 139 shows the concrete crushing on the leading side.

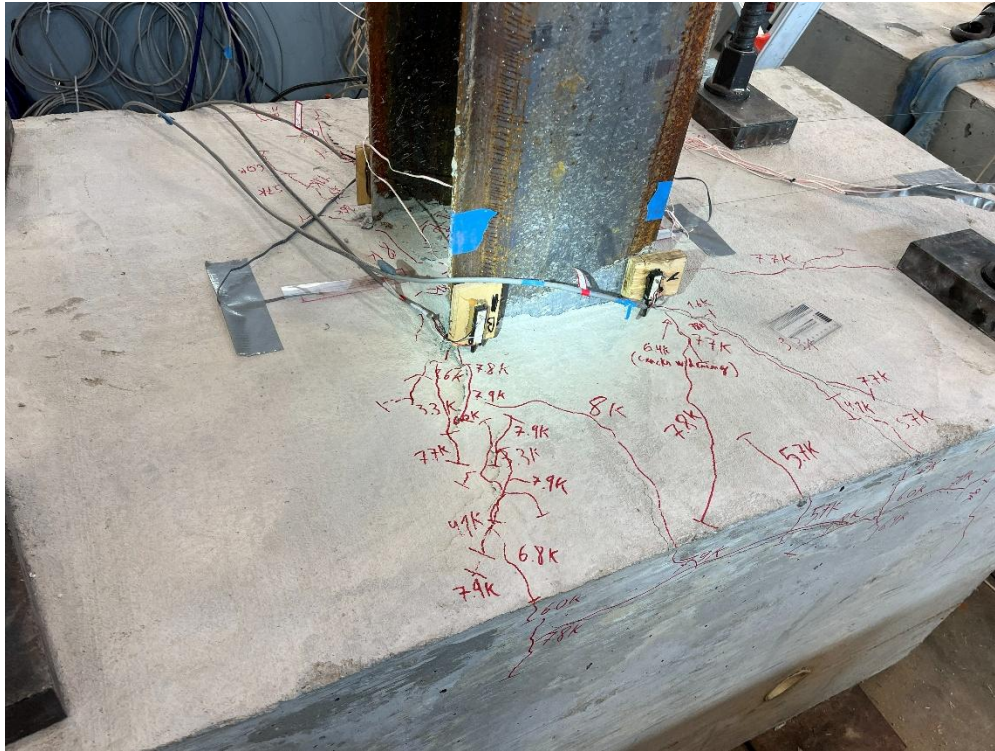


Figure 138. Specimen W-10-5 crack formation after target load of 8 kips (applied load of 6.2 kips) (load direction: from right to left).



Figure 139. Specimen W-10-5 concrete crushing on leading side after target load of 8 kips (applied load of 6.2 kips) (load direction: from right to left).

The inclinometers and tiltmeters generated rotation measurements displayed in Figure 140.

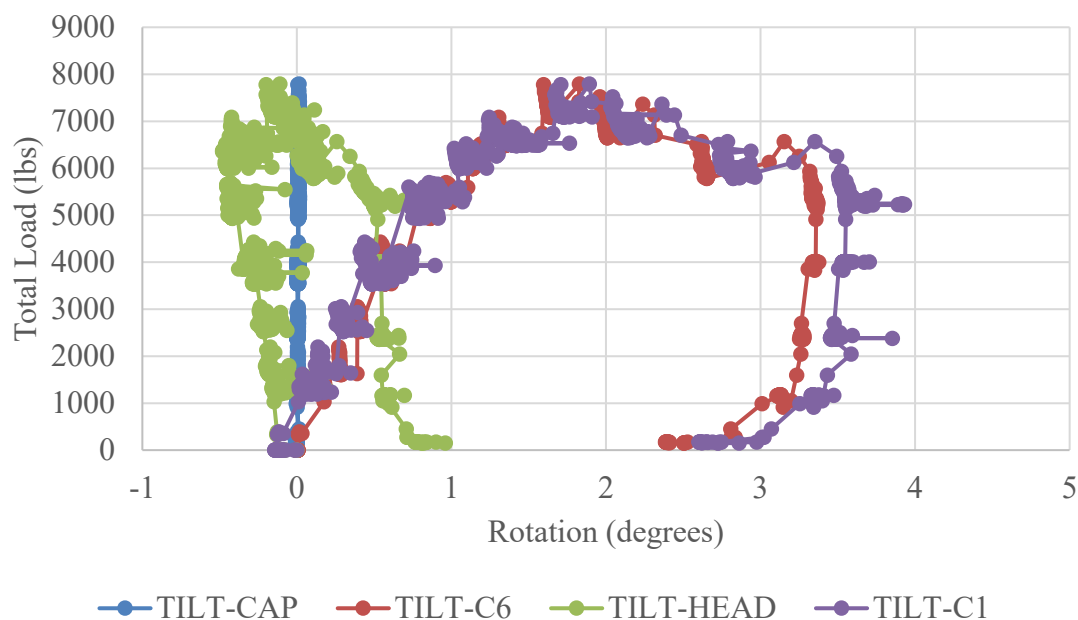


Figure 140. Specimen W-10-5, rotation measurements (positive rotation follows the load direction).

TILT-CAP remained negligible throughout the entire loading sequence, confirming that the concrete cap remained stationary.

TILT-C1 and TILT-C6 tracked each other closely throughout the entire test, including post-failure loading and during unloading, showing nearly identical behavior at all load stages and reaching approximately 1.75 degrees of rotation at failure. This behavior is consistent with what was observed in specimen W-10-7.5R and confirms that failure did not involve flange buckling, as no divergence between the two instruments developed at any load stage. It is noted that TILT-C1 and TILT-C6 were inadvertently installed at 2 inches and 7 inches from the cap surface, respectively, rather than their intended positions. This did not affect the observations, especially given that the specimen failed by concrete bearing rather than steel structural failure.

TILT-HEAD again showed counterclockwise rotation at the onset of loading, consistent with the geometric sensor attachment mechanism described in other specimens: the tiltmeter rod, welded to the base of the pile web, rotates opposite to the pile head as the pile rotates in the direction of applied load. However, W-10-5 exhibited a

departure from this behavior not observed in previous specimens. Up to approximately 6 kips, TILT-HEAD rotated counterclockwise, reaching approximately -0.44 degrees, consistent with the mechanism described above. Beyond approximately 6 kips, the rotation reversed direction, becoming clockwise and continuing in that direction through failure. This reversal is interpreted as a consequence of the concrete bearing zone breaking down sufficiently at that load level, a point when the internal cracking had progressed enough so that the concrete mass near the pile head was no longer providing the bearing resistance that had previously constrained the rod's movement. With the surrounding concrete sufficiently disrupted, the measurement rod was freed to follow the actual rotation of the pile head rather than rotate against it. This interpretation is corroborated by the SLIP D observation discussed below, which recorded an anomalous stroke compression at approximately the same load level, attributed to local concrete spalling near flange D displacing the cap surface upward, a physically consistent companion event to the internal concrete breakdown inferred from the TILT-HEAD reversal.

Spring potentiometer results are shown in Figure 141.

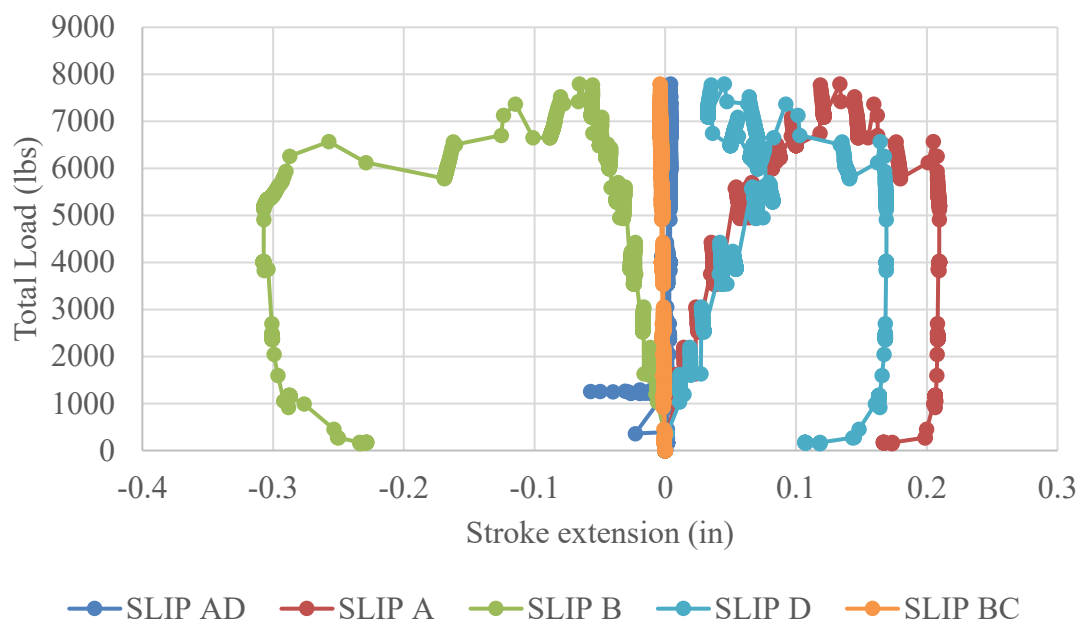


Figure 141. Specimen W-10-5, spring potentiometer measurements.

SLIP AD and SLIP BC recorded negligible displacement throughout the entire loading sequence, confirming that the concrete cap remained stationary and did not translate or rock on its support at any load stage. SLIP C malfunctioned during the test and its data were not used.

On the trailing side, SLIP A and SLIP D behaved similarly through most of the test, with stroke extensions reaching approximately 0.12 inches at failure as the tension flanges were “pulled out” of the cap surface. A notable exception occurred at approximately 6 kips, where SLIP D reversed and began compressing rather than extending, ultimately reaching a total stroke extension of only approximately 0.04 inches at failure. This anomalous behavior is attributed to local concrete spalling near flange D, where the cap surface was pushed upward by the displaced concrete mass, causing the potentiometer stroke to compress. This observation is physically consistent with the internal concrete bearing zone failure inferred from the TILT-HEAD reversal at the same load level, and together they provide mutually corroborating evidence of a local concrete failure event near the trailing side of the pile at approximately 6 kips.

On the leading side, SLIP B compressed approximately 0.07 inches at failure, consistent with the compression flange being “pushed into” the cap.

The strain gauge measurements were collected and reduced to obtain bending moments, axial loads, and stress distributions along the pile and in the cap. Strain measurements exceeding the steel yield point or 70% of the estimated concrete compressive strength were eliminated from analysis. For this specimen, strain gauges EX-C1-A, EM-I-B, and EM-H-B malfunctioned and were replaced by the opposite value of their symmetric counterparts. EM-M was not installed in this specimen. EM-I-A functioned only up to approximately 1 kip, and EM-H-A functioned only up to approximately 2 kips, after which both ceased recording. The very early loss of the embedded gauges is attributed to the greater internal movement occurring within the shallow embedment zone: at 5-inch embedment, the concrete bearing and gap-opening displacements begin at very low load levels, and the resulting interface movement was likely sufficient to disrupt gauge bond at these locations well before the specimen approached failure.

Figures 142 and 143 show the total stresses measured by the strain gauges in the steel reinforcement cage and on the concrete cap surface, respectively.

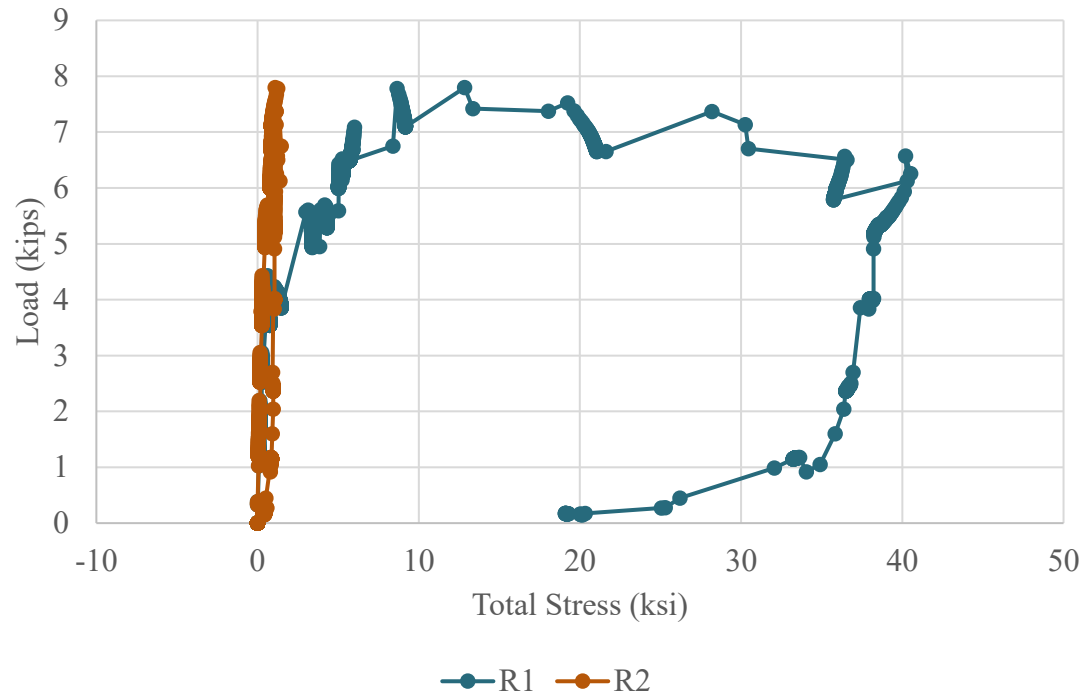


Figure 142. Specimen W-10-5, total stress measured by strain gauges R1 and R2 on steel reinforcement cage.

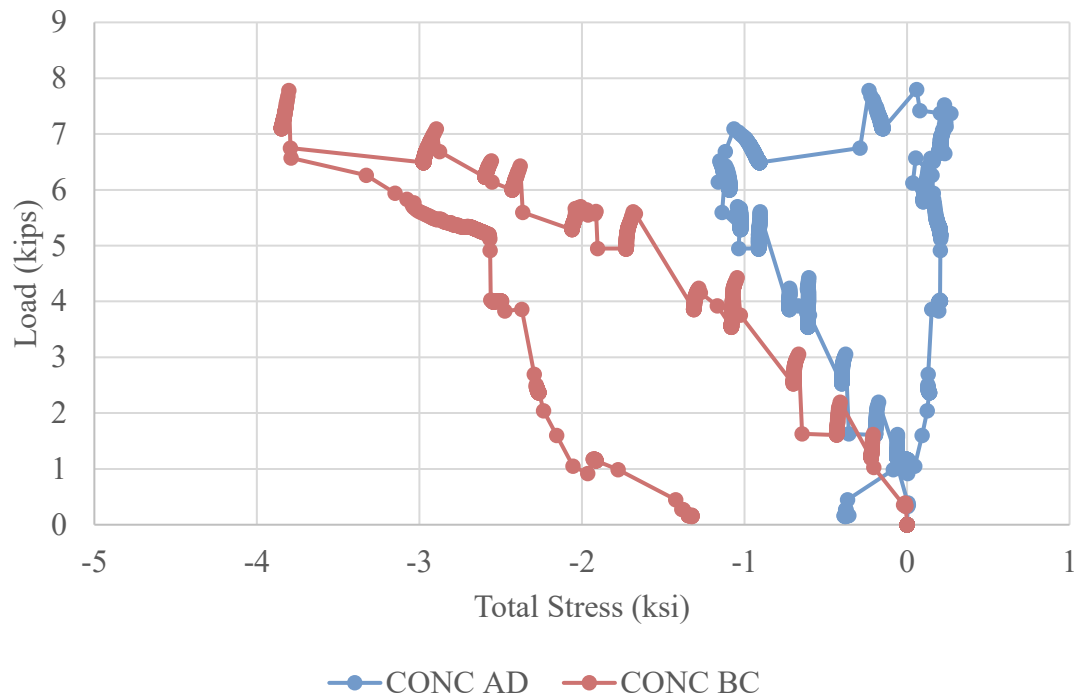


Figure 143. Specimen W-10-5, total stress measured by strain gauges CONC AD and CONC BC on the concrete cap surface.

Both reinforcement cage gauges recorded tension throughout the test. R1, on the trailing side, reached approximately 15 ksi at the failure load plateau, and continued increasing to approximately 40 ksi at the maximum recorded data point, a value approaching yield for some reinforcement grades, suggesting that the rebars may have been near yielding at that stage. R2, on the leading side, recorded a maximum of approximately 1.25 ksi in tension, substantially lower than R1. The elevated R1 stress reflects the high tensile demand on the trailing reinforcement as the concrete cap attempted to resist the pile's bearing load at this shallow embedment, and the contrast with the modest R2 reading is consistent with the asymmetric damage observed visually, where cracking and bulging were more severe on the trailing side.

The concrete surface gauges showed distinct behavior on each side. CONC AD on the trailing side recorded compression up to approximately 1.25 ksi at approximately 7 kips, after which it transitioned to apparent tension, likely reflecting the formation of a crack through the compressed zone as the concrete surface failed. CONC BC on the

leading side recorded compression up to approximately 4 ksi at the failure load, the highest leading-face compression recorded across all specimens in this test program, consistent with the concrete bearing failure mode and the progressive trend of increasing concrete stress with decreasing embedment depth observed across W-10-15, W-10-10, and W-10-7.5R.

Figure 144 shows the bending moment results for the exposed strain gauges. EX-C6-A/B showed moment values consistent with theoretical expectations throughout most of the test, providing a reliable record at that instrument level, though at higher load levels (5.5 kips and 7.5 kips) the recorded moments were somewhat lower than theoretical expectations. EX-C1-A/B recorded consistent values throughout the entire test, remaining reliable across all load stages.

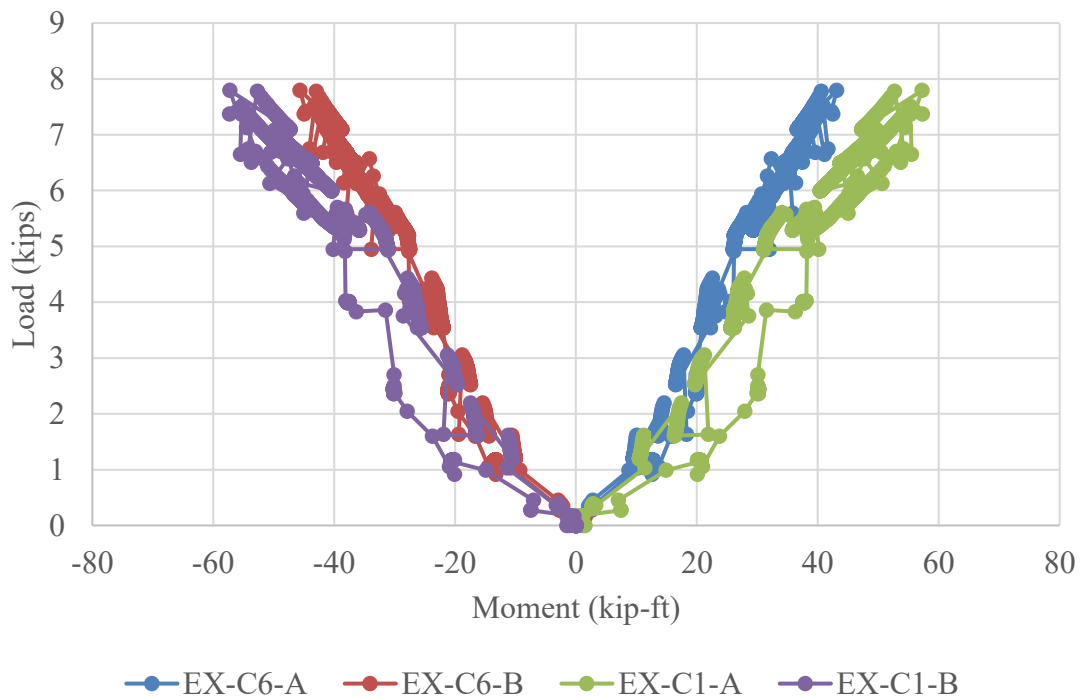


Figure 144. Specimen W-10-5, calculated bending moment for EX strain gauges.

Figure 145 shows the bending moment results for the embedded strain gauges. EM-I-A/B recorded moments compatible with the EX-C1 measurements up to approximately 1 kip, at which point the gauges began malfunctioning. EM-H-A/B recorded reasonable measurements up to approximately 2 kips, showing the expected decrease in moment with embedment depth at those early load stages.

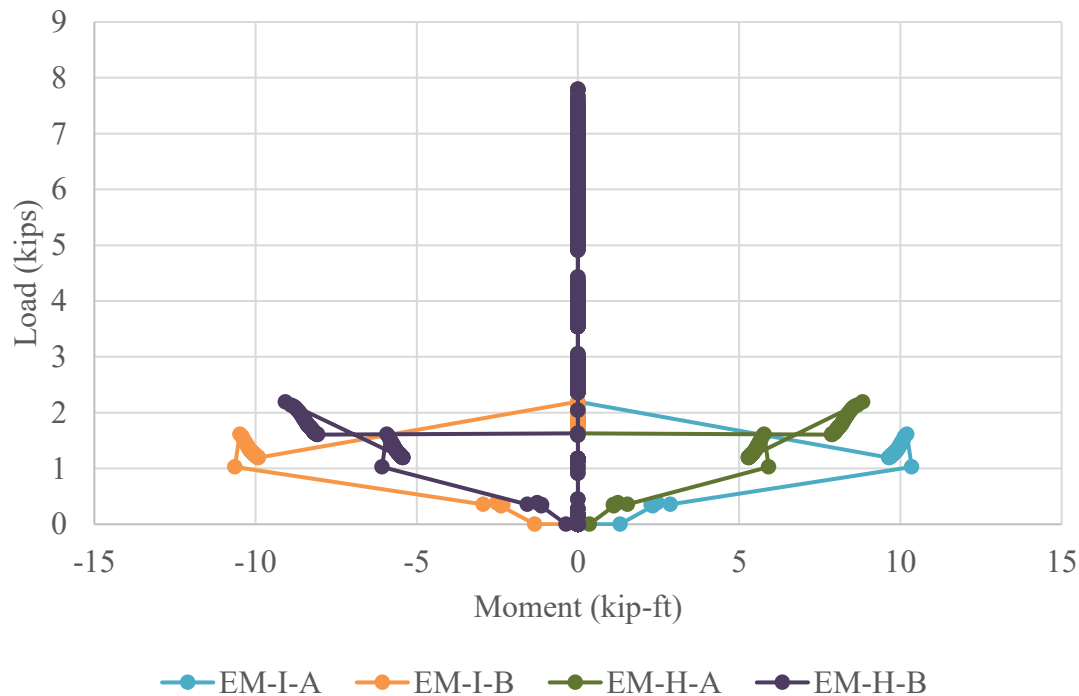


Figure 145. Specimen W-10-5, calculated bending moment for EM strain gauges.

The very early loss of both embedded gauge pairs means that the embedded moment profile for W-10-5 is poorly characterized beyond the initial load stages. This is a significant instrumentation limitation for this specimen, and quantitative conclusions about moment distribution within the embedment zone at higher load levels cannot be drawn from the available data. It is nonetheless reasonable to expect that, had the gauges continued functioning, they would have continued recording larger moments at the pile head throughout the test, as the connection was clearly transferring load into the embedment zone up to failure.

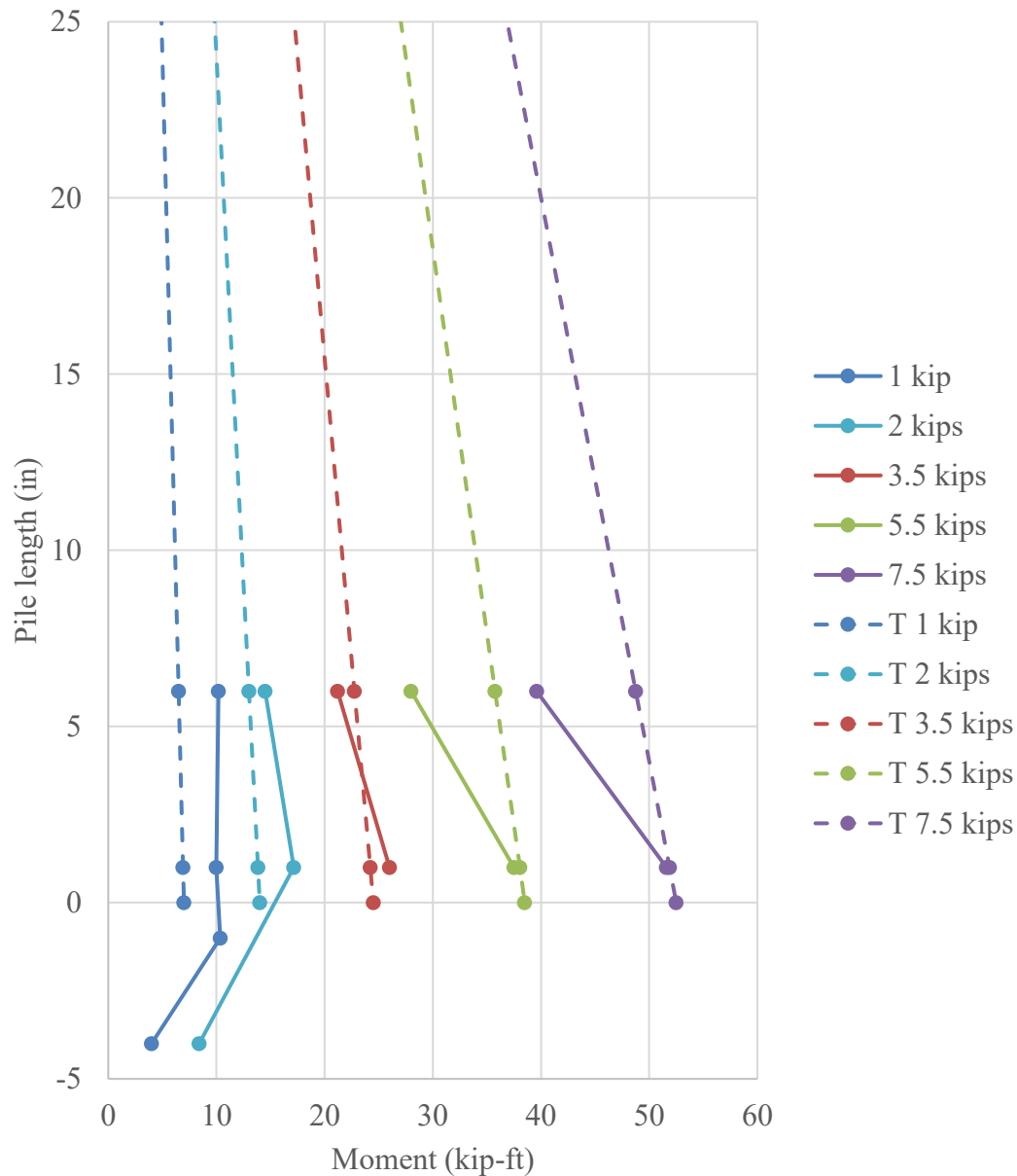


Figure 146. Specimen W-10-5 measured moments (solid lines) and theoretical moments (dashed lines) for each load stage. Zero y-axis marks the cap surface: positive is the exposed pile length, and negative is the embedded pile length.

Figure 146 shows the moment profile along the pile length for each load stage, along with a comparison to the theoretical moment based on the applied horizontal load. At lower load levels (1 kip, 2 kips, and 3.5 kips), EX-C6 and EX-C1 showed good agreement with theoretical moment expectations. At higher load levels (5.5 kips and 7.5

kips), EX-C6 recorded somewhat lower than expected moments, while EX-C1 remained consistent with theoretical expectations. EM-I recorded reliable data only at the 1 kip load stage, showing good agreement with expected moment levels at the connection interface. EM-H showed a clear decrease in moment with embedment depth at the earliest load stages where it was functional, consistent with moment dissipation into the embedment zone. The available data, limited as it is to the early load stages for the embedded gauges, confirm that moment was being transferred into the embedment zone and decreasing with depth, indicating that even at 5-inch embedment the connection retained a partial degree of fixity.

All other plots for specimen W-10-5 (wire potentiometers, strain measurements, and pile total stress results) are provided in Appendix A.

1.5. Specimen W-12-12

Specimen W-12-12 was tested on 2/10/2026. This specimen used an HP12x53 pile section, in contrast to the HP10x42 pile used in all previous specimens. Figure 147 shows the load vs. deflection curve.

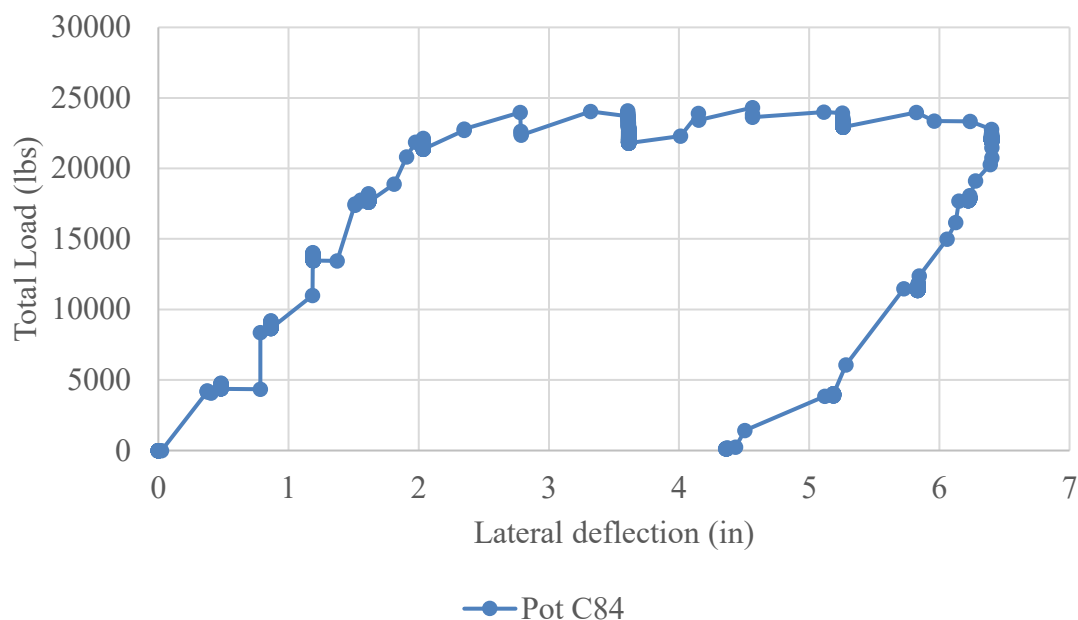


Figure 147. Specimen W-12-12 horizontal load vs lateral deflection curves.

The load vs. deflection curve for W-12-12 showed a steady load increase until approximately 17 kips at a lateral deflection of 1.61 inches, at which point the curve exhibited a slight change in slope indicating the onset of yielding and the beginning of the transition from elastic to plastic behavior. This elastic limit aligns well with the expected horizontal load of 16.8 kips corresponding to the flange local buckling moment capacity of the HP12x53 section (118.3 kip-ft), confirming that the theoretical prediction captured the observed elastic limit with good accuracy. Beyond this point, the curve continued to increase until reaching a maximum horizontal load of approximately 23.2 kips at a deflection of 3.61 inches, representing a 38% exceedance above the expected flange local buckling load. At this maximum load, the curve plateaued despite continued jack pressurization. The moment generated at the connection at this load level approaches the full plastic moment capacity of the HP12x53 section (146.24 kip-ft). Visual observations and instrumentation data both confirm that failure occurred by steel yielding and flange buckling rather than concrete bearing failure at the connection.

At the start of the test, no cracks were present. Through Stage 3 (peak total load approximately 14 kips, pile head deflection 1 inch), only a hairline crack at the steel-

concrete interface was observed, with no other cracking in the cap. At Stage 4 (peak total load approximately 18.1 kips, pile head deflection 1.375 inches), diagonal cracks appeared for the first time at all four flanges, and the steel-concrete interface gap continued to widen gradually. At Stage 5 (peak total load approximately 22.1 kips, pile head deflection 1.78 inches), the diagonal cracks grew longer, approaching the cap sides, and the interface gap continued to increase. At Stage 6 (peak total load approximately 23.5 kips, pile head deflection 3.22 inches), mill scale loss was observed on the leading flanges B and C, consistent with the onset of compression flange buckling. At Stage 7 (peak total load approximately 23.9 kips, pile head deflection 4.78 inches), no new cracks formed, only the existing cracks continued to widen, and mill scale loss extended to the trailing flanges A and D, indicating that yielding had reached the tension side of the pile cross-section. The flanges showed pronounced buckling at this stage. At Stage 8 (peak total load approximately 22.5 kips, pile head deflection 5.875 inches), flange buckling was clearly established, and a few cracks appeared on the sides of the cap with the diagonal cracks continuing to widen. The specimen was unloaded following this stage.

Post-test crack width measurements confirmed that all cracks in the concrete cap measured approximately 0.03 mm, indicating that despite pile yielding and flange buckling, concrete damage remained superficial throughout the test. Figures 148 and 149 show the cracking patterns and observed flange buckling after the completed test.

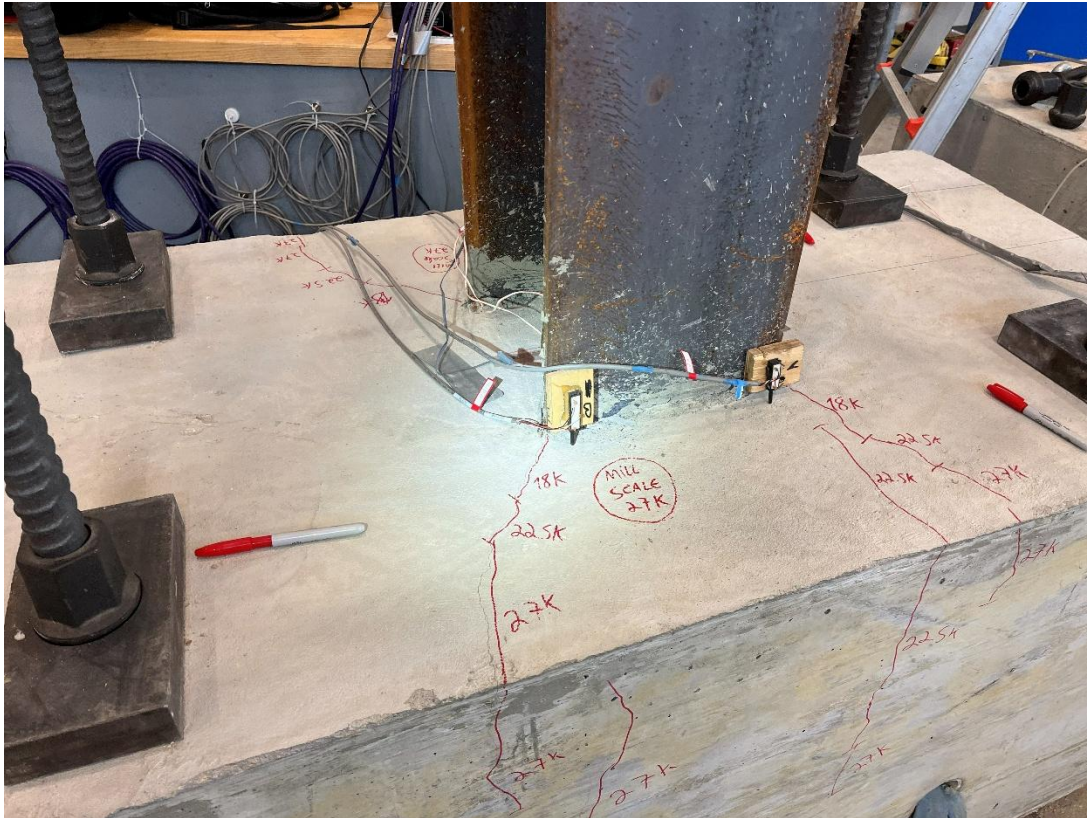


Figure 148. Specimen W-12-12 crack formation after target loads of 27 kips (applied load of 23.2 kips) (load direction: from right to left).



Figure 149. Specimen W-12-12 after final load stage (approx. 23.2 kips). Observed buckling in flanges B and C and mill scale loss.

The inclinometers and tiltmeters generated rotation measurements displayed in Figure 150.

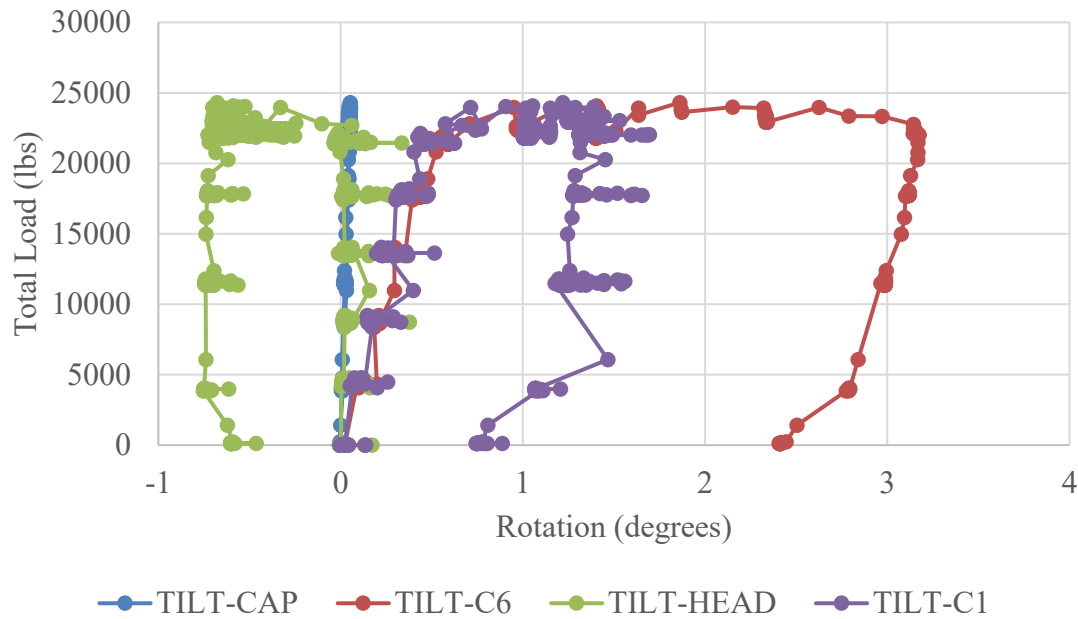


Figure 150. Specimen W-12-12, rotation measurements (positive rotation follows the load direction).

TILT-CAP showed negligible rotation throughout the entire loading sequence, confirming that the concrete cap remained stationary. A rotation of approximately 0.05 degrees was recorded at the highest load levels, slightly larger than in previous specimens but still negligible, and consistent with the higher load levels reached in this test.

TILT-C1 and TILT-C6 behaved very similarly throughout the elastic loading range and up to the maximum horizontal load, both reaching approximately 1 degree of rotation at 23.2 kips. As the load plateaued and the jack continued to be pressurized, TILT-C1 ceased accumulating rotation at approximately 1.3 degrees, while TILT-C6 continued rotating, ultimately reaching a maximum of approximately 3.1 degrees. This divergence between TILT-C1 and TILT-C6 beyond the maximum load is consistent with what was observed in specimens W-10-15 and W-10-10, and reflects the localization of inelastic deformation at the flange buckle zone between the two instrument positions. Above the buckle, TILT-C6 continued to accumulate rotation freely, while below the buckle, TILT-C1 was partially decoupled from the deformation above.

TILT-HEAD showed negligible movement throughout most of the test, similar to the behavior observed in specimen W-10-15, where the deeper embedment provided sufficient fixity to suppress excessive pile head rotation. At approximately 23 kips (the point at which the load plateau was reached) TILT-HEAD began exhibiting counterclockwise rotation, ultimately reaching approximately -0.6 degrees. As described previously, this counterclockwise reading reflects the geometric behavior of the sensor attachment: as the pile head begins to rotate in the direction of applied load, the measurement rod welded to the pile web rotates in the opposite direction, producing an apparent CCW signal at the sensor. The onset of this rotation at the failure load plateau is physically consistent with the concrete connection starting to approach its bearing capacity at that load level. The Marcakis and Mitchell (1980) method estimates the concrete bearing capacity of this connection in the range of approximately 21.8 to 27.3 kips, and the maximum achieved load of 23.2 kips falls within this range. It is therefore reasonable to interpret the TILT-HEAD signal as indicating that, while failure was ultimately controlled by steel structural buckling, the concrete at the connection began to allow internal movement near the failure load, permitting the pile head to rotate sufficiently for the rod effect to manifest.

Spring potentiometer results are shown in Figure 151.

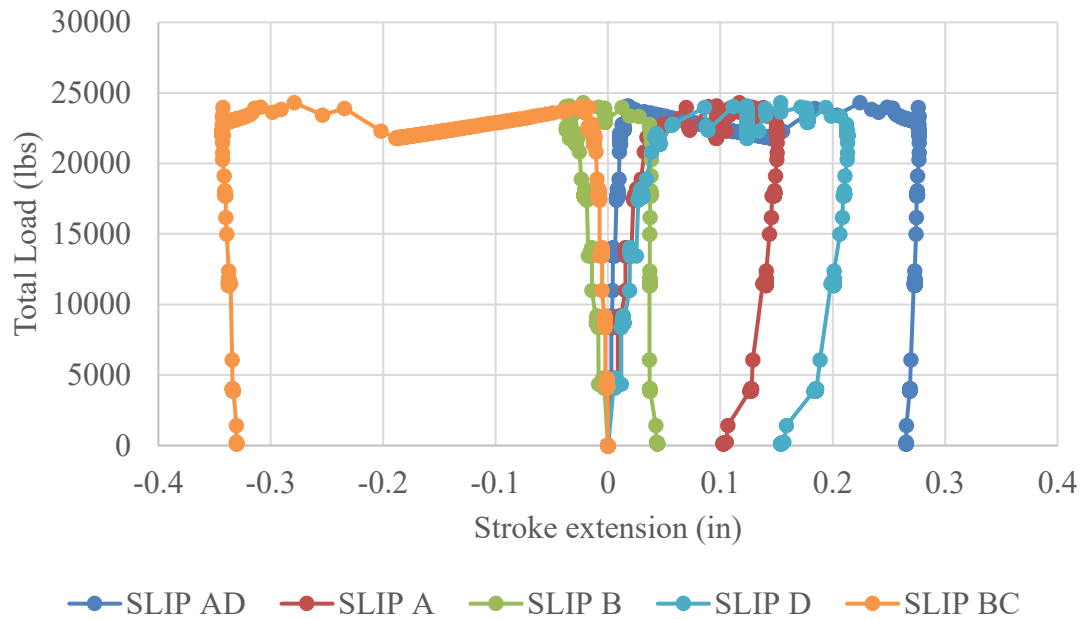


Figure 151. Specimen W-12-12, spring potentiometer measurements.

SLIP C malfunctioned during the test and its data were not used. SLIP AD and SLIP BC both recorded negligible displacement throughout the test up to the maximum achieved load. At the maximum load plateau, however, both sensors recorded a longitudinal cap movement of approximately 0.3 inches, a notable departure from the negligible cap movement observed in all previous specimens, and the largest cap displacement recorded in this test program. This anomalous movement is attributed to the substantially higher load levels reached in this specimen relative to previous tests. Importantly, this cap movement did not compromise the other test results, as the failure mode was clearly established by flange buckling prior to and concurrent with the cap displacement, and the instrumentation results are internally consistent with that failure mode.

SLIP A and SLIP D on the trailing side behaved similarly throughout the test, with stroke extensions reaching approximately 0.08 inches at the maximum load of 23.2 kips, as the trailing flanges were pulled away from the cap surface. SLIP B on the leading side compressed approximately 0.04 inches at 23.2 kips. As the load plateaued and flange buckling progressed, SLIP B reversed and began extending, due to the fact that the sensor was moving, since it was magnetically attached to the buckle area. Overall, all

flange sensors extended or compressed very little until the maximum load was achieved, consistent with the relatively stiff behavior of the deeper embedment connection up to that point.

The strain gauge measurements were collected and reduced to obtain bending moments, axial loads, and stress distributions along the pile and in the cap. Strain measurements exceeding the steel yield point or 70% of the estimated concrete compressive strength were eliminated from analysis. For this specimen, strain gauges EX-C6-A and EX-C6-B likely malfunctioned, showing strains substantially smaller than theoretical expectations. The cause is uncertain - unlike previous specimens where the concrete splash was identified as a likely explanation, no concrete splash was observed on this specimen, and the gauges were protected with duct tape prior to casting. The malfunction is attributed to a likely bonding issue during gauge installation. EM-I-B also malfunctioned and was replaced by the opposite value of its symmetric counterpart. EM-M-A showed intermittent behavior, with some data points considered reliable and retained for analysis. EM-M-B malfunctioned. EM-H-A and EM-H-B functioned reliably up to approximately 5 kips, after which both ceased recording.

Figures 152 and 153 show the total stresses measured by the strain gauges in the steel reinforcement cage and on the concrete cap surface, respectively.

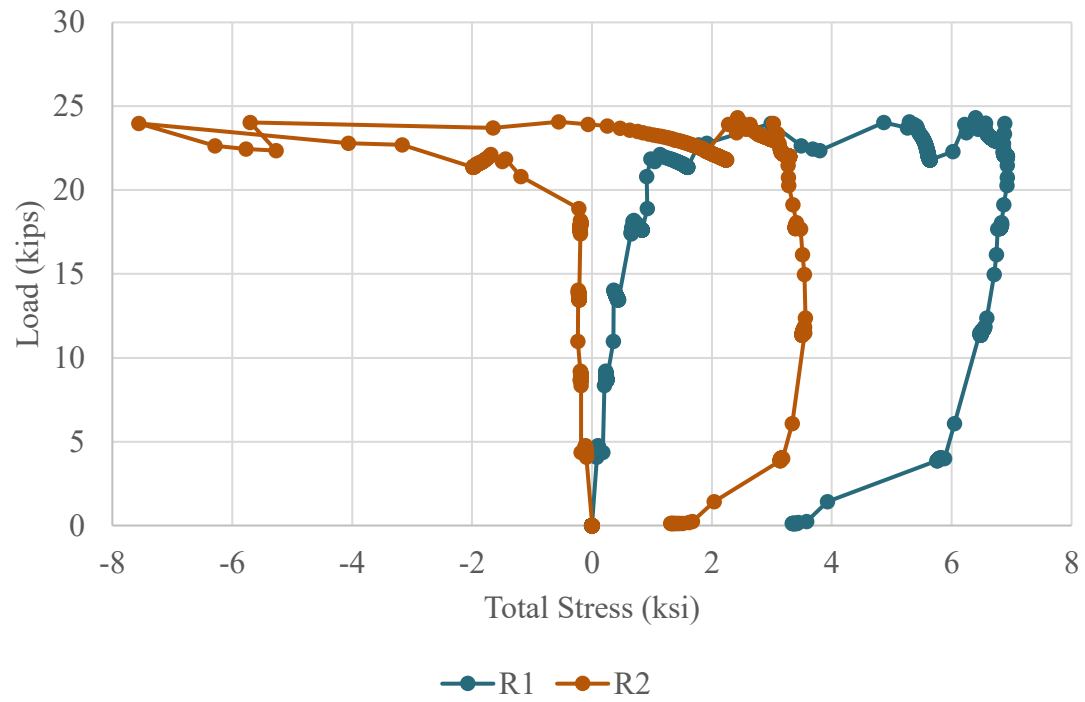


Figure 152. Specimen W-12-12, total stress measured by strain gauges R1 and R2 on steel reinforcement cage.

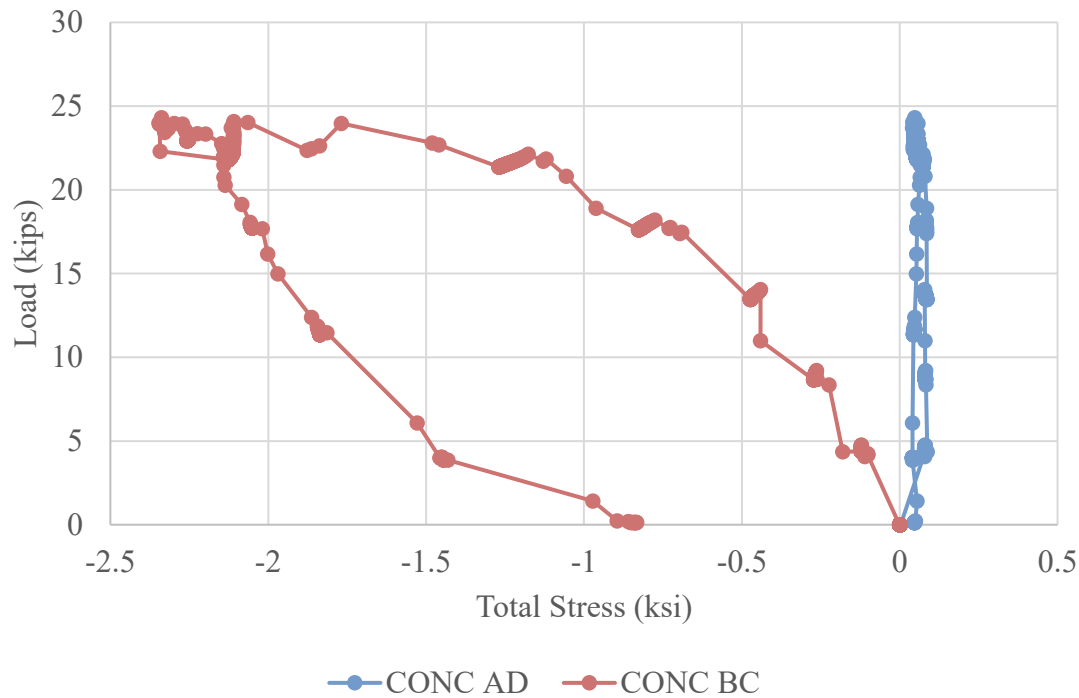


Figure 153. Specimen W-12-12, total stress measured by strain gauges CONC AD and CONC BC on the concrete cap surface.

R1 on the trailing side showed small compression behavior of approximately 210 psi throughout most of the test. As the load approached failure, the data showed a compression peak followed by a sudden transition to tension of approximately 3.2 ksi during the load plateau and into the unloading phase. R2 on the leading side showed tension behavior throughout the test, increasing steadily to approximately 3.2 ksi at the failure load plateau. The stress magnitudes in both reinforcement gauges are notably higher than those recorded in specimens W-10-15 and W-10-10, though still well below reinforcement yield stress.

The concrete surface gauges showed modest stress levels consistent with steel-controlled failure. CONC AD on the trailing side recorded small tension of approximately 65 psi, nearly negligible throughout the test. CONC BC on the leading side recorded compression up to approximately 2.4 ksi at the failure load. While this is a meaningful stress level, it is substantially below the values recorded in the concrete-failure specimens

W-10-7.5R and W-10-5, and is consistent with the interpretation that concrete bearing did not control the failure mode in this specimen.

Figure 154 shows the bending moment results for the exposed strain gauges. As noted above, EX-C6-A/B showed anomalous behavior and are considered unreliable, therefore their data are not used for quantitative conclusions. EX-C1-A/B showed measurements close to the theoretical expected moment up to approximately 14 kips, after which the sensors malfunctioned. The reliable portion of the EX-C1 data is consistent with theoretical expectations and provides a useful reference for the lower load stages.

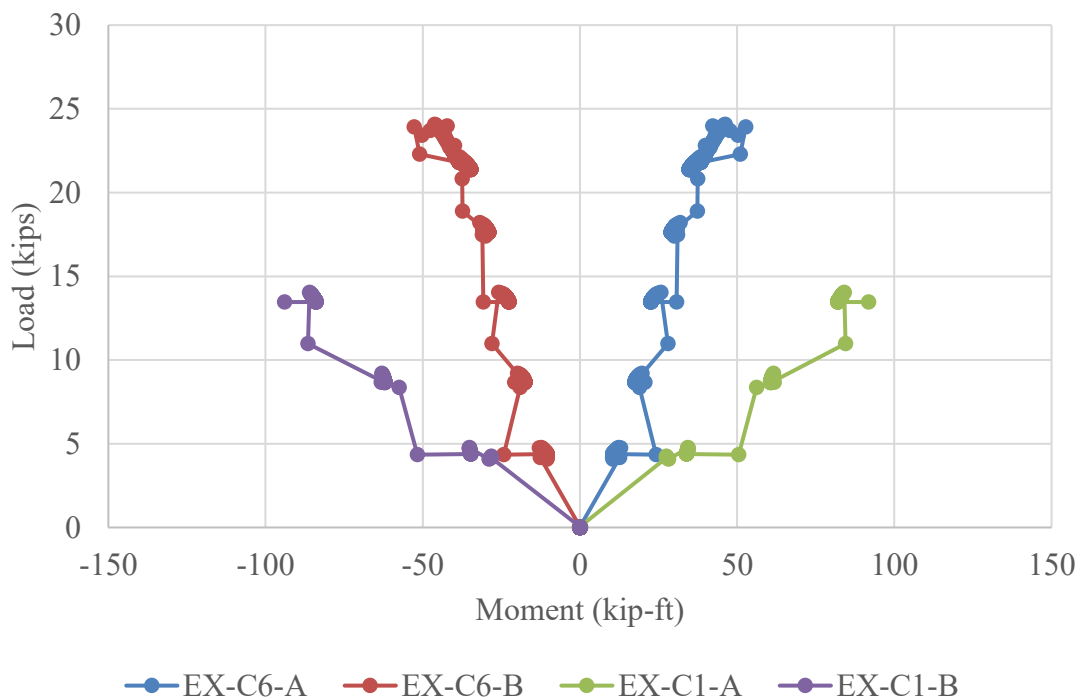


Figure 154. Specimen W-12-12, calculated bending moment for EX strain gauges.

Figure 155 shows the bending moment results for the embedded strain gauges. At the EM-I level, moments were compatible with the EX-C1 measurements at the same load stages, confirming the expected moment continuity across the steel-concrete interface and the beginning of moment decay into the embedment zone. EM-M-A showed coherent measurements during the reliable portion of the test, with data extracted up to intermediate load levels with the caveat that intermittent malfunction limits confidence in

those readings. EM-H-A/B provided reliable measurements up to approximately 5 kips before malfunctioning.

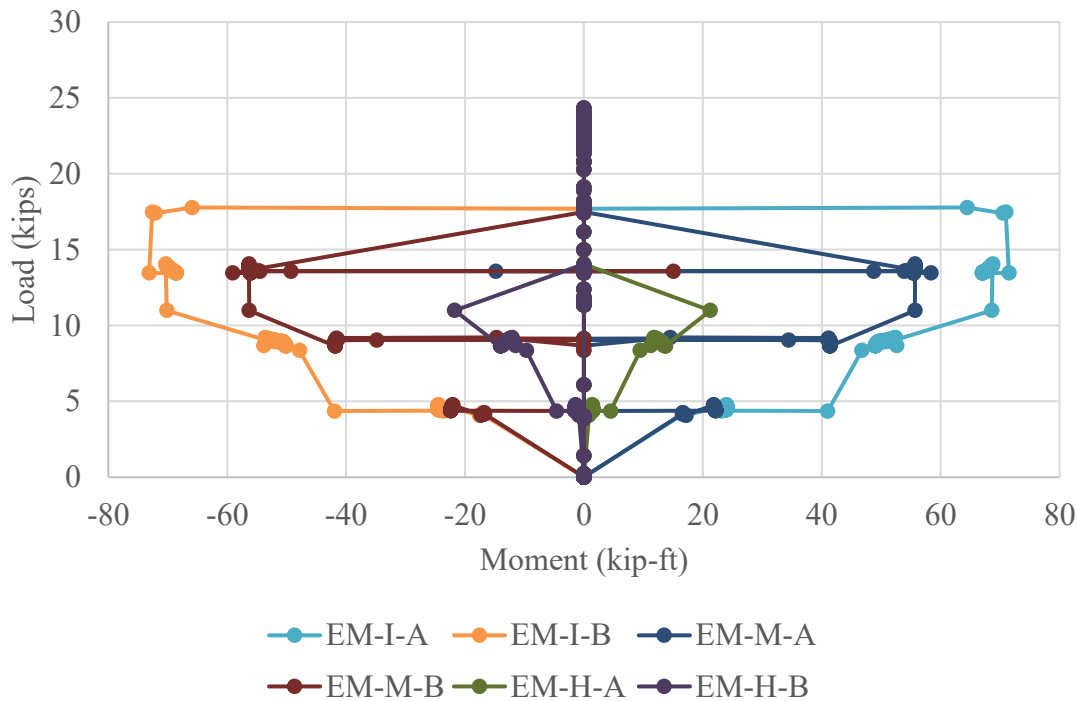


Figure 155. Specimen W-12-12, calculated bending moment for EM strain gauges.

The moment profile, in Figure 156, across the reliable gauge locations illustrates clearly how moment decreases with embedment depth. At the 6 kip load level, EX-C1 recorded approximately 41.5 kip-ft, decreasing to approximately 34 kip-ft at the EM-I level, 27 kip-ft at the EM-M level, and 6.5 kip-ft at the EM-H level - a consistent and physically coherent decay from the cap surface to the pile head. At the 12 kip load level, EX-C1 recorded approximately 74 kip-ft, decreasing to approximately 63 kip-ft at EM-I and 49 kip-ft at EM-M, while EM-H had already ceased functioning at that load stage.

The moment values at the pile head at low load levels confirm that the 12-inch embedment generates measurable moment transfer and partial fixity at the connection, consistent with the behavior observed in the deeper HP10x42 embedment specimens.

All other plots for specimen W-12-12 (wire potentiometers, strain measurements, and pile total stress results) are provided in Appendix A.

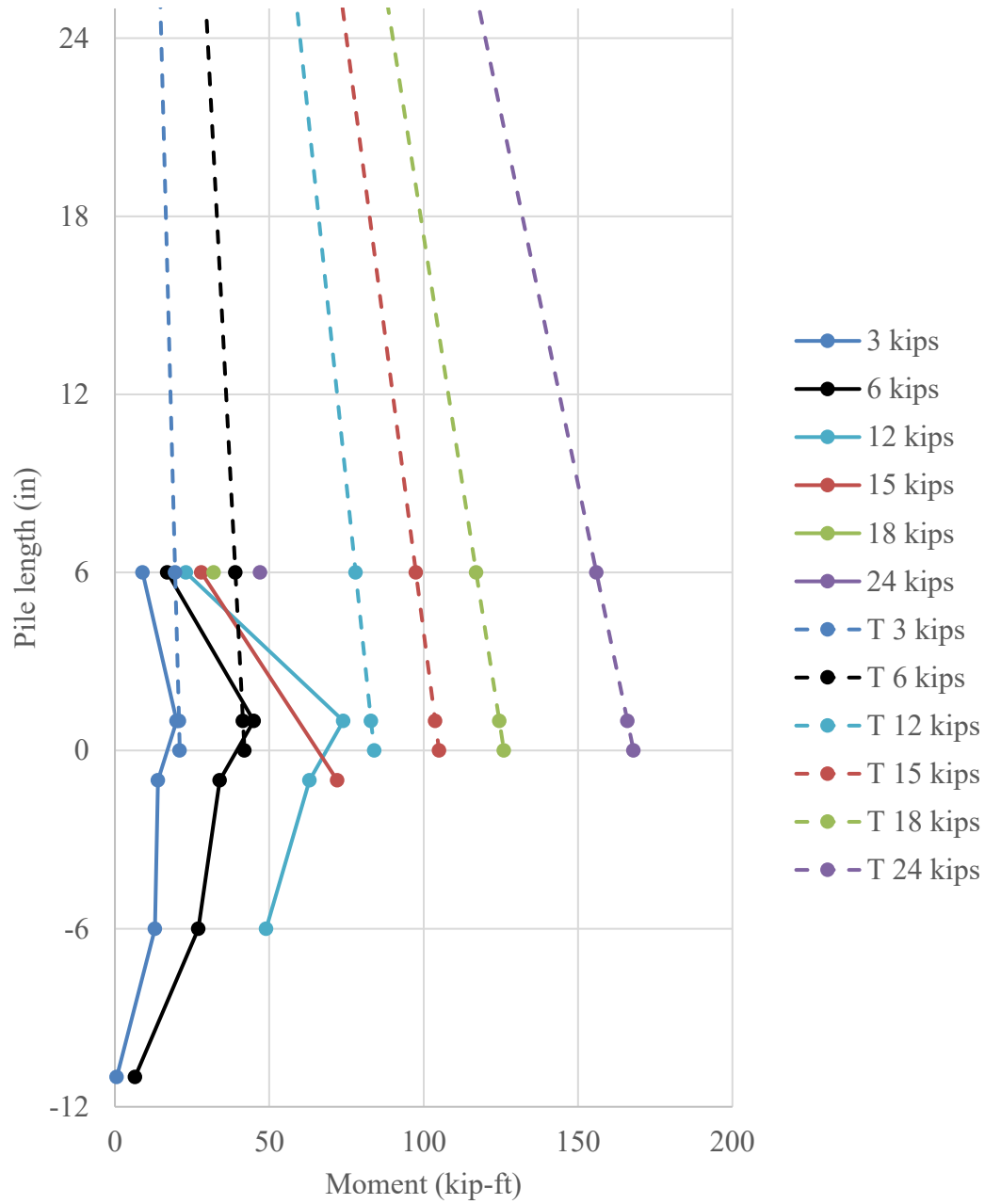


Figure 156. Specimen W-12-12 measured moments (solid lines) and theoretical moments (dashed lines) for each load stage. Zero y-axis marks the cap surface: positive is the exposed pile length, and negative is the embedded pile length.

1.6. Specimen W-12-6

Specimen W-12-6 was tested on 2/06/2026. Figure 157 shows the load vs. deflection curve.

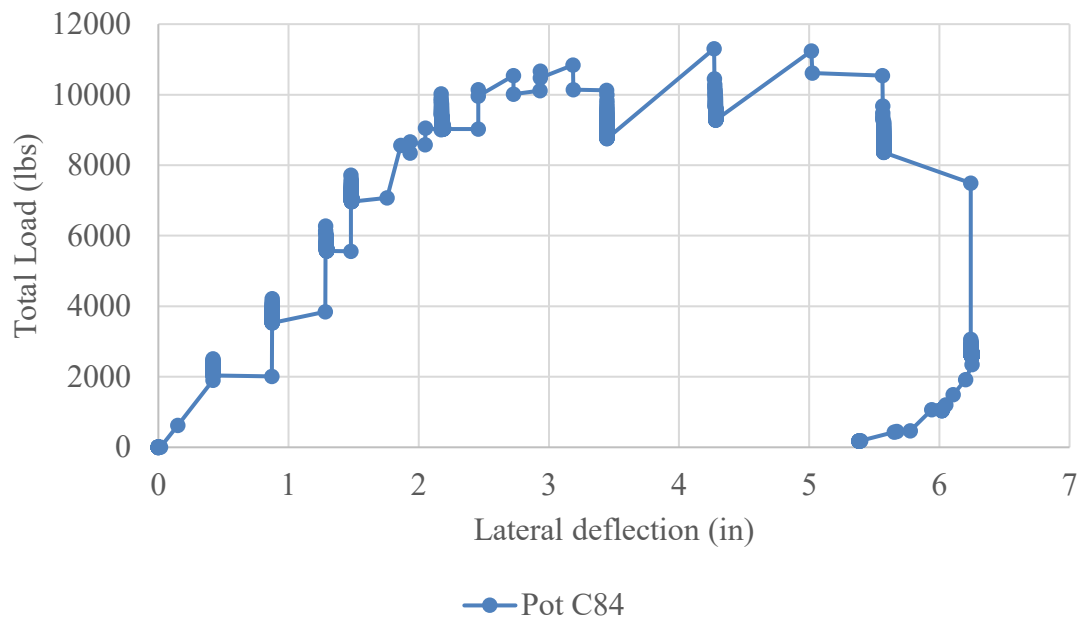


Figure 157. Specimen W-12-6 horizontal load vs lateral deflection curves.

The load vs. deflection curve for W-12-6 showed a steady load increase until approximately 7.8 kips at a lateral deflection of 1.8 inches, at which point the curve slightly changed its slope, indicating the transition from elastic to plastic behavior. This load level is notably close to the upper bound of the Marcakis and Mitchell (1980) predicted concrete bearing capacity of 7.2 kips, suggesting that the concrete bearing zone began to soften at approximately that load and that the observed onset of nonlinearity reflects the initiation of concrete bearing failure rather than any steel structural limit.

The maximum achieved horizontal load was approximately 10 kips at a deflection of 3.1 inches, representing a 38% exceedance above the upper bound of the Marcakis and Mitchell (1980) predicted range of 5.75 to 7.2 kips. As discussed for previous specimens, this exceedance is consistent with the known conservatism of the Marcakis and Mitchell (1980) equation, particularly for shallow embedments. The maximum load

remains well below any threshold associated with steel yielding onset, flange local buckling, or full plastic moment of the HP12x53 section. Visual observations and instrumentation data both confirm that failure occurred by concrete cracking and crushing at the connection.

At the start of the test, a few hairline cracks were noted, including a minor diagonal crack initiating at flange D and a hairline-width gap at the steel-concrete interface on the trailing side. Through Stage 2 (peak total load approximately 4.2 kips, pile head deflection 0.72 inches), the diagonal crack at flange D increased in length and the interface gap widened, but no new cracks appeared. At Stage 3 (peak total load approximately 6.1 kips, pile head deflection 1.06 inches), diagonal cracks appeared at all four flanges, and the interface gap continued to increase. At Stage 4 (peak total load approximately 7.6 kips, pile head deflection 1.28 inches), the diagonal cracks continued to grow and the gap widened further. At Stage 5 (peak total load approximately 10.0 kips, pile head deflection 1.91 inches), the diagonal cracks widened noticeably, concrete spalling was observed at one of the flanges, and the connection appeared to be failing. At Stage 6 (peak total load approximately 10.0 kips, pile head deflection 3.13 inches), the diagonal cracks widened substantially, concrete spalling was observed in the leading zone, and the specimen was considered to have reached failure. The load did not increase beyond approximately 10 kips despite continued pressurization. Stages 7 and 8 were conducted to confirm failure, with continued cracking and crack widening observed at each stage.

Post-test crack width measurements showed large crack widths ranging from 1 mm to 10 mm, with the widest cracks observed at flange D, consistent with the concrete bearing failure mode. Figures 158, 159 and 160 show the cracking patterns observed during and after the test.

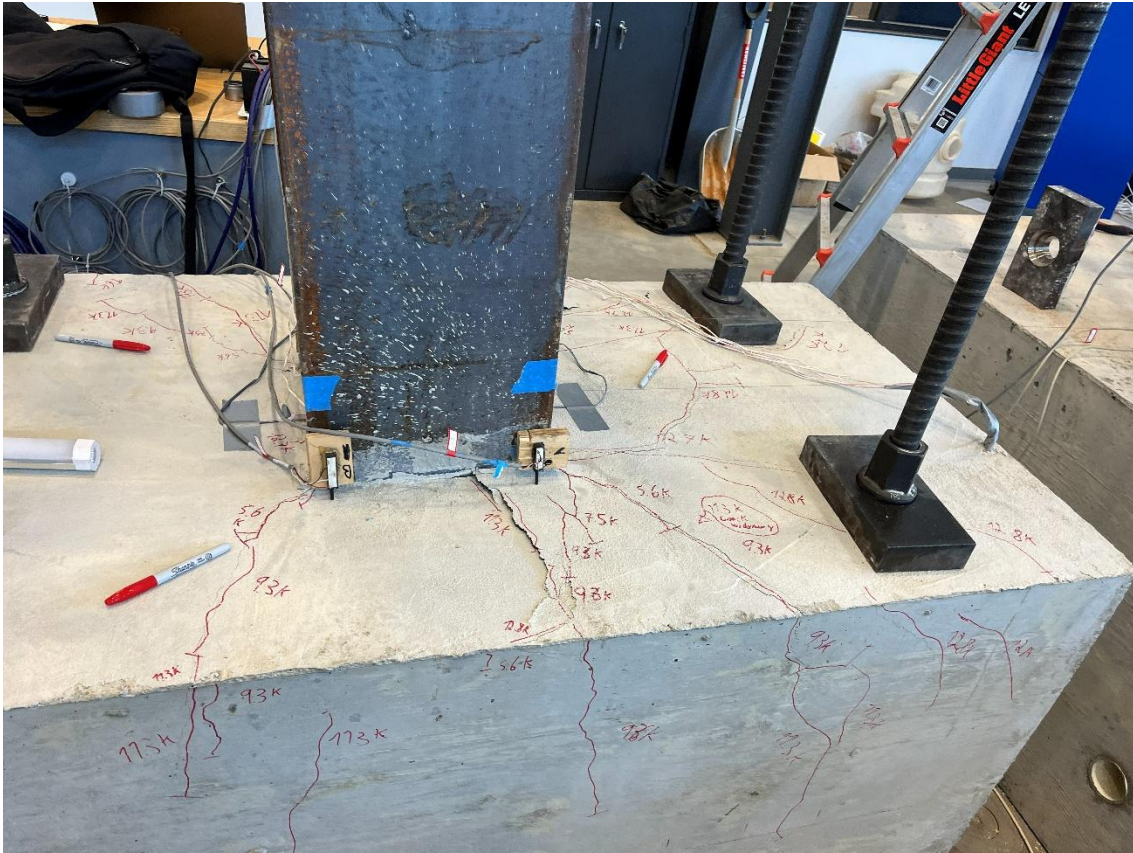


Figure 158. Specimen W-12-6 crack formation after finished test (load direction: from right to left).



Figure 159. Specimen W-12-6 after final load stage (approx. 10 kips). Observed concrete crushing on the leading side.

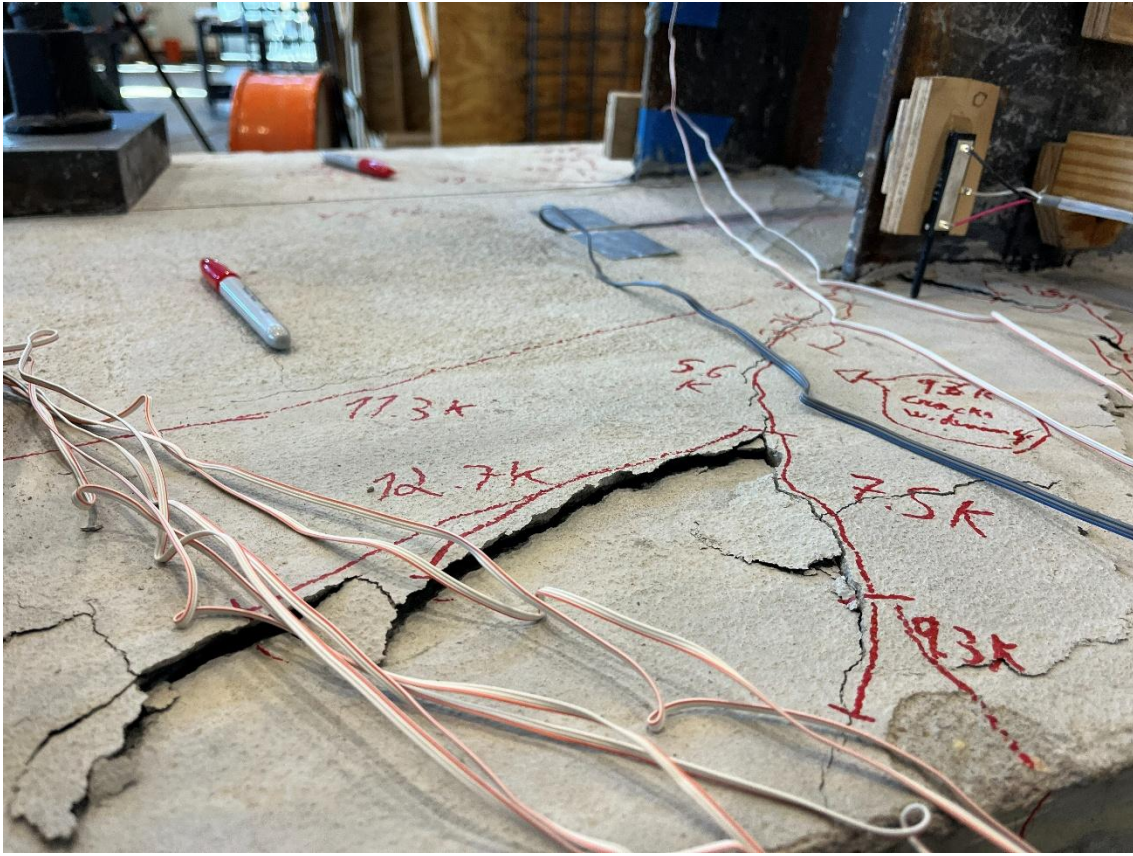


Figure 160. Specimen W-12-6 after final load stage (approx. 10 kips). Observed excessive crack opening (trailing side).

The inclinometers and tiltmeters generated rotation measurements displayed in Figure 161.

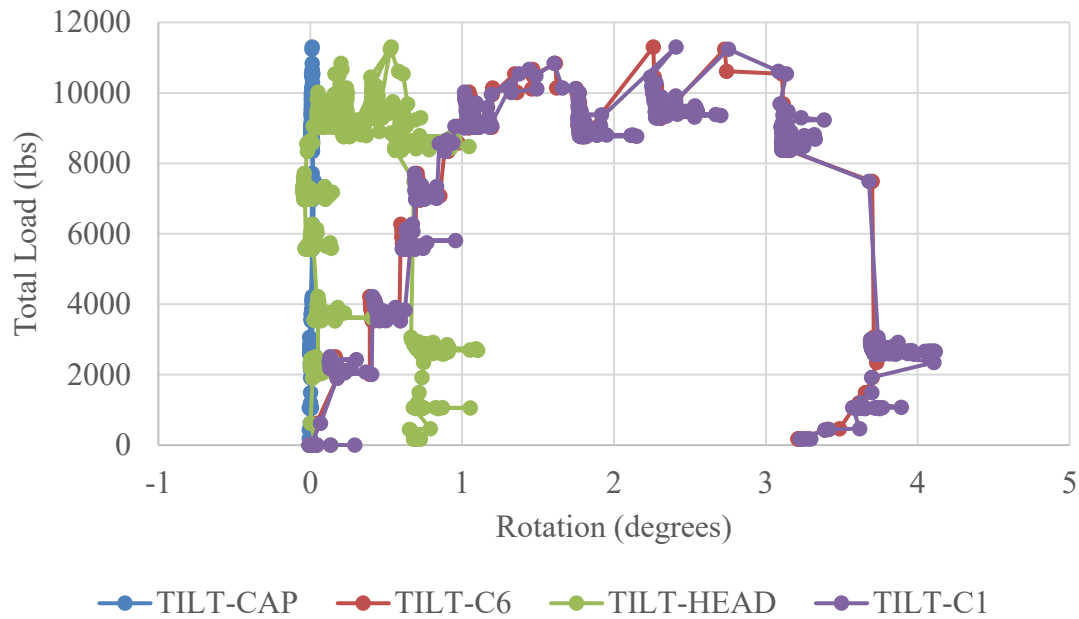


Figure 161. Specimen W-12-6, rotation measurements (positive rotation follows the load direction).

TILT-CAP showed negligible rotation throughout the entire loading sequence, confirming that the concrete cap remained stationary.

TILT-C1 and TILT-C6 exhibited matching behavior throughout the entire load test, both reaching approximately 1.5 degrees at peak load and retaining approximately 3.25 degrees of residual rotation upon unloading. Consistent with the behavior observed in specimens W-10-7.5R and W-10-5, no divergence developed between the two instruments at any load stage, confirming that no flange buckling occurred between the sensor positions and reinforcing the conclusion that failure was controlled by concrete cracking and crushing rather than steel structural failure.

TILT-HEAD showed complex behavior throughout the test. At the onset of loading, a minor initial clockwise rotation of approximately 0.07 degrees was recorded before the sensor behavior became more complex. As load increased, TILT-HEAD transitioned to a small counterclockwise rotation of approximately -0.04 degrees at around 5.5 kips, consistent with the geometric rod mechanism described in previous specimens, where the tiltmeter rod welded to the pile web rotates opposite to the pile head rotation.

At approximately 8 kips, near the point at which the concrete bearing zone began to lose capacity, a second inflection occurred and the rotation reversed again, becoming clockwise and reaching approximately 0.5 degrees at peak load. This final reversal is consistent with what was observed in specimen W-10-5: as the concrete bearing resistance degraded sufficiently, the internal concrete mass began to move more freely, releasing the constraint on the measurement rod and allowing it to follow the actual clockwise rotation of the pile head rather than rotate against it.

Spring potentiometer results are shown in Figure 162.

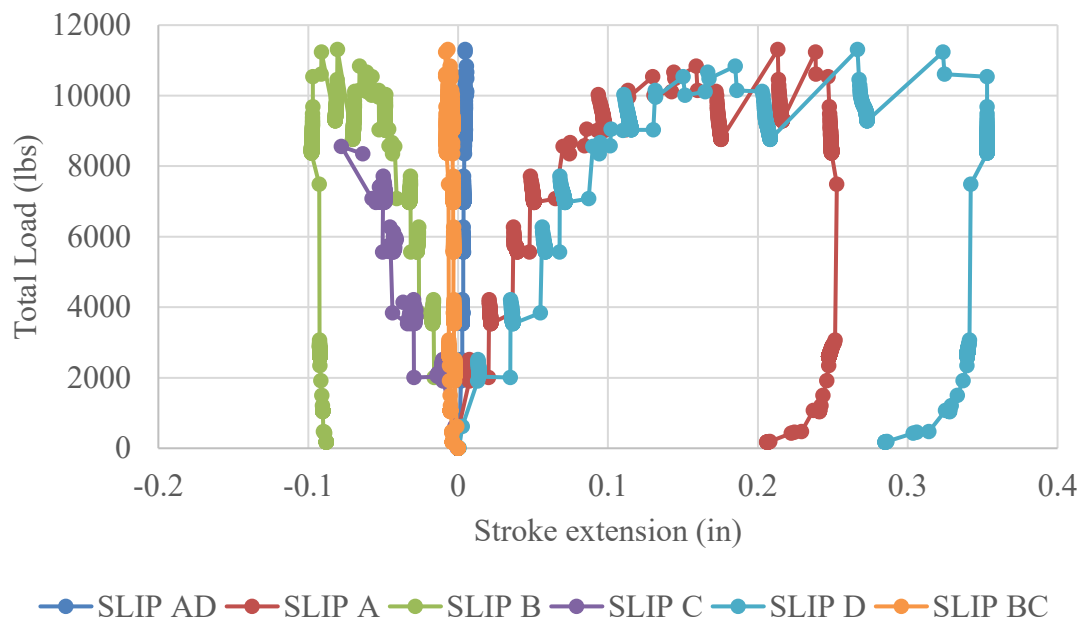


Figure 162. Specimen W-12-6, spring potentiometer measurements.

SLIP AD and SLIP BC recorded negligible displacement throughout the loading sequence, confirming that the concrete cap remained stationary.

On the trailing side, SLIP A and SLIP D both extended their strokes by approximately 0.15 inches at peak load, as the trailing flanges were pulled away from the cap surface. On the leading side, SLIP B compressed approximately 0.075 inches at peak load, and SLIP C showed a compression of approximately 0.075 inches before malfunctioning at the 8 kip load stage. The symmetry between the leading-side sensors

and the similarity between trailing-side sensors indicate consistent pile rotation behavior up to failure.

The strain gauge measurements were collected and reduced to obtain bending moments, axial loads, and stress distributions along the pile and in the cap. Strain measurements exceeding the steel yield point or 70% of the estimated concrete compressive strength were eliminated from analysis. For this specimen, strain gauges EX-C6-A and EX-C6-B malfunctioned, showing strains substantially smaller than theoretical expectations, attributed to a likely bonding issue during gauge installation consistent with the behavior observed in specimen W-12-12. EX-C1-A also malfunctioned. EX-C1-B functioned well throughout the test and served as the primary source of reliable exposed-pile moment data for this specimen. EM-I-A and EM-I-B malfunctioned entirely, providing no usable data at that level. EM-H-A and EM-H-B functioned reliably up to approximately 2.5 kips, after which both ceased recording.

As with specimen W-10-5, the early loss of the embedded gauges is attributed to the greater internal movement occurring within the shallow embedment zone: at 6-inch embedment, concrete bearing and gap-opening displacements begin at low load levels, and the resulting interface movement was likely sufficient to disrupt gauge bond well before the specimen approached failure. The EM-M level was not installed in this specimen.

Figures 163 and 164 show the total stresses measured by the strain gauges in the steel reinforcement cage and on the concrete cap surface, respectively.

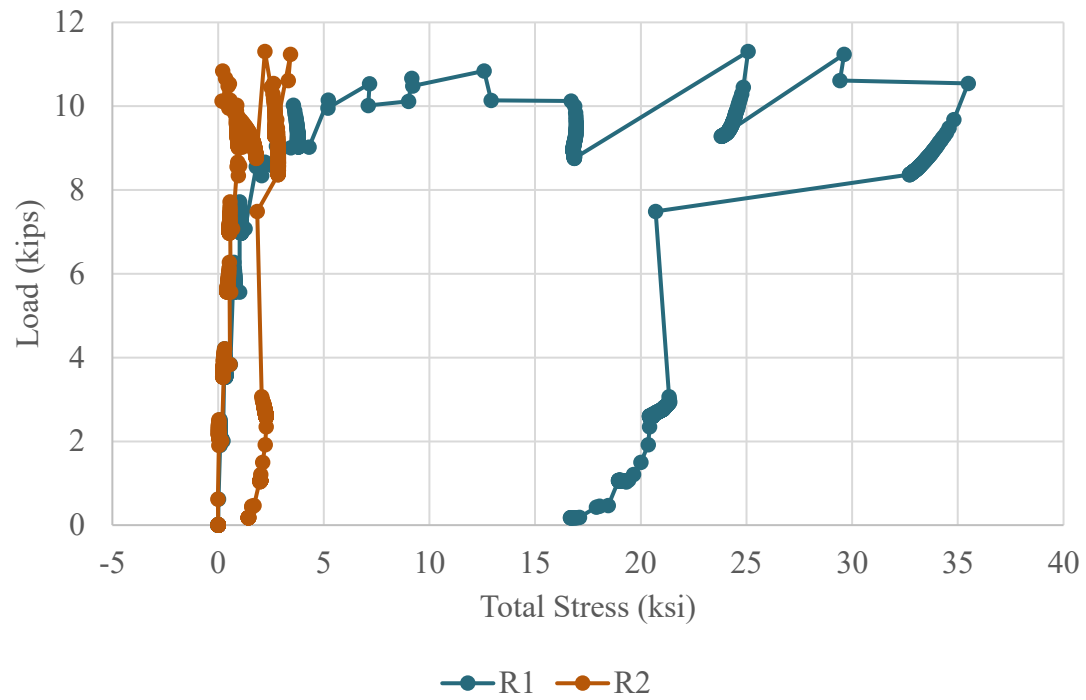


Figure 163. Specimen W-12-6, total stress measured by strain gauges R1 and R2 on steel reinforcement cage.

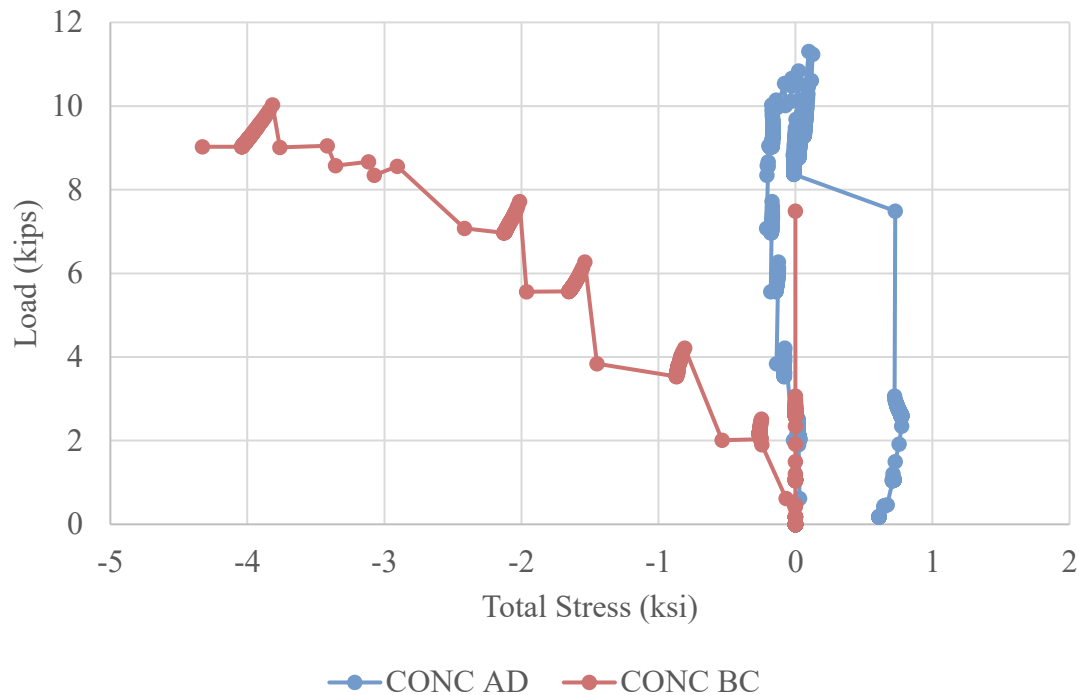


Figure 164. Specimen W-12-6, total stress measured by strain gauges CONC AD and CONC BC on the concrete cap surface.

Both reinforcement cage gauges recorded tension throughout the test. R1, on the trailing side, reached approximately 10 ksi at peak load. R2, on the leading side, also recorded tension throughout, at lower magnitudes than R1. The elevated rebar stress levels relative to the steel-failure specimens are consistent with the greater concrete demand at this shallow embedment, though both values remain below reinforcement yield stress.

The concrete surface gauges showed compression on both the leading and trailing faces. CONC BC on the leading side reached approximately 4.5 ksi in compression at the failure load, the highest leading-face compression, approximately 73% of f'_c for this specimen ($f'_c = 6,192$ psi). CONC AD on the trailing side also showed compression behavior, at lower magnitudes than the leading side. These results are consistent with the concrete bearing failure mode and continue the trend of increasing concrete bearing stress with decreasing embedment depth observed across the test program.

Figure 165 shows the bending moment results for the exposed strain gauges. EX-C6-A/B showed anomalous behavior and are considered unreliable, so their data are not used for quantitative conclusions. EX-C1-B recorded moment values consistent with theoretical expectations based on the applied horizontal load throughout the test, providing the main reliable record on the exposed portion of the pile.

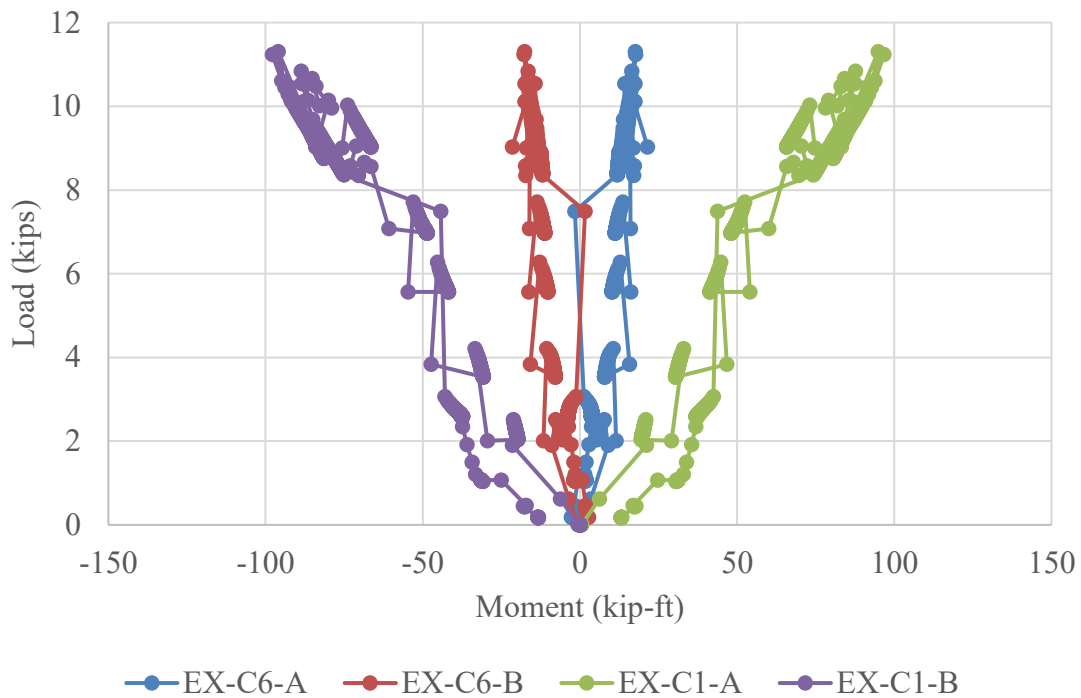


Figure 165. Specimen W-12-6, calculated bending moment for EX strain gauges.

Figure 166 shows the bending moment results for the embedded strain gauges. EM-I-A/B malfunctioned throughout the entire test and provided no usable data. EM-H-A/B recorded reliable measurements up to approximately 2.5 kips, showing moment values consistent with the expected decay into the embedment zone: approximately 0.25 kip-ft at the 1 kip load stage and approximately 1.25 kip-ft at the 2 kip load stage. These values are small, as expected at the pile head for a shallow embedment at low load levels, and confirm that moment transfer into the embedment zone was occurring even at early stages of the test.

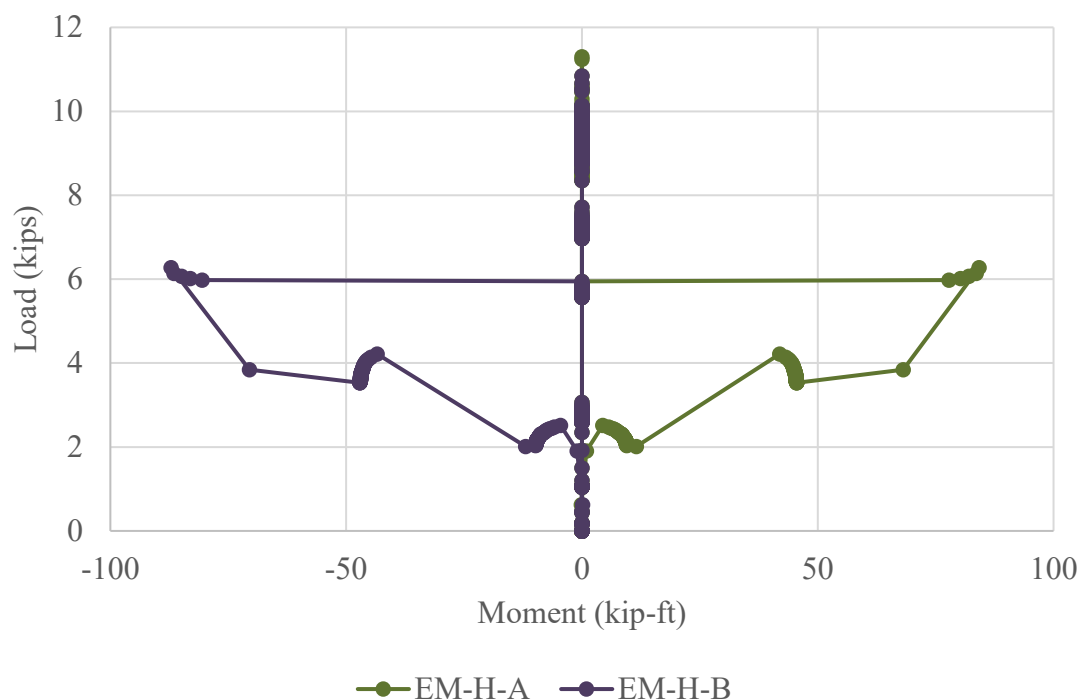


Figure 166. Specimen W-12-6, calculated bending moment for EM strain gauges.

Figure 167 shows the moment profile along the pile length for each load stage, along with a comparison to the theoretical moment based on the applied horizontal load. At lower load levels (1 kip and 2 kips), all functioning gauges showed good agreement with theoretical moment expectations. At higher load levels (4, 6, 8, and 10 kips), EX-C6 did not record reliable moments, while EX-C1-B remained consistent with theoretical expectations. The EM-I level provided no data, and EM-H recorded the small but physically coherent moment values noted above at the earliest load stages. The available data confirm that moment was being transferred into the embedment zone and decreasing with depth, indicating that even at 6-inch embedment the connection retained a partial degree of fixity at low load levels, consistent with the partial fixity trend observed across all specimens in this program.

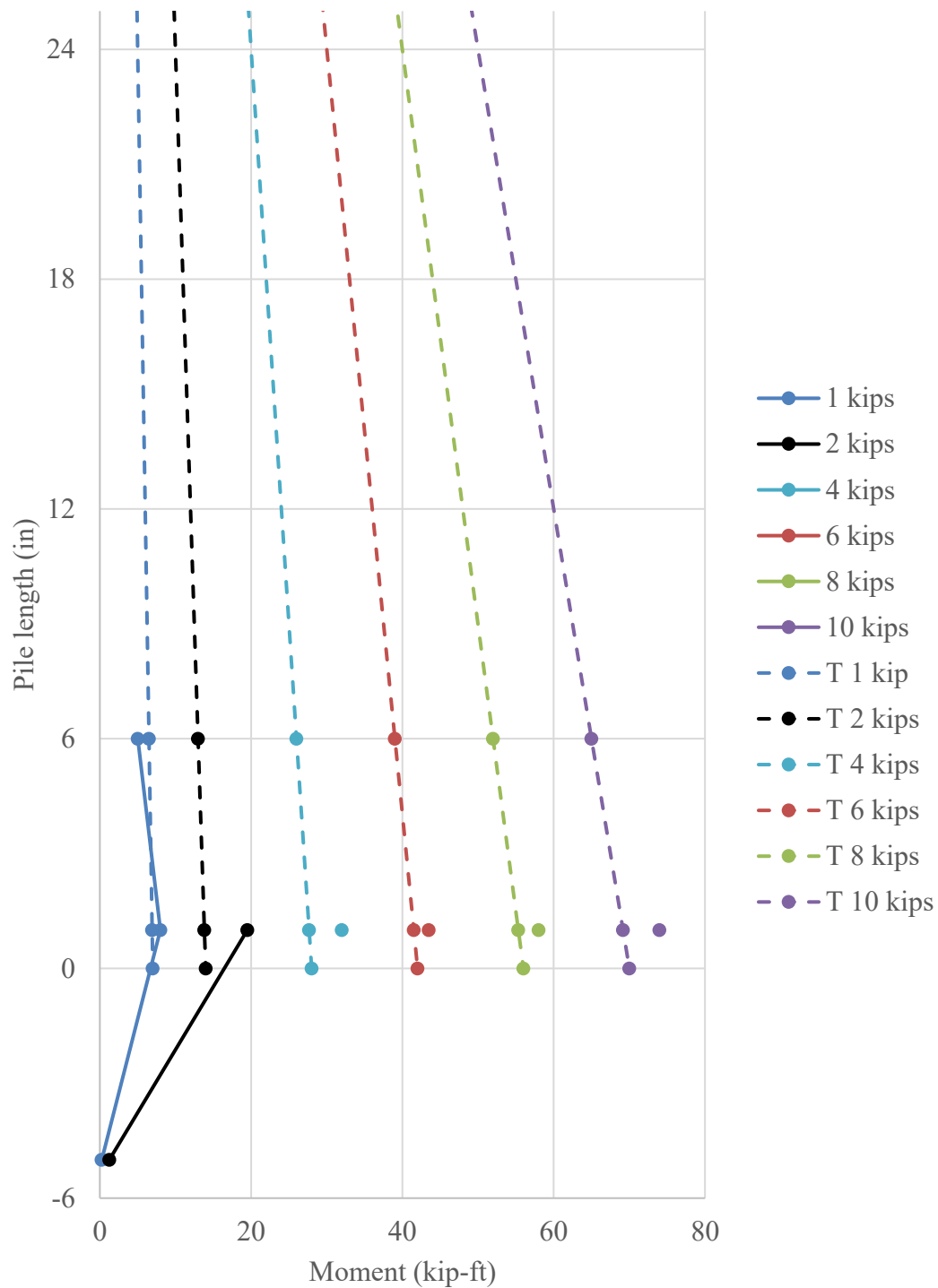


Figure 167. Specimen W-12-6 measured moments (solid lines) and theoretical moments (dashed lines) for each load stage. Zero y-axis marks the cap surface: positive is the exposed pile length, and negative is the embedded pile length.

All other plots for specimen W-12-6 (wire potentiometers, strain measurements, and pile total stress results) are provided in Appendix A.

1.7. Specimen S-10-10

Specimen S-10-10 was tested on 2/18/2026. Unlike all previous specimens, which were loaded to bend the pile about its weak axis, this test bent the pile about its strong axis, with the concrete cap positioned identically to previous tests. This configuration simulates a bent cap subjected to loading in the bridge longitudinal direction rather than the transverse direction. As a result of the strong-axis orientation, flanges A and B are on the trailing side of the loading direction, and flanges C and D are on the leading side, a different convention from the one used in all previous specimens.

Figure 168 shows the load vs. deflection curve.

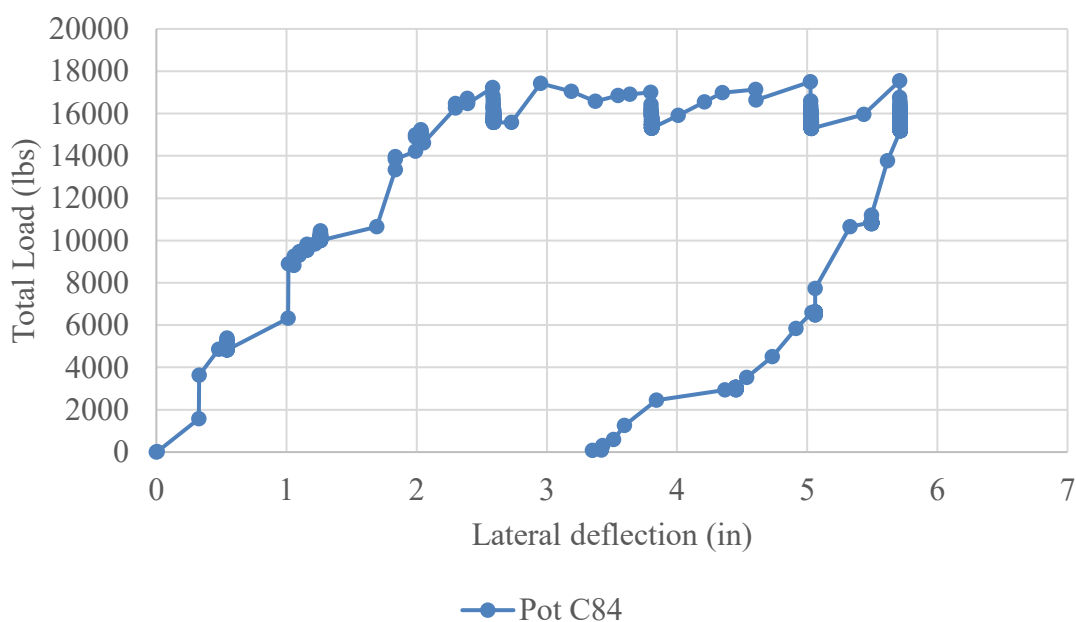


Figure 168. Specimen S-10-10 horizontal load vs lateral deflection curves.

The curve showed a steady load increase until approximately 14-15 kips at a lateral deflection of approximately 2 inches, at which point the curve quickly changed slope. The specimen reached a maximum horizontal load of 16.3 kips at a deflection of

approximately 3 inches, after which the load plateaued despite continued jack pressurization until a maximum deflection of 5.7 inches was reached. The system was then unloaded.

The structural moment capacity of the HP10x42 bent about its strong axis, per the AISC Steel Manual (2023), would require a maximum horizontal load of approximately 30.5 kips to achieve the onset of steel yielding. The maximum achieved load of 16.3 kips falls well below this threshold, confirming that steel structural failure did not occur.

According to the Marcakis and Mitchell (1980) method, the concrete bearing capacity of this connection ranges from approximately 12.8 to 19.3 kips. The observed failure load of 16.3 kips falls within this range, a notably different outcome from all previous specimens, where the observed failure load exceeded the upper bound of the Marcakis and Mitchell (1980) predicted range.

This result is also worth comparing to specimen W-10-10, which shares the same embedment depth and pile section. In W-10-10, the maximum load under weak-axis bending was approximately 14-15 kips, but failure was controlled by steel flange buckling rather than concrete bearing. In S-10-10, the strong-axis configuration generated a similar maximum load of 16.3 kips, but failure was controlled by concrete bearing, as the pile has substantially greater steel structural capacity about its strong axis. The similar load magnitudes at different failure modes across the two specimens highlight how bending axis orientation affects the controlling failure mechanism at the same embedment depth.

Visual observations and instrumentation data both confirm that failure occurred by concrete cracking and crushing at the connection, and are seen in figures 169, 170 and 171.

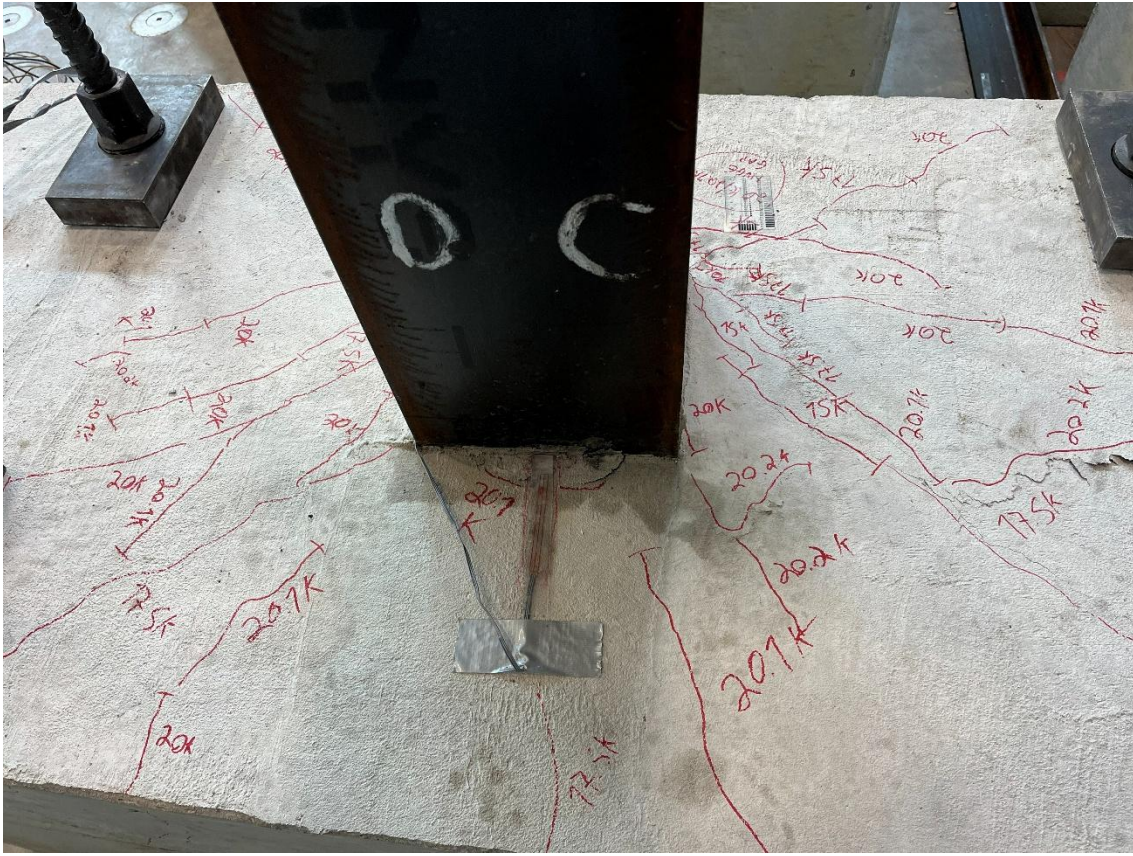


Figure 169. Specimen S-10-10 crack formation after target load 20.2 kips (actual load 16.3 kips) (load direction: from top to bottom in the photo).

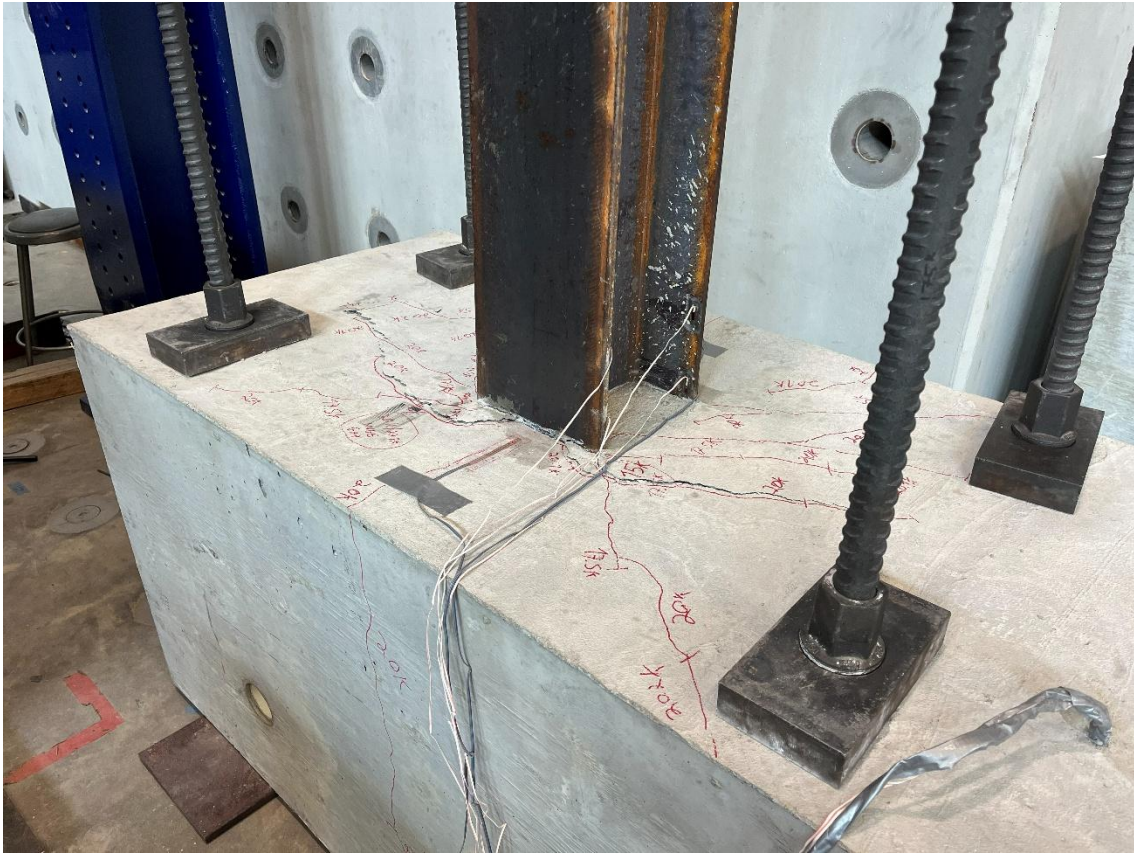


Figure 170. Specimen S-10-10 crack formation on the trailing side after target load 20.2 kips (actual load 16.3 kips) (load direction: from bottom left corner to top right corner).

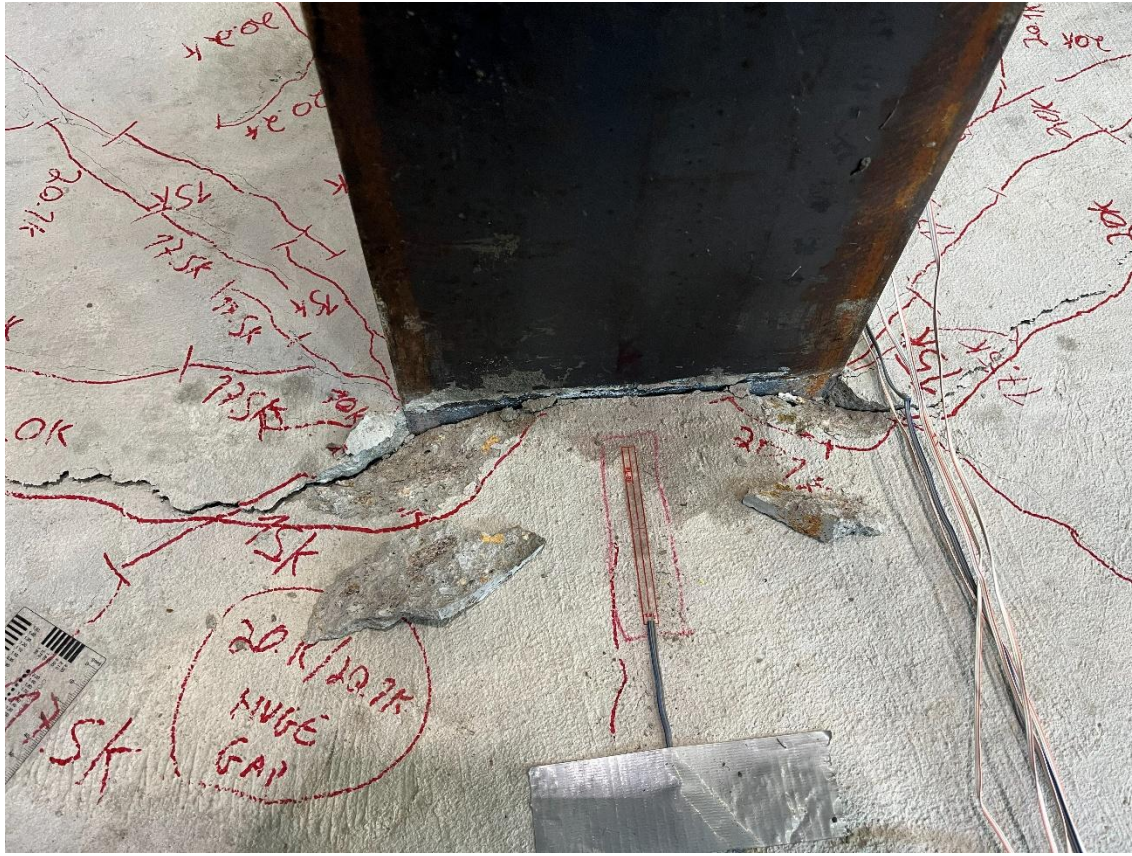


Figure 171. Specimen S-10-10 crack formation on the trailing side after target load 20.2 kips (actual load 16.3 kips) (load direction: from bottom to top).

At Stage 1 (peak total load approximately 5.3 kips), hairline cracks appeared at the ends of flanges A and D, indicating early separation at the steel-concrete interface on the trailing side.

At Stage 2 (peak total load approximately 10.3 kips), additional cracks indicating pullout appeared at the inside and outside faces of flanges A and B, and a diagonal crack initiated at flange B. A notable feature of the crack morphology in this specimen was observed at Stage 3 (peak total load approximately 15.1 kips): the diagonal cracks propagated in the same direction as the applied load, rather than opposite to it as seen in all previous weak-axis specimens. This is attributed to the geometry of strong-axis bending, with the entire flange faces of A and B moving in the load direction, the concrete mass deforms in that direction, and the resulting diagonal cracks follow accordingly.

At Stage 4 (peak total load approximately 17.0 kips), the failure mechanism evolved further: diagonal cracks running opposite to the load direction appeared on flanges A and B, and additional diagonal cracks initiated at flanges A and D. The interface gap on the outside of the trailing flanges became large. Large "wing-shaped" cracks developed behind flanges A and B, a crack morphology distinct from the diagonal corner cracks observed in all weak-axis specimens.

These wing-shaped cracks are interpreted as reflecting the concrete being pulled away from the full flange face as the trailing flanges move in the load direction: unlike weak-axis bending where the flange tips initiate local bearing, followed by the web, in strong-axis bending the entire flange width engages the concrete simultaneously, producing a wider zone of tensile separation behind the flange face.

At Stages 5 through 7 (peak loads approximately 16.3-17.0 kips), cracking continued to propagate and widen, the wing-shaped cracks became very large, and no signs of steel yielding were observed.

Post-test crack width measurements ranged from 0.06 mm to 5 mm, consistent with the concrete bearing failure mode.

The inclinometers and tiltmeters generated rotation measurements displayed in Figure 172.

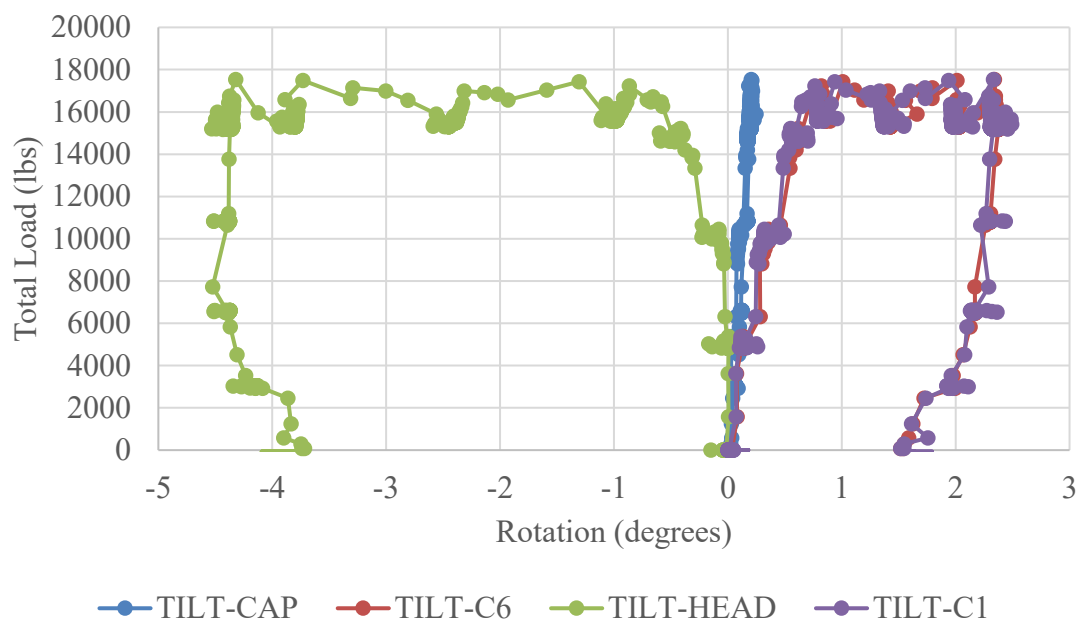


Figure 172. Specimen S-10-10, rotation measurements (positive rotation follows the load direction).

TILT-CAP showed a small rotation of approximately 0.2 degrees, larger than in all previous specimens where TILT-CAP was essentially negligible. This is physically expected: in the strong-axis loading configuration, the lateral load acts transversely on the cap, and the cap cross-section is less stiff about this axis than about the axis engaged in weak-axis pile bending tests. The rotation remained stable throughout the test and did not affect the validity of the results.

TILT-C1 and TILT-C6 behaved very similarly throughout the entire load test, consistent with the behavior observed in all concrete-failure specimens, confirming that no steel structural yielding or flange buckling occurred between the two sensor positions. Both instruments reached approximately 1 degree of rotation at the maximum load stage.

TILT-HEAD again exhibited counterclockwise rotation, consistent with the geometric rod mechanism described in Section 1.2. The counterclockwise rotation became more pronounced at approximately 10 kips and reached approximately -1 degree at the maximum load of 16.3 kips.

Spring potentiometer results are shown in Figure 173.

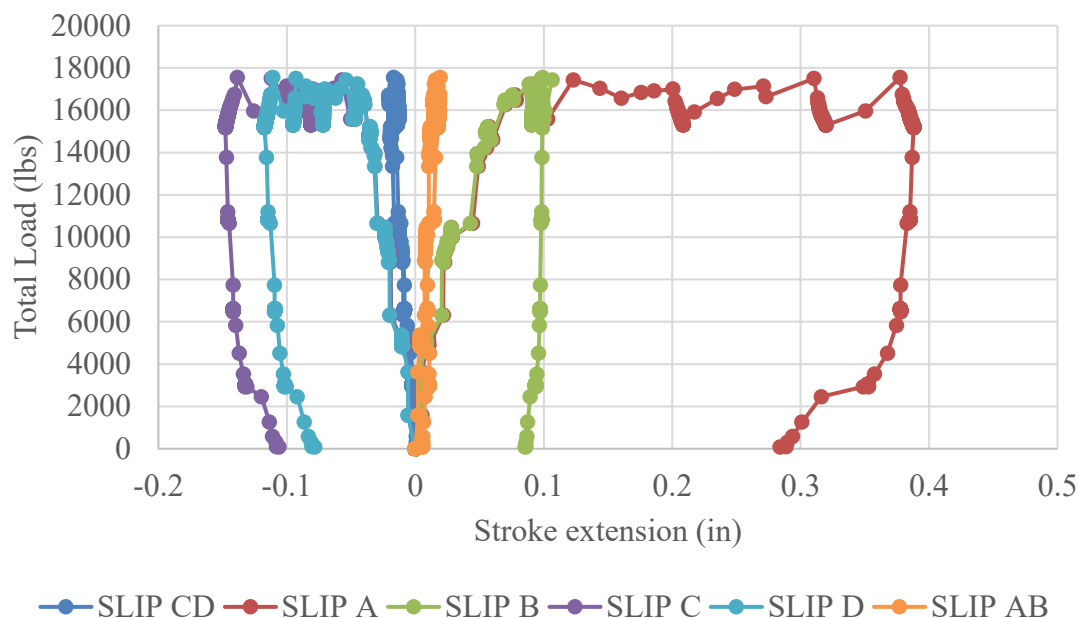


Figure 173. Specimen S-10-10, spring potentiometer measurements.

SLIP AB and SLIP CD recorded small and approximately symmetric cap movement of approximately 0.015 inches, confirming that the concrete cap remained essentially stationary throughout the test.

On the trailing side, SLIP A and SLIP B both extended their strokes by approximately 0.1 inches at the maximum load stage, as the trailing flanges were pulled away from the cap surface.

On the leading side, SLIP C and SLIP D showed stroke compression of approximately 0.05 inches at the maximum load stage, as the leading flanges were pushed into the cap.

The asymmetry between trailing-side extension (0.1 inches) and leading-side compression (0.05 inches) is consistent with the concrete bearing failure mode, where cracking and gap opening on the trailing side allowed greater flange displacement than the concrete crushing on the leading side permitted in compression.

The strain gauge measurements were collected and reduced to obtain bending moments, axial loads, and stress distributions along the pile and in the cap. Strain measurements exceeding the steel yield point or 70% of the estimated concrete

compressive strength were eliminated from analysis. For this specimen, strain gauges EX-C6-A/D and EX-C1-A/D malfunctioned and are not used for quantitative conclusions, though they retained sufficient qualitative signal to indicate moment trends. EM-I-D and EM-H-A/D were entirely non-functional. EM-I-A functioned up to approximately 5.5 kips. EM-M-A functioned up to approximately 5.5 kips. EM-M-D was non-functional.

Figures 174 and 175 show the total stresses measured by the strain gauges in the steel reinforcement cage and on the concrete cap surface, respectively.

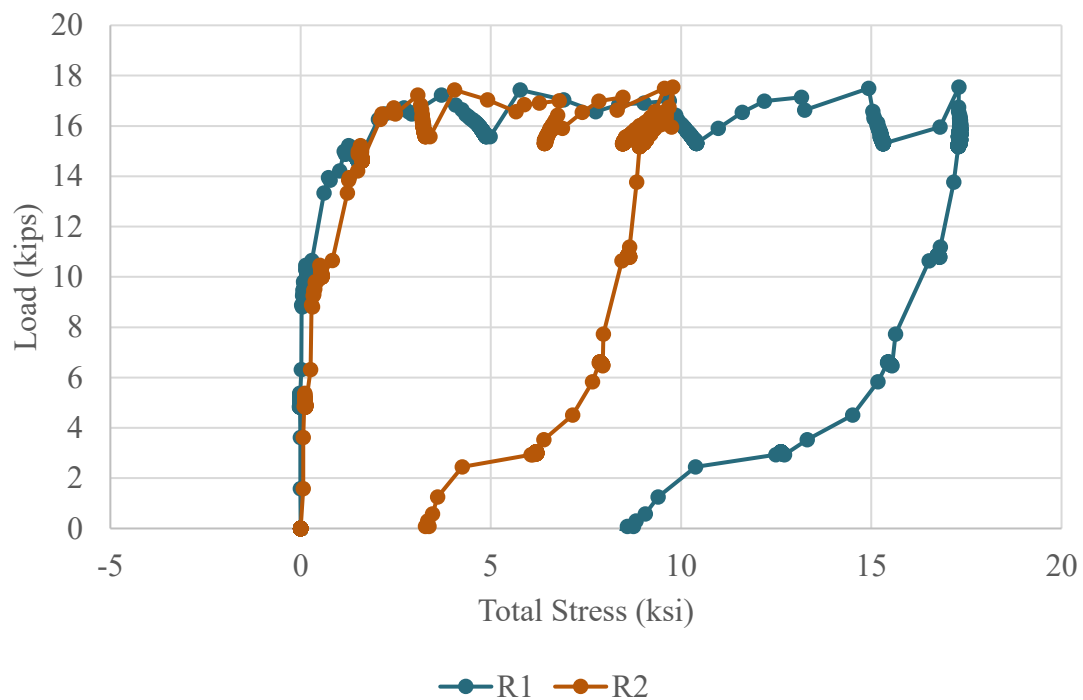


Figure 174. Specimen S-10-10, total stress measured by strain gauges R1 and R2 on steel reinforcement cage.

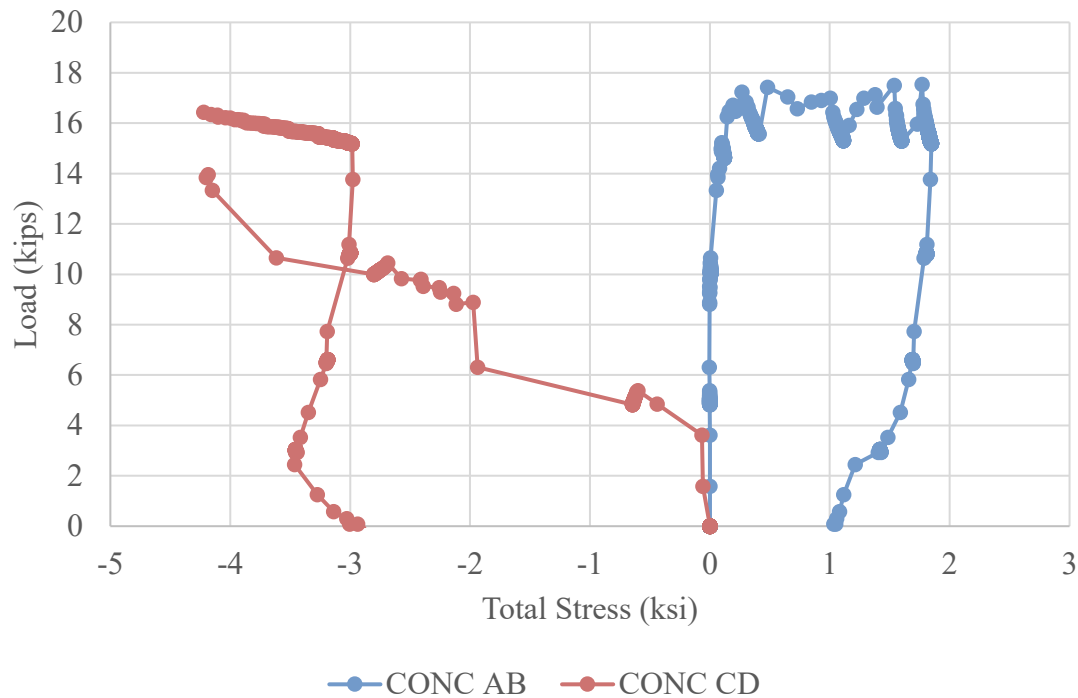


Figure 175. Specimen S-10-10, total stress measured by strain gauges CONC AD and CONC BC on the concrete cap surface.

In this specimen, R1 and R2 are oriented orthogonally to the load direction rather than on the leading and trailing faces as in previous specimens. This orientation differs from all previous tests and should be noted when comparing reinforcement stress results across specimens. Despite this different orientation, both R1 and R2 recorded tension throughout the test, reaching approximately 4 ksi at the maximum load stage. These values are substantially higher than those recorded in the steel-failure specimens W-10-15 and W-10-10, and are consistent with the concrete bearing failure mode generating significant tensile demand in the cap reinforcement.

The concrete surface gauges also showed notable behavior. CONC AB, on the trailing side, recorded tension of approximately 397 psi at the maximum load stage, approaching but not exceeding the estimated concrete tensile rupture strength, consistent with the trailing side being pulled away from the pile. CONC CD, on the leading side, showed negligible behavior up to approximately 4 kips, then recorded compression increasing to approximately 4.2 ksi at the 14 kip load stage. Beyond this point, a crack

formed through the gauge location and no further data were recorded. The 4.2 ksi compression on the leading face is among the highest concrete surface compression values recorded in this test program and is consistent with the concrete bearing failure mode in this specimen.

Figure 176 shows the bending moment results for the exposed strain gauges. EX-C6-A/D and EX-C1-A/D malfunctioned and are not used for quantitative conclusions. Their data nonetheless show qualitative trends consistent with expected moment behavior, providing a general indication that bending moment increased with load in the expected direction.

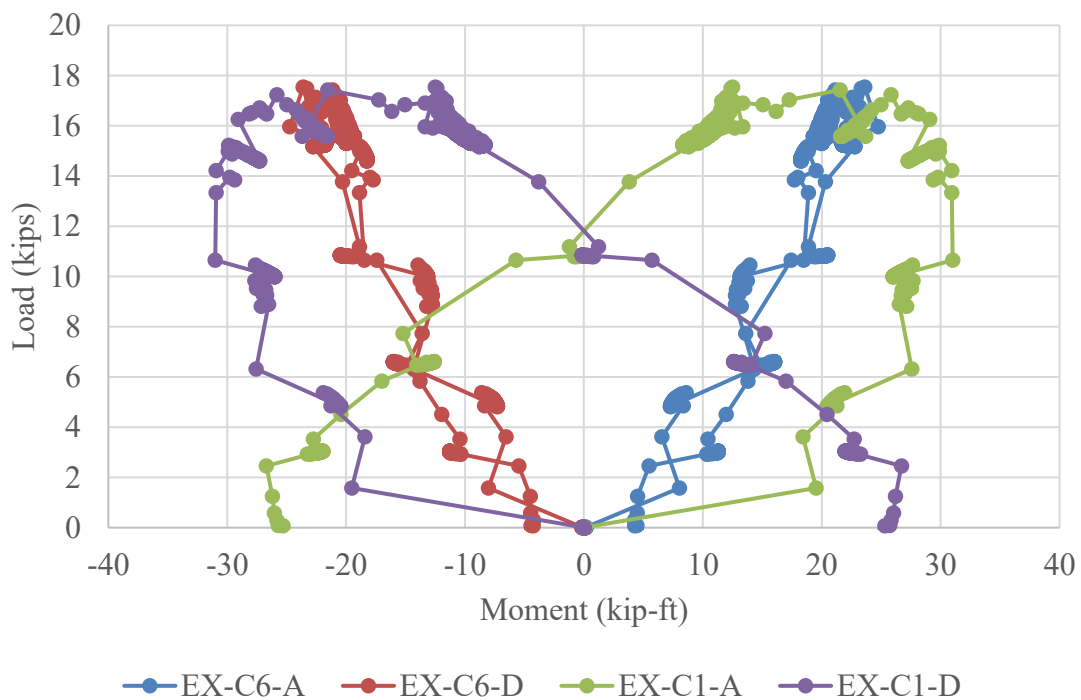


Figure 176. Specimen S-10-10, calculated bending moment for EX strain gauges.

Figure 177 shows the bending moment results for the embedded strain gauges. EM-H-A/D were entirely non-functional. EM-I-A and EM-M-A provided reliable data up to approximately 5.5 kips, showing moment dissipation with embedment depth consistent with theoretical expectations. The data are limited to lower load stages but remain physically coherent.

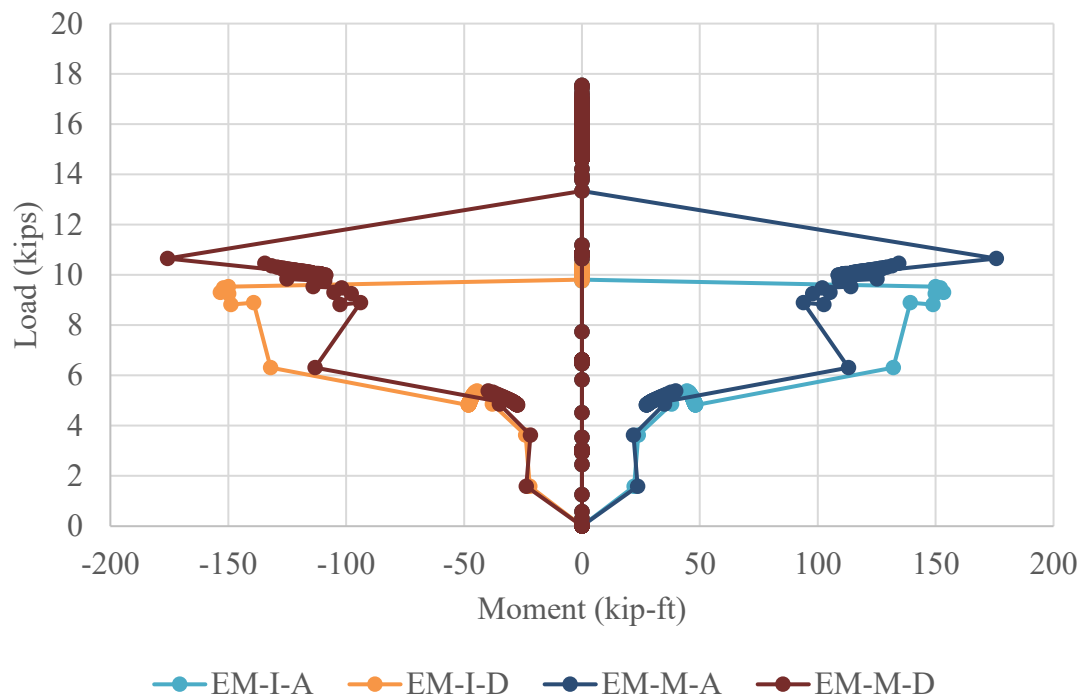


Figure 177. Specimen S-10-10, calculated bending moment for EM strain gauges.

Figure 178 shows the moment profile along the pile length for each load stage, along with a comparison to the theoretical moment based on the applied horizontal load. At the 1 kip load stage, EM-I recorded approximately 12 kip-ft and EM-M recorded approximately 5 kip-ft. At the 2 kip load stage, EM-I recorded approximately 16 kip-ft and EM-M recorded approximately 12 kip-ft. At the 4 kip load stage, EM-I recorded approximately 28 kip-ft and EM-M recorded approximately 24 kip-ft. These values show a consistent decrease in moment with embedment depth at each load stage, confirming that moment was being transferred into the embedment zone and dissipating with depth. The data are limited to load levels up to approximately 5.5 kips due to gauge malfunction at higher loads, and quantitative conclusions about moment distribution at higher load levels cannot be drawn from the available data. The available data nonetheless confirm that the 10-inch strong-axis embedment provided partial fixity at the connection, consistent with the partial fixity trend observed across all specimens in this program.

All other plots for specimen S-10-10 (wire potentiometers, strain measurements, and pile total stress results) are provided in Appendix A.

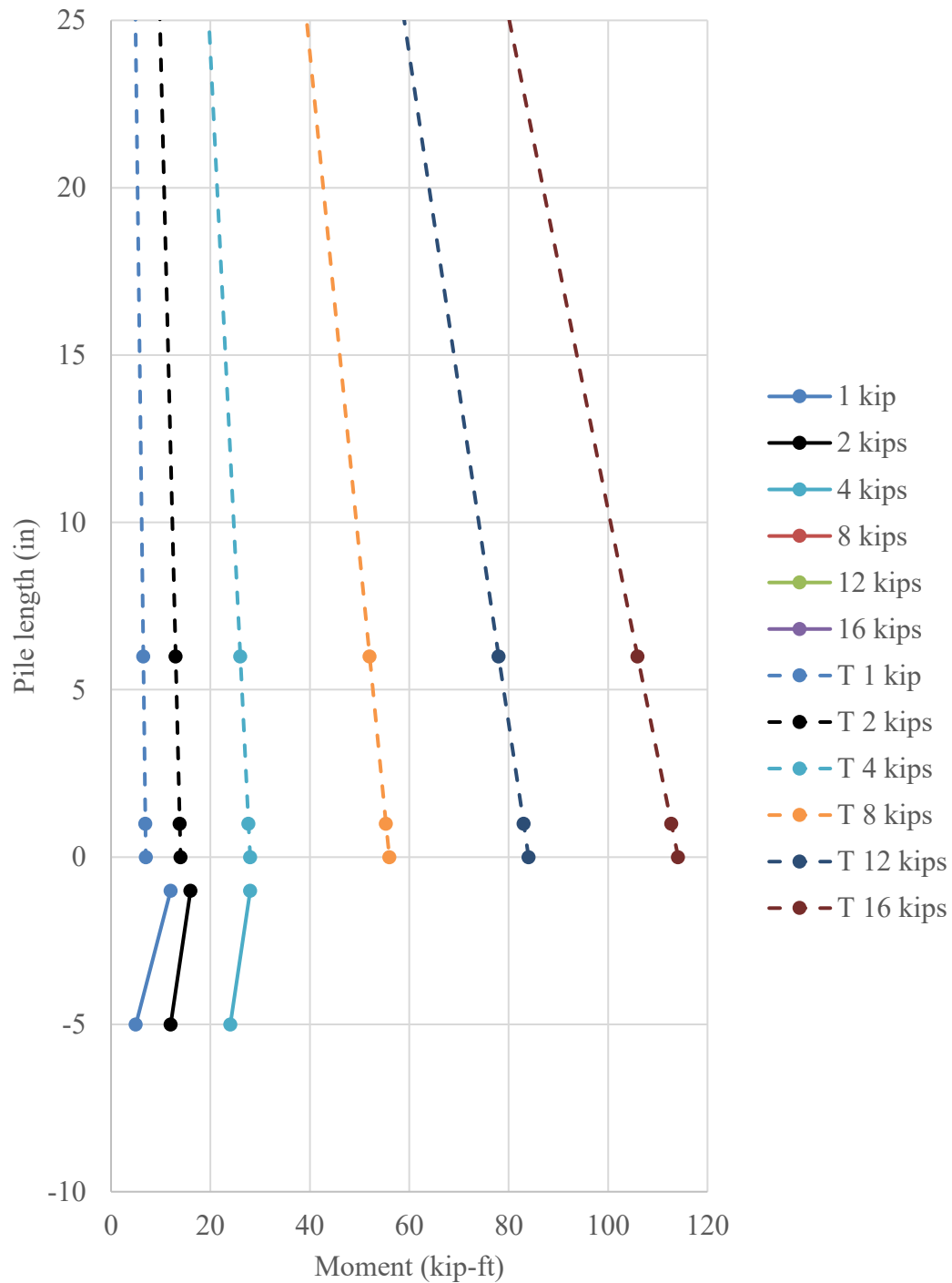


Figure 178. Specimen S-10-10 measured moments (solid lines) and theoretical moments (dashed lines) for each load stage. Zero y-axis marks the cap surface: positive is the exposed pile length, and negative is the embedded pile length.

2. Discussion

Table 15 summarizes the results across all specimens. Figure 179 shows the plot of Maximum achieved load vs. ratio embedment/depth for all tested specimens.

Table 15. Summary of Test Results

Specimen	Embed. Depth (in)	Failure mode	Max. horizontal load corresponding to steel structural capacity (kips)	Marcakis and Mitchell (1980) range (kips)	Maximum achieved load (kips)	Post-test crack width
W-10-15	15	Steel yielding	13.1	27.7 - 41.6	16	0.03 mm or less
W-10-10	10	Steel yielding	13.1	12.8 - 19.2	15	0.08 mm or less
W-10-7.5 (1st try)	7.5	Concrete bearing	13.1	7.3 - 11.1	-	-
W-10-7.5 (2nd try)	7.5	Concrete bearing	13.1	7.3 - 11.1	11.5	-
W-10-7.5R	7.5	Concrete bearing	13.1	7.3 - 11.1	14	exceeded 1 mm at several points
W-10-5	5	Concrete bearing	13.1	3.4 - 5.1	7.5	exceeded 1-2 mm at

						several points
W-12-12	12	Steel Yielding	16.9	21.9 - 27.3	23.2	0.03 mm or less
W-12-6	6	Concrete bearing	16.9	5.8 - 7.2	10	1 mm to 10 mm (widest at flange D)
S-10-10	10	Concrete bearing	30.6	12.8 - 19.2	16.3	0.06 mm to 5 mm

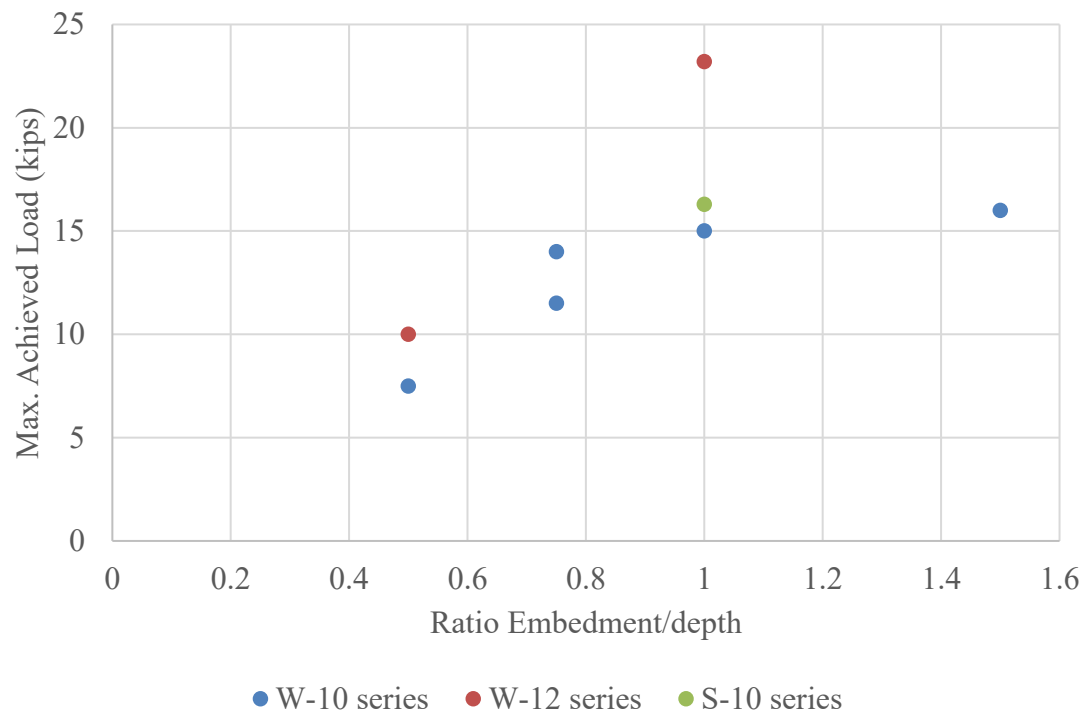


Figure 179. Maximum achieved load vs. ratio embedment/depth.

2.1. W-10 series

The four W-10 specimens - W-10-15, W-10-10, W-10-7.5R, and W-10-5 - used the same HP10x42 pile section and the same cap geometry, with embedment depth as the primary variable.

2.1.1. Failure Mode Transition

At 15-inch embedment, W-10-15 failed by steel structural failure: flange local buckling on the leading side, yielding extending to the trailing flanges, and post-test crack widths of 0.03 mm or less confirming negligible concrete damage.

At 10-inch embedment, W-10-10 also failed by steel structural failure, but the elevated leading-face concrete stress (2.5 ksi versus 0.9 ksi in W-10-15) and the proximity of the observed failure load to the Marcakis and Mitchell (1980) predicted concrete bearing range both indicate that the concrete connection was approaching its capacity at the same time the steel was reaching its structural limit. The 10-inch embedment is therefore best characterized as a transitional case: steel failure still controlled, but the concrete was no longer far from its own limit.

At 7.5-inch embedment, W-10-7.5R crossed the transition: failure was defined by concrete cracking and crushing, with post-test crack widths exceeding 1 mm and no evidence of flange buckling.

At 5-inch embedment, W-10-5 failed similarly by concrete bearing, but more abruptly, since the load plateau was not sustained, the failure was sudden and brittle relative to the more ductile behavior of the deeper-embedment specimens, and cracking initiated at loads as low as 1.5 kips.

The data therefore suggest that the transition from steel-controlled to concrete-controlled failure occurs between 7.5 and 10 inches of embedment for the HP10x42 pile in this cap configuration, with the 10-inch case occupying a boundary region where both mechanisms are nearly simultaneously active.

2.1.2. Maximum Horizontal Loads

A notable feature of the W-10 results is that the maximum horizontal loads for W-10-15, W-10-10, and W-10-7.5R are similar: approximately 16 kips, 15 kips, and 14 kips,

respectively. This near-coincidence is not accidental but reflects the proximity of two independent capacity limits that converge in this embedment range. The HP10x42 pile's steel structural capacity (controlled by flange local buckling and full plastification) corresponds to approximately 13 to 16 kips horizontal load for the test geometry used. The concrete bearing capacity of the connection for the 7.5-inch embedment, estimated by Marcakis and Mitchell (1980), falls in a similar range. When embedment is sufficient for the concrete connection to outlast the steel, the steel capacity controls. When it is insufficient, the concrete controls first. In the 7.5 to 10-inch range, both limits are close enough that the controlling mechanism changes without a meaningful change in the total load carried. W-10-5, by contrast, sits well below both steel limits, since its 7.5 kip maximum load is governed entirely by the concrete connection at a much lower level. Therefore, the drop in capacity is correspondingly large.

The practical implication is that reducing embedment from 15 to 7.5 inches had little effect on peak load for this pile section, while fundamentally changing the failure mechanism and the post-peak behavior from ductile to brittle. This distinction matters more for structural performance than the capacity numbers alone would suggest.

2.1.3. Pile Head Behavior

The TILT-HEAD instrumentation produced a systematic and physically meaningful progression across the four specimens that warrants explicit discussion as a cross-specimen pattern rather than a series of individual observations. Figure 180 shows the measurements made for all W-10 specimens.

In W-10-15, TILT-HEAD registered negligible rotation throughout the test. The 15-inch embedment provided sufficient fixity to suppress any measurable pile head rotation, consistent with the embedded pile head being essentially fixed against rotation by the surrounding concrete.

In W-10-10, TILT-HEAD began showing counterclockwise rotation at approximately 9 kips, opposite to the applied load direction. As discussed in Section 1.2, this reflects the geometric behavior of the sensor attachment: the tiltmeter rod, welded to the base of the pile web, rotates opposite to the pile head as the pile rotates under load, in the manner of an object balancing on a cylinder. The onset of this signal at 9 kips

coincided with visible pile battering and gap formation at the interface, indicating that the pile head had begun to rotate sufficiently for the geometric rod effect to manifest. The 10-inch embedment, while still providing significant fixity, was no longer sufficient to fully suppress pile head rotation at higher load levels.

In W-10-7.5R, the same CCW signal appeared, but at a larger magnitude (approximately -1 degree at failure versus -0.5 degrees in W-10-10), consistent with the shorter embedment providing less rotational restraint and allowing the pile head to rotate more freely as a rigid body within the cap.

In W-10-5, the pattern diverged: TILT-HEAD initially showed the expected CCW rotation up to approximately 6 kips, then reversed to clockwise rotation through failure. This reversal was interpreted as the concrete bearing zone breaking down sufficiently to release the constraint on the measurement rod, allowing it to follow the actual pile head rotation. Corroborating evidence from SLIP D - which showed anomalous compression at the same load level due to local concrete spalling - supports this interpretation.

TILT-HEAD behavior traces the transition from higher levels of partial fixity (W-10-15), to lower levels of partial fixity with measurable pile head rotation (W-10-10 and W-10-7.5R), to internal concrete failure releasing the connection entirely (W-10-5). The progression is physically coherent and provides qualitative confirmation of the failure mode transition identified from load-deflection data and crack observations.

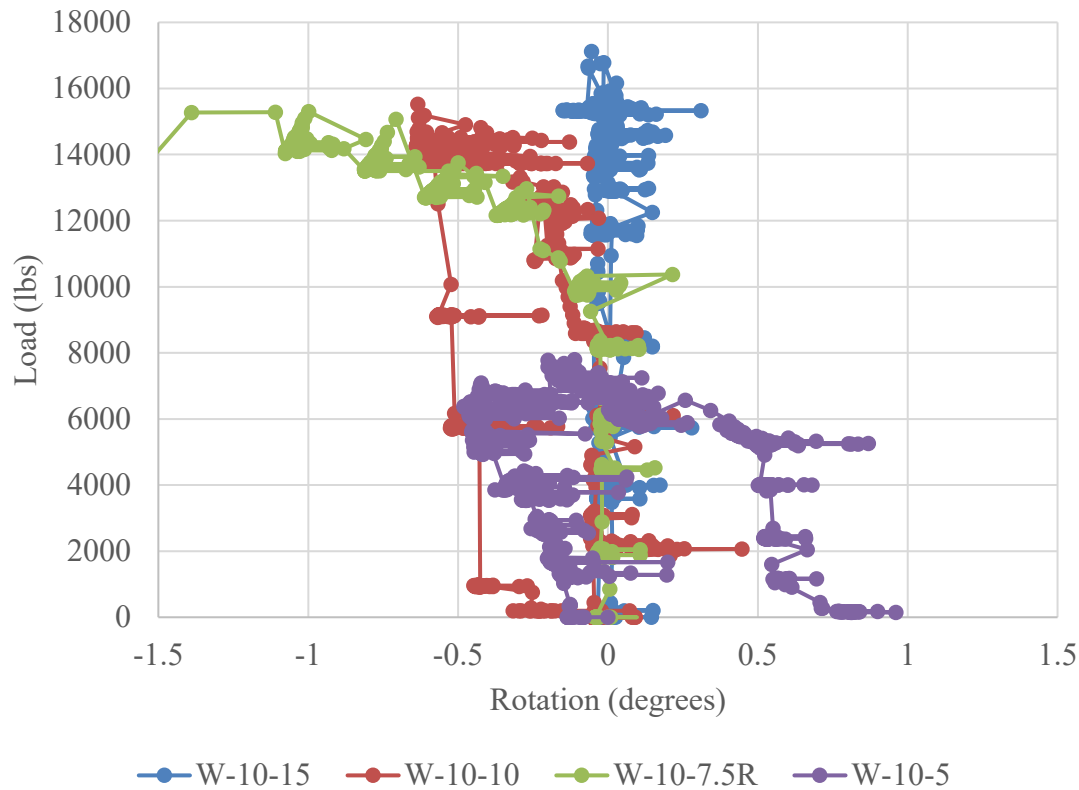


Figure 180. W-10 series TILT-HEAD measurements.

2.1.4. Moment Distribution and Connection Fixity

The embedded strain gauge data, despite the instrumentation limitations discussed in each specimen section, consistently confirm that all four embedment depths transferred moment into the embedded zone and that the moment decreased with depth.

The magnitude of moment at the pile head (EM-H level) varied across specimens in a way that broadly reflects the fixity trend: 22 kip-ft in W-10-15, 15 kip-ft in W-10-10, and 25 kip-ft in W-10-7.5R (the last value attributed to specimen-to-specimen variability rather than a reversal of the trend). For W-10-5, the embedded gauges ceased functioning at very low load levels due to early interface movement, precluding direct comparison at failure loads.

The data confirm that even at 5-inch embedment, moment was being transferred into the cap at early load stages, indicating that no embedment depth tested produced a

fully pinned condition. All connections exhibited partial fixity, with the degree of fixity increasing with embedment depth.

2.2. W-12 series

The W-12 series comprised two specimens (W-12-12 and W-12-6) using the HP12x53 pile section. Despite the limited number of specimens, the results are internally consistent and show the same pattern observed in the W-10 series.

W-12-12, with 12-inch embedment, failed by steel yielding and flange buckling, with post-test concrete crack widths of 0.03 mm - identical to the steel-failure specimens in the W-10 series. TILT-HEAD showed negligible movement until the failure load plateau, at which point the CCW signal appeared, consistent with the concrete connection approaching its bearing capacity at that load level. The leading-face concrete surface stress (2.4 ksi) was elevated but well below the values recorded in concrete-failure specimens, and the Marcakis and Mitchell (1980) predicted range captures the failure load, placing W-12-12 in a transitional position analogous to W-10-10: steel failure controlled, but the concrete was near its limit simultaneously.

W-12-6, with 6-inch embedment, failed by concrete cracking and crushing, with post-test crack widths ranging from 1 to 10 mm. The TILT-HEAD reversal from CCW to clockwise rotation near the failure load, and its correspondence with the onset of concrete bearing softening, mirrors the behavior observed in W-10-5 and W-10-7.5R. The leading-face concrete surface compression reached 4.5 ksi - approximately 73% of f'_c for that specimen - consistent with proximity to bearing failure.

2.3. Comparison between W-10 and W-12 series

Comparing the two pile series shows both similarities and an important structural difference due to their pile section properties.

The failure mode progression (steel-controlled at deeper embedment, concrete-controlled at shallower embedment) is consistent across both series. The transition appears to occur at a similar ratio of embedment depth to pile section depth rather than at a fixed absolute embedment. For the HP10x42 (nominal depth approximately 10 inches), the transition occurs between 7.5 and 10 inches, corresponding to roughly 0.75

to 1.0 times the pile depth. For the HP12x53 (nominal depth approximately 12 inches), the transition appears to occur between 6 and 12 inches, a wider range given the limited number of specimens, but roughly consistent with the same ratio magnitude.

The maximum load levels differ substantially between the two series, reflecting the larger section modulus and moment capacity of the HP12x53. W-12-12 reached 23.2 kips at failure, nearly 50% above the W-10 steel-failure specimens. On the other hand, W-12-6 reached approximately 10 kips, somewhat above W-10-7.5R and W-10-5. The concrete bearing capacity also scales with pile section width, as would be expected from the Marcakis and Mitchell (1980) formulation.

One common feature across both series is the conservatism of the Marcakis and Mitchell (1980) predictions for the concrete-failure specimens: in every case where concrete bearing governed, the observed failure load exceeded the predicted upper bound. This conservatism was consistent across both pile sections and both embedment depths where concrete failure occurred, suggesting it reflects a systematic characteristic of the equation rather than specimen-specific variability. A more detailed evaluation of the equation's performance is reserved for a subsequent chapter.

The moment distribution data in W-12-12, while limited to lower load stages due to gauge malfunction at higher loads, showed a physically coherent decay from the cap surface to the pile head, consistent with partial fixity. The moment values at the 6 kip load stage (EX-C1: 41.5 kip-ft; EM-I: 34 kip-ft; EM-M: 27 kip-ft; EM-H: 6.5 kip-ft) represent the clearest picture of the full embedded moment profile obtained in this program for any specimen, confirming the expected behavior pattern across gauge levels.

2.4. Strong Axis Test: S-10-10

Specimen S-10-10 introduced a different loading configuration: strong-axis bending of the HP10x42 pile. This load test represents a bent cap loaded in the bridge longitudinal direction.

The most significant finding is that the Marcakis and Mitchell (1980) predicted range (12.8 to 19.3 kips) captured the observed failure load of 16.3 kips, in contrast to all weak-axis concrete-failure specimens where the actual load exceeded the predicted

upper bound. This suggests that the Marcakis and Mitchell (1980) equation may perform less conservatively for strong-axis bending than for weak-axis bending.

A plausible explanation is that the bearing geometry changes between the two configurations: in weak-axis bending, the pile flanges engage the concrete at their tips and the web engages progressively, while in strong-axis bending the entire width of the flanges (both trailing and leading sides) bear simultaneously against the concrete.

The effective bearing area and stress distribution assumed in the Marcakis and Mitchell (1980) formulation may map more accurately onto the strong-axis geometry, reducing the conservatism that is consistently observed in the weak-axis tests. This will be further discussed in the subsequent chapters.

The crack morphology in S-10-10 was distinctly different from all weak-axis specimens. Rather than diagonal corner cracks radiating from individual flange tips, the specimen developed wing-shaped cracks behind the trailing flanges - a pattern consistent with the full flange width pulling away from the concrete simultaneously, as discussed in Section 1.7. This morphological difference reflects the geometric reality of strong-axis engagement and provides additional confirmation that the internal load transfer mechanism differs meaningfully between the two bending orientations.

The comparison with W-10-10 is particularly instructive. Both specimens failed at similar load levels (approximately 14-16 kips), but for entirely different reasons: W-10-10 reached its steel structural capacity under weak-axis bending, while S-10-10 reached its concrete bearing capacity under strong-axis bending. The pile's strong-axis steel capacity is roughly twice its weak-axis capacity, so the concrete connection became the controlling limit under strong-axis loading at an embedment depth that was borderline for weak-axis loading. This illustrates that the controlling failure mode for a given connection depends not only on embedment depth but on the bending orientation, and that connections designed adequately for bridge transverse loading may behave quite differently under longitudinal loading.

2.5. General Observations and Synthesis

Failure mode was governed primarily by embedment depth relative to pile section dimensions: deeper embedments produced steel structural failure with ductile post-peak

behavior, while shallower embedments produced concrete bearing failure with progressively more brittle response. A transitional zone exists in both pile series, where the two capacity limits converge and governing mode becomes sensitive to local variability.

No specimen behaved as a pinned connection. Every embedment depth tested, including the shallowest, produced measurable moment transfer at the pile head and a decreasing moment profile with depth, confirming partial fixity across the full range of specimens.

Ductility decreased consistently with embedment depth. Steel-failure specimens sustained load plateaus over large deflections, while shallow concrete-failure specimens lost load rapidly after reaching peak.

Finally, strain gauge malfunctions affected completeness of moment profile data in several specimens, but primary instrumentation remained reliable and provided mutually consistent evidence supporting the failure mode conclusions for each specimen.

Chapter 6:

Connection Capacity Analysis

1. Introduction

The experimental load tests results in Chapter 5 produced observed failure moments for eight of the eleven load tests. The remaining three (W-10-15 1st and 2nd tries and W-10-7.5 1st try) were terminated before failure due to equipment limitations and, for this reason, are not included in this connection capacity analysis. This chapter evaluates the predictive accuracy of existing analytical methods for estimating the moment capacity of plain H-pile embedment connections using those eight observed failure moments as the reference.

Five methods from the literature are considered, along with adaptations that modify geometric input assumptions: Marcakis and Mitchell (1980), Mattock and Gaafar (1982), Shama et al. (2002a), Kappes et al. (2021), and Hammett (2017) Method 4. In total, fourteen method variants are evaluated. The primary metric for evaluation is the predicted-to-actual ratio - the ratio of the method's predicted moment capacity to the observed failure moment. A ratio above 1.0 indicates the method overestimates capacity and is unconservative. A ratio below 1.0 indicates underestimation and is conservative. Ratios near 1.0 indicate accurate prediction.

The analysis is organized by failure mode. Since specimens W-10-10 and W-12-12 failed by steel pile yielding, the bearing-based capacity methods are not applicable to them and are discussed separately. The remaining concrete-bearing specimens form the primary comparison dataset and are analyzed in detail by bending axis orientation. Pre-damaged specimens are addressed in a separate section.

2. Moment Capacity Prediction Methods

As discussed, Marcakis and Mitchell (1980), Mattock and Gaafar (1982), Shama et al. (2002a), Kappes et al. (2021), and Hammett (2017) Method 4 are used in this chapter. Additionally, in published literature these methods have also been used with

some adaptations, such as variable changes, among others. Below is a list of the 14 variants used in this chapter, and their abbreviations.

- **MM:** Marcakis and Mitchell (1980). Original theory as published by the authors
- **MM-pw:** Marcakis and Mitchell (1980), adapted by using pile width instead of effective width.
- **MM-H:** Marcakis and Mitchell (1980) modified by Harries et al. (2000), where they consider concrete cover spalling.
- **MM-H-pw:** Marcakis and Mitchell (1980) modified by Harries et al. (2000), where they consider concrete cover spalling, but also adapted to use pile width instead of effective width.
- **MG:** Mattock and Gaafar (1982). Original theory as published by the authors.
- **MG-cc:** Mattock and Gaafar (1982), adapted to use width of confined concrete region instead of cap width.
- **MG-2.5pw:** Mattock and Gaafar (1982), adapted to use 2.5 times the pile width, instead of cap width.
- **MG-H:** Mattock and Gaafar (1982), modified by Harries et al. (2000), where they consider concrete cover spalling.
- **MG-H-cc:** Mattock and Gaafar (1982), modified by Harries et al. (2000), where they consider concrete cover spalling, but also adapted to use width of confined concrete region, instead of cap width.
- **MG-H-2.5pw:** Mattock and Gaafar (1982), modified by Harries et al. (2000), where they consider concrete cover spalling, but also adapted to use 2.5 times the pile width, instead of cap width.
- **SH:** Shama et al. (2002a), original theory as published by the authors.
- **SH-s:** Shama et al. (2002c), simplified equation of the Shama et al. (2002a) work.

- **KP:** Kappes et al. (2021), original theory as published. Originally created for steel tube piles, it was also argued that it could be considered for steel H-piles, with some considerations.
- **HM4:** Hammett (2017), specifically method 4, as originally published.

3. Steel-Controlled Failures

Three specimens failed by steel pile yielding: W-10-10 ($L_e/d = 1.0$, undamaged), W-12-12 ($L_e/d = 1.0$, undamaged), and W-10-15 3rd try ($L_e/d = 1.5$, pre-damaged). Tables 16 and 17 present the predicted concrete bearing capacities and the corresponding predicted-to-actual ratios for all methods alongside the structural plastic moment.

The structural plastic moment of the HP10x42 section is 107 kip-ft and of the HP12x53 is 146 kip-ft. Predicted-to-actual ratios for the structural plastic moment are 1.06, 0.91, and 1.02 for W-10-10, W-12-12, and W-10-15 3rd try respectively, confirming it as an accurate predictor when steel controls. Notably, the structural plastic moment predicts the failure moment of W-10-15 3rd try with a ratio of 1.02 despite the specimen having been pre-damaged by two prior loading events, suggesting that, at $L_e/d = 1.5$, the connection concrete capacity far exceeded the pile's structural plastic moment regardless of prior damage, so steel failure is the inevitable controlling mechanism.

Table 16. Predicted Connection Moment Capacity (kip-ft) for steel-governed specimens.

Specimen	W-10-10	W-10-15 3 rd try	W-12-12
Specimen condition	Undamaged	Pre- damaged	Undamaged
Moment (kip-ft) at failure	101	105	160
Failure mode	Steel yielding	Steel yielding	Steel yielding
MM	192	313	250

MM-pw	137	224	215
MM-H	124	237	175
MM-H-pw	88	169	150
MG	240	486	356
MG-cc	129	260	191
MG-2.5pw	188	382	316
MG-H	153	364	247
MG-H-cc	82	195	132
MG-H-2.5pw	120	286	219
SH	201	146	319
SH-s	104	175	165
KP	179	321	291
HM4	120	207	194

Table 17. Predicted-to-actual ratio for steel-governed specimens.

Specimen	W-10-10	W-10-15 3rd try	W-12-12
Specimen condition	Undamaged	Pre-damaged	Undamaged
Moment (kip-ft) at failure	101	105	160
Failure mode	Steel yielding	Steel yielding	Steel yielding
MM	1.90	2.98	1.56
MM-pw	1.36	2.13	1.34
MM-H	1.22	2.25	1.09
MM-H-pw	0.87	1.61	0.94
MG	2.37	4.63	2.22

MG-cc	1.27	2.48	1.19
MG-2.5pw	1.87	3.64	1.97
MG-H	1.52	3.47	1.54
MG-H-cc	0.81	1.86	0.83
MG-H-2.5pw	1.19	2.73	1.37
SH	1.99	1.39	2.00
SH-s	1.03	1.67	1.03
KP	1.77	3.05	1.82
HM4	1.19	1.97	1.22

Although the bearing-based methods cannot be validated against a concrete failure that did not occur, their predictions still have some interpretive value. For these specimens, the predicted concrete bearing capacity establishes how much margin existed between the observed steel failure and a potential concrete failure. In other words, how close the connection was to a failure mode transition. A method that predicts concrete capacity just above the pile's structural plastic moment is signaling proximity to the transition zone. On the other hand, a method that predicts concrete capacity well below it is informing that concrete would have governed, contradicting the observed behavior.

At $L_e/d = 1.0$, the predicted margins vary substantially across methods. For W-10-10, MM gives 192 kip-ft and KP gives 179 kip-ft, suggesting the concrete had considerable remaining capacity when the steel yielded. MM-pw predicts 137 kip-ft, a close margin (28%) to the steel plastic moment of 107 kip-ft. MM-H-pw (88 kip-ft) and MG-H-cc (82 kip-ft) predict concrete capacity below the steel plastic moment entirely, incorrectly implying that concrete would have controlled at $L_e/d = 1.0$ for the HP10x42. For W-12-12, the same two methods again produce the smallest margins: MG-H-cc predicts 132 kip-ft, below the HP12x53 plastic moment of 146 kip-ft, and MM-H-pw predicts 150 kip-ft, barely above it.

At $L_e/d = 1.5$, every method, including the most conservative variants, predicts concrete capacity well above the 107 kip-ft steel plastic moment for W-10-15 3rd try. Even MG-H-cc, which incorrectly predicted concrete failure at $L_e/d = 1.0$, gives 195 kip-ft at $L_e/d = 1.5$, meaning an 82% margin above the steel threshold.

The progression across embedment depths is itself informative: at $Le/d = 1.0$, two methods incorrectly predict the governing failure mode. At $Le/d = 1.5$, none do. This is directly consistent with the failure mode transition documented in Chapter 5, where Le/d close to 1.0 marks the approximate boundary between concrete- and steel-governed behavior for the pile sections tested.

The methods that most severely underestimate concrete capacity (MM-H-pw and MG-H-cc) are thus the ones most likely to misidentify the governing mechanism near this transition zone, a limitation that should be considered when applying them to connections where the failure mode is not known in advance.

4. Concrete-Bearing Failures - Weak Axis

The three weak-axis specimens that failed by concrete bearing provide the primary dataset for evaluating the capacity prediction methods: W-10-7.5R ($Le/d = 0.75$), W-10-5 ($Le/d = 0.5$), and W-12-6 ($Le/d = 0.5$). Tables 18 and 19 present the predicted capacities and predicted-to-actual ratios for all fourteen method variants. Table 19 also summarizes, across the three specimens, the mean ratio and standard deviation for each method.

Table 18. Predicted Connection Moment Capacity (kip-ft) for load tests in undamaged specimens

Specimen	W-10-7.5R	W-10-5	W-12-6
Specimen condition	Undamaged	Undamaged	Undamaged
Moment (kip-ft) at failure	98	51	70
Failure mode	Concrete cracking/crushing	Concrete cracking/crushing	Concrete cracking

			/crushin g
MM	101	38	58
MM-pw	72	27	50
MM-H	55	14	26
MM-H-pw	39	10	22
MG	136	60	92
MG-cc	73	32	49
MG-2.5pw	107	47	81
MG-H	73	21	41
MG-H-cc	39	11	22
MG-H-2.5pw	57	17	36
SH	119	56	102
SH-s	54	20	37
KP	97	40	71
HM4	62	23	43
Structural Plastic Moment	107	107	146

Table 19. Ratio of Predicted Connection Moment Capacity (kip-ft) to Moment at Failure
for load tests in undamaged specimens

Specimen	W-10- 7.5R	W-10-5	W-12-6	Mean	Std. Dev
Specimen condition	Undama ged	Undama ged	Undama ged		
Moment (kip-ft) at failure	98	51	70		

Failure mode	Concret e cracking /crushin g	Concret e cracking /crushin g	Concret e cracking /crushin g		
MM	1.03	0.75	0.83	0.87	0.144
MM-pw	0.74	0.54	0.71	0.663	0.108
MM-H	0.56	0.27	0.37	0.4	0.147
MM-H-pw	0.40	0.20	0.32	0.307	0.101
MG	1.39	1.17	1.31	1.29	0.111
MG-cc	0.75	0.63	0.70	0.693	0.06
MG-2.5pw	1.09	0.92	1.16	1.057	0.123
MG-H	0.75	0.42	0.58	0.583	0.165
MG-H-cc	0.40	0.23	0.31	0.313	0.085
MG-H-2.5pw	0.59	0.33	0.52	0.48	0.135
SH	1.21	1.09	1.45	1.25	0.183
SH-s	0.55	0.39	0.52	0.487	0.085
KP	0.99	0.78	1.01	0.927	0.127
HM4	0.63	0.46	0.62	0.57	0.095
Structural Plastic Moment	1.09	2.10	2.09	-	-

Figure 181 shows the predicted-to-actual ratio for each method and each specimen graphically, with a horizontal reference line at 1.0.

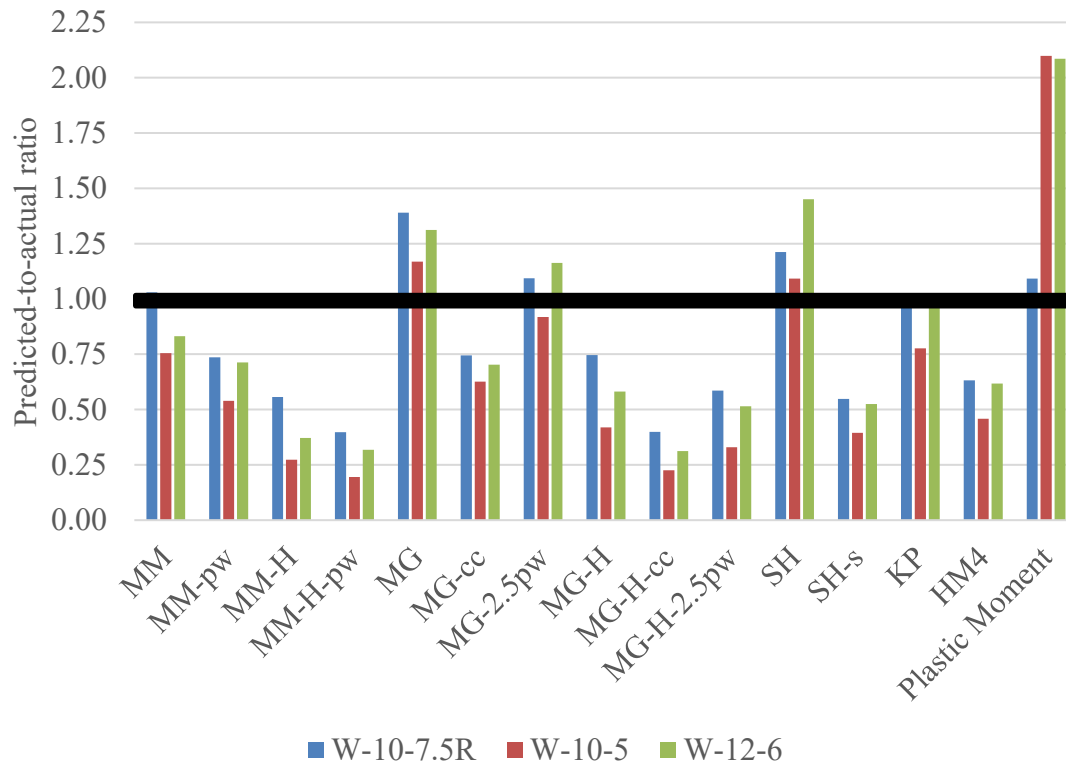


Figure 181. Predicted-to-actual moment capacity ratio for weak-axis concrete-bearing failure specimens: W-10-7.5R, W-10-5, and W-12-6.

For specimen W-10-7.5R ($L_e/d = 0.75$, failure moment = 98 kip-ft), the closest predictions come from KP (ratio = 0.99) and MM (ratio = 1.03), both within 3% of the actual failure moment. MG and SH overestimate capacity by 39% and 21% respectively and are unconservative. The Harries-modified variants with pile width substitution (MM-H-pw and MG-H-cc) reach only 0.40, representing a severe underestimate. The remaining methods fall in between, with most variants on the conservative side.

For specimen W-10-5 ($L_e/d = 0.5$, failure moment = 51 kip-ft), no method achieves close agreement. The two nearest predictions are MG-2.5pw (ratio = 0.92) and SH (ratio = 1.09), which are within 10% of the actual value, although neither can be considered reliably accurate. Most methods substantially underestimate the failure moment: MM-H-pw and MG-H-cc reach only 0.20 and 0.23, respectively. The consistent underestimation across nearly all methods at this shallow embedment suggests that existing formulations

do not adequately capture the bearing mechanism at $L_e/d = 0.5$ for the HP10x42, where the bearing zone is short and highly localized.

For specimen W-12-6 ($L_e/d = 0.5$, failure moment = 70 kip-ft), predictive accuracy improves noticeably relative to W-10-5 despite sharing the same L_e/d ratio. KP achieves a ratio of 1.01 and MM gives 0.83. SH is the most unconservative at 1.45, and MG reaches 1.31. MG-H-cc and MM-H-pw remain the most conservative at 0.31 and 0.32. The contrast between W-10-5 and W-12-6 at the same embedment ratio reveals a pile section effect in predictive accuracy: methods that systematically underestimate for the HP10x42 at $L_e/d = 0.5$ perform considerably better for the HP12x53 at the same L_e/d . This is likely related to differences in flange width and web geometry between the two sections, which affect the bearing area in ways the methods do not fully represent.

Among the fourteen variants in Table 19, KP achieves the mean ratio closest to 1.0 at 0.927, with a standard deviation of 0.127. It is the only method that combines reasonable mean accuracy with results consistently on the conservative side on average, making it the most balanced predictor across this dataset. MM has a comparable mean of 0.870 but is occasionally unconservative, as it slightly overestimates at $L_e/d = 0.75$ (ratio = 1.03) and its standard deviation of 0.144 reflects more variability across the three specimens. MG-2.5pw also has a close mean of 1.057, the closest unconservative average, but it has a tendency to overestimate capacity.

MG and SH are persistently unconservative with means of 1.29 and 1.25 respectively, and should not be applied without modification to H-pile plain embedment connections. The Harries-modified variants (MM-H, MM-H-pw, MG-H, MG-H-cc, MG-H-2.5pw) are excessively conservative across the board, with means ranging from 0.31 to 0.58. While conservatism is not inherently problematic from a safety standpoint, the degree of underestimation (40-70%) suggests these modifications overcorrect for spalling effects in a way that is not calibrated to H-pile connection geometry. HM4 and SH-s are also consistently conservative with means near 0.49-0.57, limiting their utility for practical capacity estimation.

A secondary trend is visible across embedment depths: nearly every method shows degraded accuracy at $L_e/d = 0.5$ compared to $L_e/d = 0.75$. Methods that perform well at W-10-7.5R tend to underpredict at W-10-5 and W-12-6, suggesting that the stress

block geometries assumed in these formulations (derived largely from deeper embedment configurations) do not scale correctly to very shallow embedment where the bearing length is short and the capacity is modest. This systematic bias at shallow embedment warrants caution when applying any of these methods to connections with L_e/d approaching 0.5.

5. Concrete-Bearing Failures - Strong Axis

Specimen S-10-10 ($L_e/d = 1.0$, failure moment = 110 kip-ft) is the only strong-axis test performed. It achieved concrete-bearing failure. Table 20 presents the predicted-to-actual ratios for all methods.

Table 20. Predicted Connection Moment Capacity (kip-ft) and Ratio of Predicted Connection Moment Capacity to Moment at Failure for specimen S-10-10

Specimen	S-10-10	
Specimen condition	Undamaged	
Moment (kip-ft) at failure	110	
Failure mode	Concrete cracking/crushing	
	Mom Cap (kip-ft)	Ratio
MM	157	1.43
MM-pw	112	1.02
MM-H	101	0.92
MM-H-pw	72	0.66
MG	231	2.10
MG-cc	124	1.13
MG-2.5pw	182	1.65
MG-H	147	1.34

MG-H-cc	79	0.72
MG-H-2.5pw	116	1.05
SH	165	1.50
SH-s	85	0.77
KP	156	1.41
HM4	99	0.90
Plastic Moment	237	2.15

The two methods closest to the observed failure moment are MM-pw (ratio = 1.02) and MG-H-2.5pw (ratio = 1.05), both slightly unconservative. MM-H gives 0.92, the most accurate conservative prediction.

Most other methods significantly overestimate or underestimate the capacity. MG reaches a ratio of 2.10, and MM-H-pw reaches 0.66. The methods that use the pile width or a limited bearing width as an input tend to perform better for the strong-axis configuration.

The dataset consists of a single specimen, which is insufficient to distinguish method performance from specimen variability. Additional experimental data on strong-axis H-pile connections is required.

6. Pre-Damaged Specimens

Specimen W-10-7.5 2nd try is the only pre-damaged specimen that failed by concrete bearing and can therefore be compared against the capacity prediction methods. Table 21 presents the predicted capacities and predicted-to-actual ratios for all fourteen variants.

Table 21. Predicted Connection Moment Capacity (kip-ft) and Ratio of Predicted Connection Moment Capacity to Moment at Failure for specimen W-10-7.5 2nd try

Specimen	W-10-7.5 2nd try
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Specimen condition	Pre-damaged	
Moment (kip-ft) at failure	85	85
Failure mode	Concrete cracking/crushing	Concrete cracking/crushing
	Mom Cap (kip-ft)	Ratio
MM	110	1.30
MM-pw	79	0.93
MM-H	60	0.70
MM-H-pw	43	0.50
MG	138	1.63
MG-cc	74	0.87
MG-2.5pw	109	1.28
MG-H	74	0.87
MG-H-cc	40	0.47
MG-H-2.5pw	58	0.69
SH	97	1.15
SH-s	59	0.69
KP	103	1.21
HM4	68	0.79
Plastic Moment	107	1.26

None of the methods reviewed in this chapter were developed for pre-damaged connections. All assume an intact concrete bearing zone, and their predictions should be interpreted accordingly. As estimates of the undamaged capacity at this geometry, not of the reduced capacity resulting from prior loading.

The observed failure moment of 85 kip-ft compares to 98 kip-ft for the undamaged W-10-7.5R at the same embedment depth, a reduction of approximately 13%. By

contrast, the prior loading event reduced rotational stiffness by approximately 65% (from 7,869 to 2,654 kip-ft/rad), as is further explored in the next chapter. This asymmetry is physically meaningful: concrete micro-damage in the bearing zone degrades the elastic stiffness of the connection far more severely than it degrades ultimate bearing capacity. The cracked bearing zone loses much of its ability to resist small deformations elastically while retaining most of its ability to mobilize compressive resistance at the larger deformations associated with failure.

Among the fourteen methods, MM-pw produces the closest prediction at a ratio of 0.93, slightly conservative and within 7% of the actual failure moment, despite not being calibrated for a damaged connection. KP, which was the most accurate method for undamaged weak-axis specimens, overestimates the pre-damaged capacity with a ratio of 1.21, which is expected given that the method was calibrated against intact connections.

The dataset consists of a single specimen, and the apparent accuracy of MM-pw are very likely coincidental. Additional experimental data on pre-damaged connections is required before any method can be reliably applied to this condition.

7. Summary

In this test program, three specimens failed by steel pile yielding (W-10-10, W-12-12, and W-10-15 3rd try) and five failed by concrete bearing (W-10-7.5R, W-10-5, W-12-6, S-10-10 and W-10-7.5 2nd try).

For steel-governed failures, the structural plastic moment predicted all three accurately, including the pre-damaged W-10-15 3rd try, confirming that this level of prior damage to the concrete does not reduce moment capacity when steel governs. For weak-axis concrete-bearing failures, Kappes et al. (2021) and Marcakis and Mitchell (1980) were the most balanced predictors, while Mattock and Gaafar (1982) and Shama et al. (2002a) were consistently unconservative and the Harries-modified variants excessively conservative. For the single strong-axis specimen, no significant conclusions can be drawn from one test. For the pre-damaged specimen, prior loading reduced moment capacity by only 13% while rotational stiffness dropped 66%, and no method was developed to account for this condition.

Across the full dataset, no single method performs well in all scenarios. Kappes et al. (2021) was specifically calibrated for steel tube pile connections but shows the best overall balance for steel H pile weak-axis concrete-bearing failures. However it still performs poorly for W-10-5 at $L_e/d = 0.5$. Marcakis and Mitchell (1980), the oldest method in this comparison, shows reasonable accuracy for weak-axis bending at moderate embedment depths, but becomes unconservative for the strong-axis configuration. Mattock and Gaafar (1982), limited to 2.5 times the pile width was the closest unconservative method on average, but is not suitable for design without a reduction factor. The absence of a universally reliable method reflects that none of these formulations were originally developed specifically for HP pile sections embedded in bent caps under weak-axis bending, and all carry geometric assumptions that do not map cleanly onto this connection type.

The dataset size is a fundamental limitation of the conclusions that can be drawn. A larger experimental program with replicated specimens across a wider range of embedment depths and pile sections would be needed to establish method performance with statistical confidence.

Despite these limitations, a few practical observations are possible. Methods incorporating the Harries et al. (2000) spalling modification consistently underestimate capacity by 40-70% across all specimens and configurations. The degree of conservatism is large enough to be practically counterproductive, since a designer relying on MM-H-pw or MG-H-cc would significantly underutilize the available connection capacity. These modifications appear to overcorrect for spalling effects in a way that was not calibrated to H-pile geometry, and their use for this connection type is not supported by the experimental evidence presented here. Similarly, the original MG and SH formulations are persistently unconservative and carry safety implications if applied directly to design without reduction.

For practitioners working with plain HP pile embedment connections in the absence of better-calibrated methods, Kappes et al. (2021) methodology offers the most defensible starting point for weak-axis bending at embedment ratios between $L_e/d = 0.5$ and 0.75, with the understanding that it tends toward slight conservatism on average and may underestimate capacity at very shallow embedment.

Chapter 7:

Experimental Rotational Stiffness

1. Introduction

As observed by different authors (Rollins and Stenlund, 2010; Kappes et al, 2021, Rakestraw et al, 2024, and others), full rotational restraint does not exist since the piles and deform and rotate inside the embedded portion of the cap, as well as the connection concrete itself suffers deformations, allowing internal movement.

As seen in the literature review, rotational stiffness is the parameter that determines the fixity of pile-to-cap connection behavior. A pure theoretical pinned condition would mean a rotational stiffness equal to zero. A pure theoretical fixed condition would mean a rotational stiffness trending to infinity. However, in reality, neither case exists.

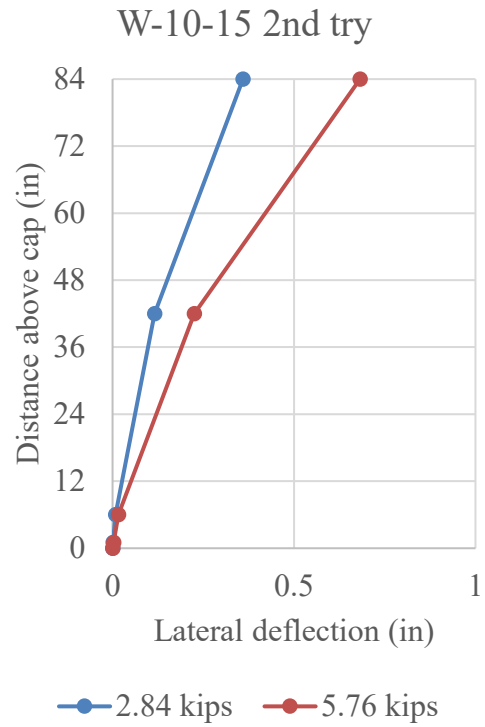
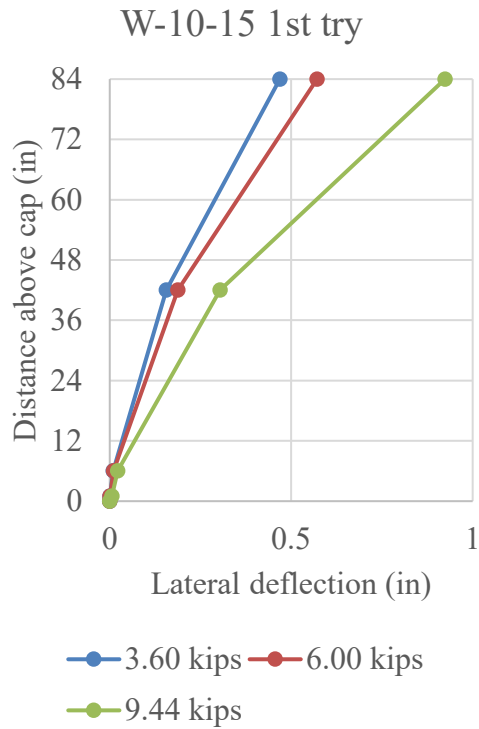
The rotational stiffness analysis is based on all the results seen in chapter 5. The acting moment at the connection was determined by multiplying the measured applied horizontal load by the lever arm (7 ft). The rotation at the connection interface can be calculated based on the tiltmeter measurements or potentiometer displacements at the portion of the pile that behaves as a rigid body. For each specimen, the results are plotted in a curve of moment vs. rotation. The rotational stiffness is then calculated as the slope of the linear elastic region of the curve.

1.1. Rigid Body Behavior

In order to analyze rotational stiffness, it is important to determine if the pile is behaving as a rigid body during the load tests. This way, one can rely not only on the inclinometer sensors but also on the displacement potentiometers to calculate the pile rotation at the connection level.

To verify rigid body behavior, a method is to plot lateral deflection versus distance from cap at different load levels. Figures 182, 183 and 184 show the measurements of

potentiometers POT C1, POT C6, POT C42 and POT C84 versus their respective distances from the cap, at multiple load levels.



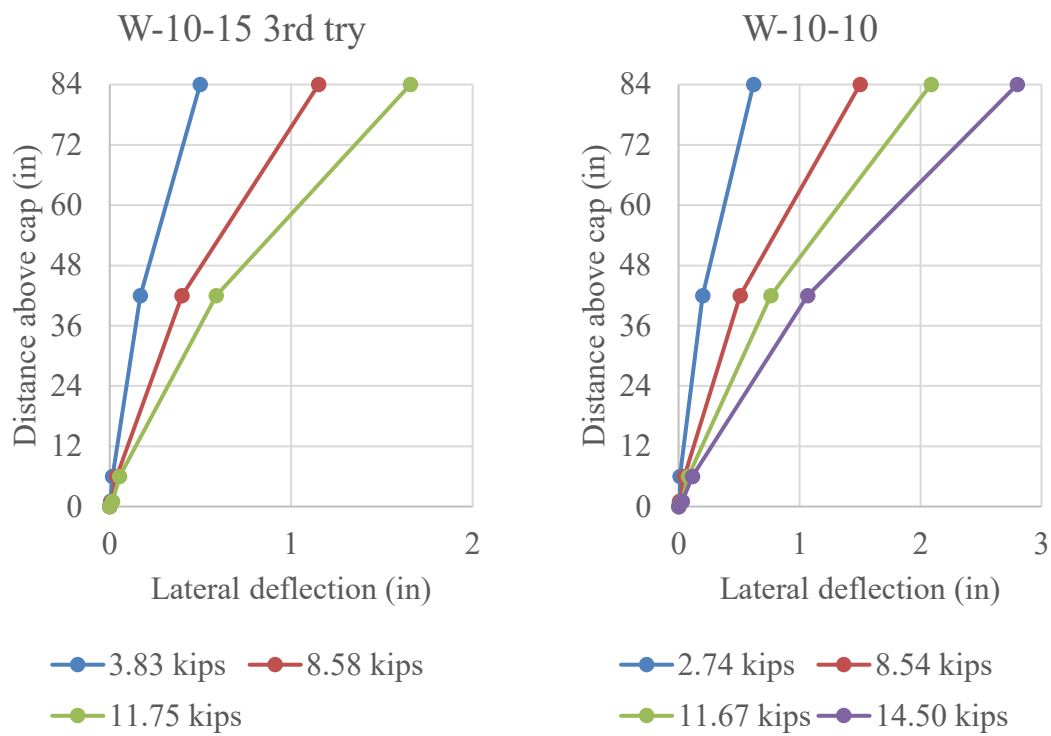


Figure 182. Sample deflection vs distance from cap for specimens W-10-15 1st, 2nd and 3rd try, and W-10-10 (applied load at 84 in level).

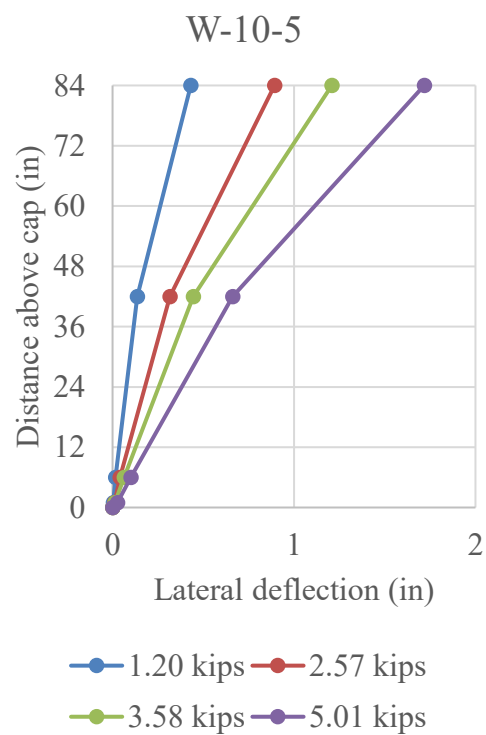
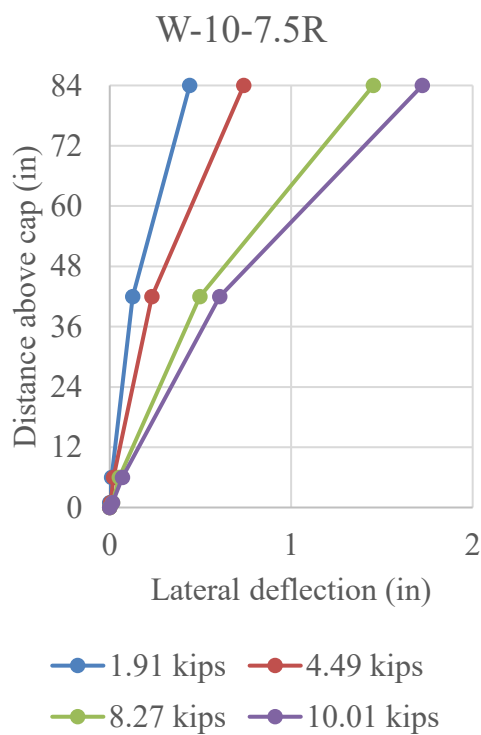
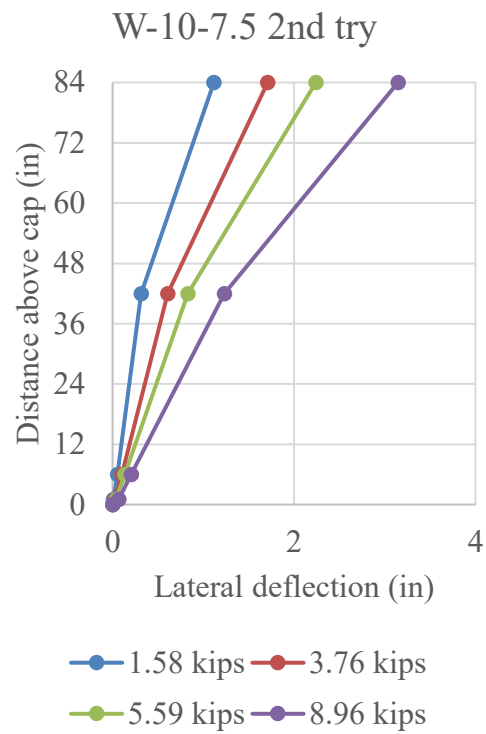
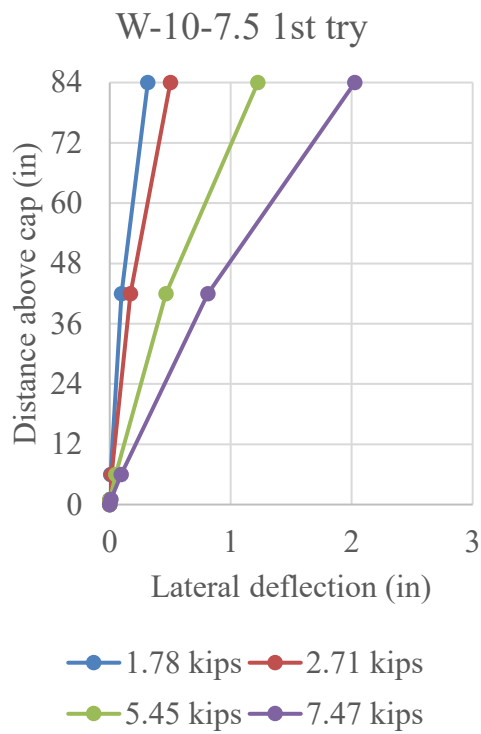
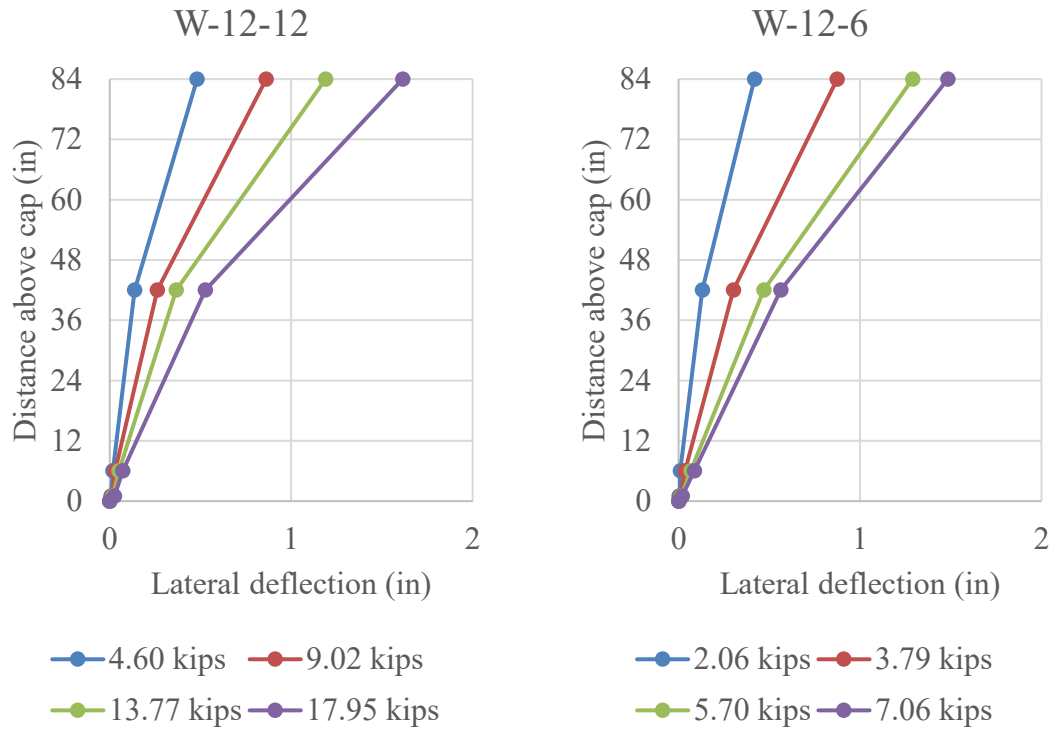


Figure 183. Sample deflection vs distance from cap for specimens W-10-7.5 1st and 2nd tries, W-10-7.5R and W-10-5 (applied load at 84 in level).



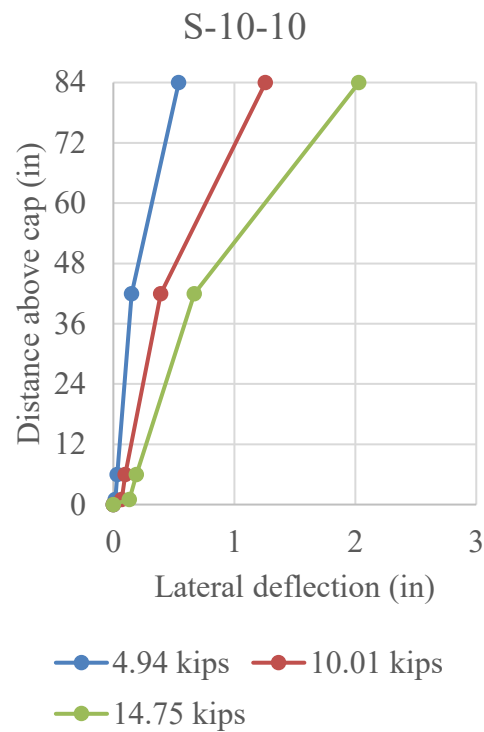


Figure 184. Sample deflection vs distance from cap for specimens W-12-12, W-12-6 and S-10-10

As observed, for all specimens, the displacements at C1, C6, and C42 were found to be mutually consistent and collinear, indicating that the pile behaves as an approximately rigid body over the lower 42 inches. The displacement at C84, however, consistently fell beyond the line defined by the three lower instruments, indicating that bending of the pile becomes non-negligible in the upper portion of the free length. For this reason, Pot C84 was deemed unsuitable for estimating the rotation at the pile-cap interface, since its displacement includes a bending component that would be incorrectly attributed to rigid body rotation.

For all specimens except S-10-10, the collinear displacement profile extrapolates back to a near-zero intercept at the cap face (ranging from 0.008 to 0.018 in.), meaning lateral translation of the system is negligible. In this case, the interface rotation is taken from Inclinator C1 as the primary measurement, with Pot C1 serving as a secondary cross-check.

For specimen S-10-10, the same three pots remain collinear, but the extrapolated intercept at the cap face is approximately 0.12 in., indicating a meaningful lateral translation of the system during the test. Because any potentiometer-based rotation estimate includes this translation component, Pot C1 is unreliable for S-10-10. For this specimen, the interface rotation is taken exclusively from Inclinometer C1, corrected by the TILT_CAP inclinometer.

1.2. Specimen W-10-15

As discussed in chapter 5, specimen W-10-15 required 3 attempts to succeed, due to hydraulic jack malfunction on the first 2 tries. Figure 185 shows the results for the 3 tries.

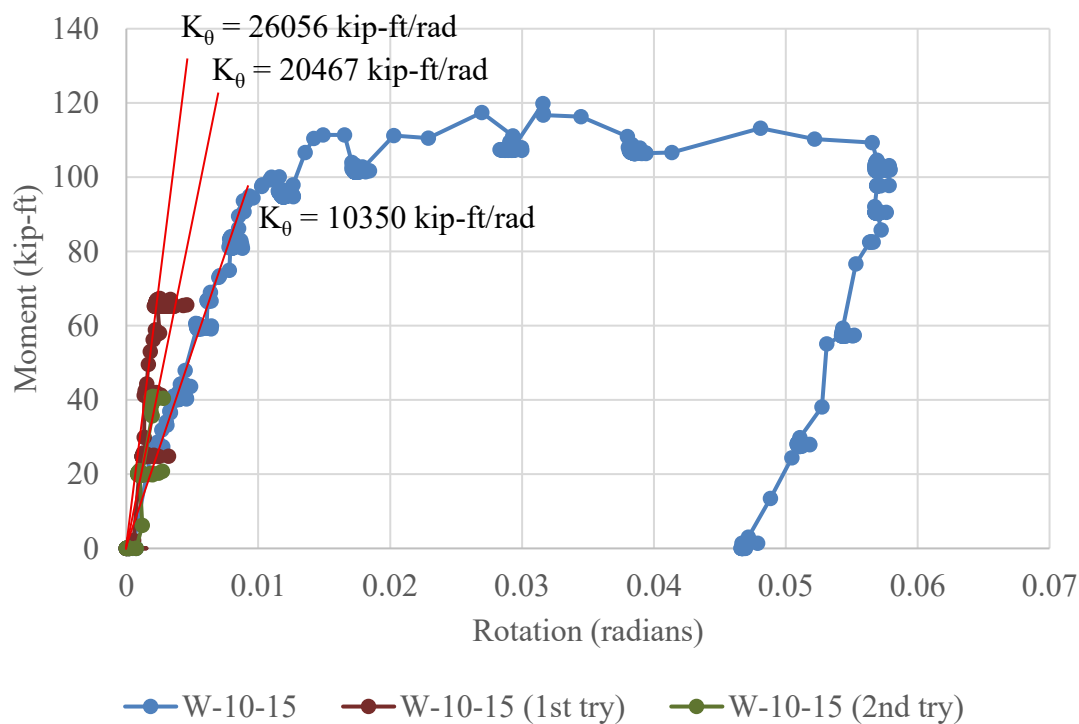


Figure 185. Moment-rotation plot of specimen W-10-15.

As observed in Figure 185, the multiple tries had different stiffnesses. The three curves are visually close at low moment levels (until approximately 25 kip-ft, horizontal load 3.5 kips) due to the scale of the plot, but have distinct slopes from the onset.

The 1st and 2nd tries had to be terminated before failure, and the third try ran through failure and post-peak. The linear elastic region was identified visually as the portion of the curve where the moment-rotation response maintained an approximately constant slope, from the origin up to a certain moment level. For tries 1 and 2, the entire curve remained within the linear range, For try 3, the linear region was maintained until about 85 kip-ft (horizontal load 12.1 kip).

The first try showed a rotational stiffness of approximately $K_{\theta} = 26056$ kip-ft/rad. The load test went up to a maximum moment at the connection of 66.96 kip-ft (horizontal load of 9.6 kips) and maximum rotation of 0.002 radians (0.13°) at the connection level.

The second try showed a rotational stiffness of approximately $K_{\theta} = 20467$ kip-ft/rad, 21% below the first try. The load test reached a maximum moment at the connection of 40.65 kip-ft (horizontal load of 5.8 kips) and a maximum rotation of 0.002 radians (0.13°).

The third and final try, this time a completed test, showed a rotational stiffness of approximately $K_{\theta} = 10350$ kip-ft/rad, 50% below the second try, and 60% below the first try. The linear part of the curve was maintained until about 85 kip-ft, then smoothly transitioned with increasing moment and rotation until reaching maximum moment. Upon reaching failure, the maximum moment at the connection was 110 kip-ft (horizontal load of 15.7-16 kips) and maximum rotation of 0.014 radians (0.80°). In post failure, the specimen kept a moment of approximately 105-115 kip-ft, while the hydraulic jack kept being pressurized, without any noticeable increase in load. After a few pressurization attempts and upon reaching a rotation of 0.056 radians (3.2°), unloading began.

The stiffness degradation from try 1 to 3 (26056 to 20467 to 10350 kip-ft/rad) is physically interpretable as cumulative micro-damage to the connection concrete from each loading event. Tries 1 and 2 both loaded the connection to meaningful moment levels (66.96 kip-ft and 40.65 kip-ft), respectively, while remaining entirely in the elastic range. Those loads were sufficient to initiate minor cracking at the steel-concrete interface, reducing subsequent stiffness. This is consistent with what was observed in chapter 5: even at elastic load levels in W-10-15, the trailing-side interface gap was widening. However, their stiffness measurements reflect early-cycle elastic response at load levels below the onset of nonlinearity observed in try 3.

Therefore, the rotational stiffness for the W-10-15 3rd complete try ($K_\theta = 10350$ kip-ft/rad) should be considered the stiffness of a pre-loaded (damaged) connection, not an undamaged one. The stiffness found in try 1 ($K_\theta = 26056$ kip-ft/rad) is likely the closest approximation to the undamaged initial stiffness, however this load test suffered from hydraulic jack imbalance and it only reached intermediate moment levels, so its boundary conditions are uncertain.

1.3. Specimen W-10-10

Unlike specimen W-10-15, specimen W-10-10 was completed in a single load test, providing a straightforward moment-rotation dataset without pre-loading history. Figure 186 shows the moment-rotation curve.

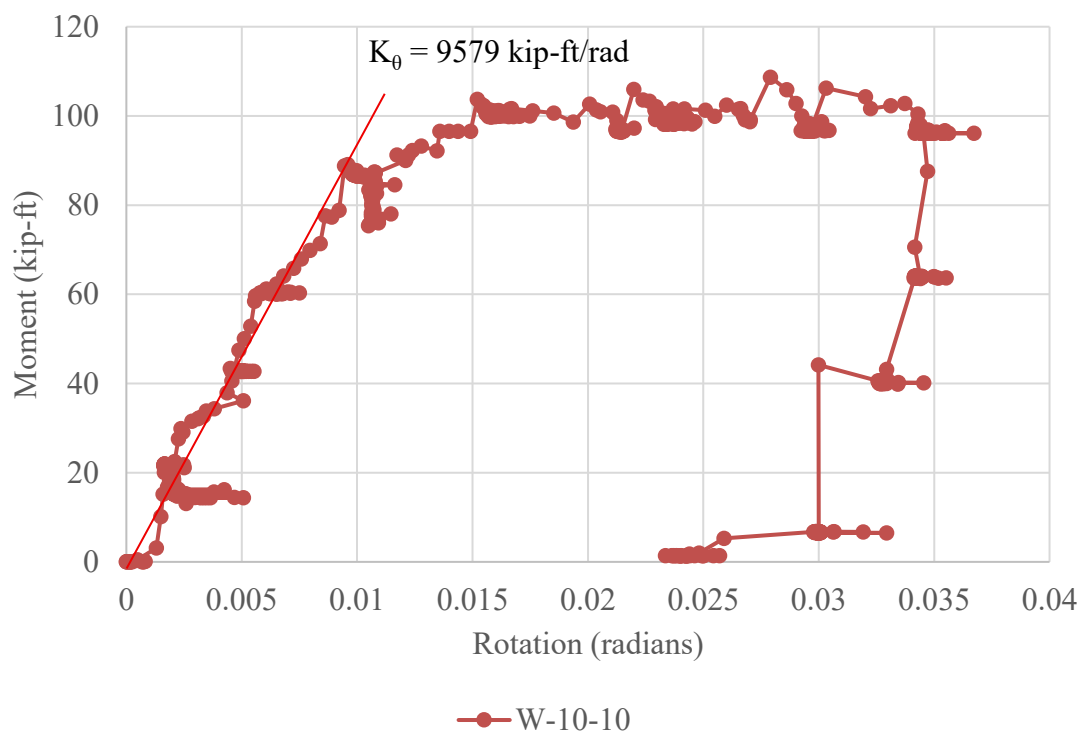


Figure 186. Moment-rotation plot of specimen W-10-10.

As observed, the curve rises linearly from the origin, transitions smoothly into a post-elastic region, reaches a moment plateau near failure, and then unloads. The linear

elastic region was identified visually as the portion of the curve where the moment-rotation response maintained an approximately constant slope, from the origin up to approximately 85 kip-ft (horizontal load 12.1 kips, rotation 0.009 radians or 0.51°), yielding a rotational stiffness of approximately $K_{\theta} = 9579$ kip-ft/rad.

Beyond the linear region, the curve smoothly transitioned from elastic to plastic behavior, reaching a maximum moment of approximately 100-108 kip-ft (horizontal load 14.3-15.4 kips) at a rotation of 0.016 radians (0.91°). In post-failure, the specimen maintained a moment of approximately 100-108 kip-ft while the hydraulic jack continued to be pressurized, without any noticeable increase in load. Upon reaching a rotation of 0.034 radians (1.94°), unloading began.

The linear elastic region of W-10-10 terminates at approximately the same moment level as specimen W-10-15 (try 3), approximately 85 kip-ft in both cases. This level corresponds closely to the onset of steel yielding predicted by the pile's theoretical flange local buckling moment of 92 kip-ft for the HP10x42 section. This shows that capacity, in both cases, is governed by the steel structural capacity for these embedment depths. However, the W-10-10 rotational stiffness ($K_{\theta} = 9579$ kip-ft/rad) is 7%, 53% and 63% lower than W-10-15 3rd (damaged), 2nd (damaged) and 1st (undamaged) tries, respectively. This is consistent with the shorter embedment providing less rotational restraint at the connection.

A systematic comparison across all specimens is presented in Section 2.

1.4. Specimens W-10-7.5 and W-10-7.5R

As discussed in Chapter 5, specimen W-10-7.5 required two load attempts: the first was interrupted early due to an inappropriate reaction frame, and the second was conducted on the same, pre-loaded specimen. Specimen W-10-7.5R was a fresh specimen of identical geometry tested in a single complete load test. Figure 187 shows the moment-rotation curves for all three load tests.

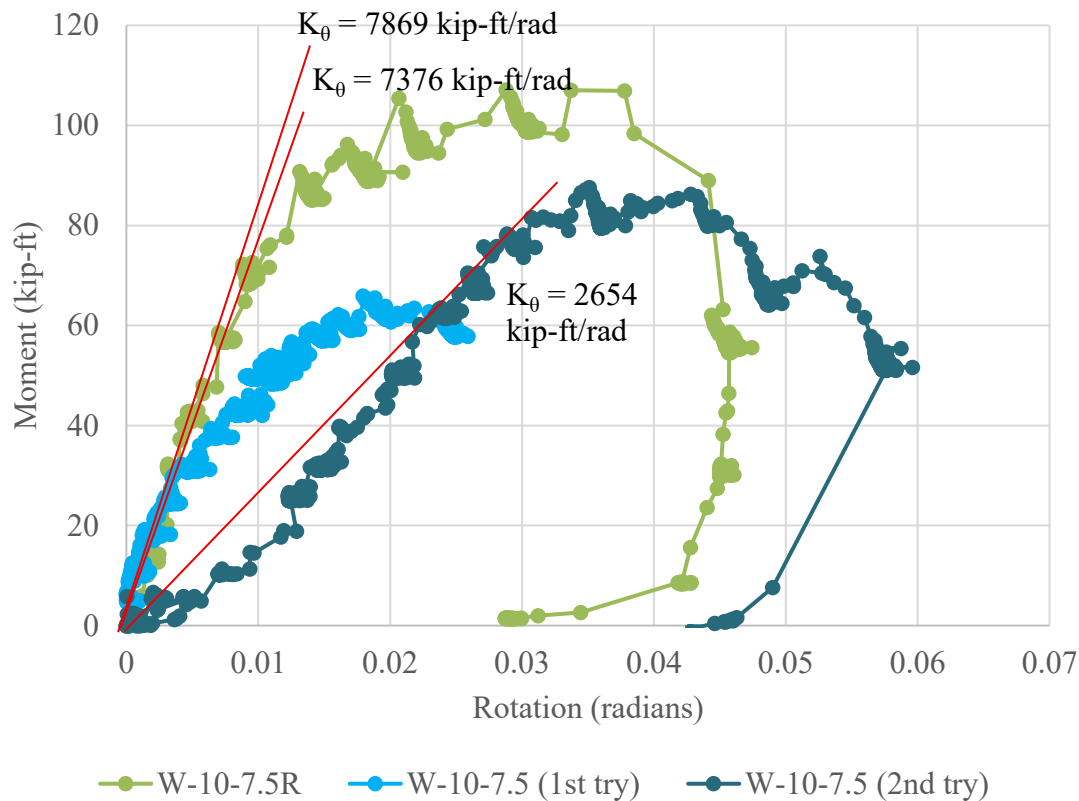


Figure 187. Moment-rotation plot of specimens W-10-7.5 and W-10-7.5R

For W-10-7.5, first try, the linear elastic region was extremely short and was identified visually from the origin up to approximately 18 kip-ft (horizontal load 2.5 kips, rotation 0.002 radians or 0.11°), yielding a rotational stiffness of approximately $K_0 = 7376$ kip-ft/rad.

It is noted that the linear region terminated at a substantially lower moment level than observed in specimens W-10-15 and W-10-10, both of which showed linear behavior up to approximately 85 kip-ft. This earlier departure from linearity is likely attributable, at least in part, to the inappropriate reaction frame used in this try, which may have introduced unintended forces or displacements that caused the measured response to deviate from true pile-to-cap behavior at low load levels. As a result, the stiffness value from this try carries uncertainty beyond the specimen variability already discussed for W-10-15. After the linear region, the curve continued to change slope progressively, and the

test was interrupted at approximately 62 kip-ft (horizontal load 8.8 kips, rotation 0.021 radians or 1.20°).

For W-10-7.5, second try, the linear elastic region extended from the origin up to approximately 78 kip-ft (horizontal load 11.1 kips, rotation 0.029 radians or 1.66°), yielding a rotational stiffness of approximately $K_{\theta} = 2654$ kip-ft/rad. This indicates a drop of approximately 64% relative to the first try. This stiffness degradation follows the same pattern observed in specimen W-10-15, and is physically consistent with cumulative damage to the connection concrete accumulated during the first load test. Beyond the linear region, the curve quickly changed slope and reached a maximum moment of approximately 85 kip-ft (horizontal load 12.1 kips) at a rotation of 0.036 radians (2.06°). Unlike the deeper-embedment specimens, the moment plateau was not sustained and the load dropped quickly after the peak, consistent with the concrete bearing failure mode described in Chapter 5.

For W-10-7.5R, tested as a fresh specimen in a single load test, the linear elastic region extended from the origin up to approximately 68 kip-ft (horizontal load 9.7 kips, rotation 0.01 radians or 0.57°), yielding a rotational stiffness of approximately $K_{\theta} = 7869$ kip-ft/rad. Beyond the linear region, the curve transitioned from elastic to plastic behavior and reached a maximum moment of approximately 100 kip-ft (horizontal load 14.2 kips) at a rotation of 0.03 radians (1.71°). Post-failure shows that the plateau was not able to be sustained and quickly dropped after the peak.

The rotational stiffness of W-10-7.5R ($K_{\theta} = 7869$ kip-ft/rad) is notably similar to that of W-10-7.5, first try ($K_{\theta} = 7376$ kip-ft/rad), with only a 7% difference, likely due to specimen-to-specimen variability. This agreement between two independent undamaged measurements at the same embedment depth supports the $K_{\theta} \approx 7300$ -7800 kip-ft/rad range as representative of the undamaged 7.5-inch embedment connection. The substantially lower stiffness of the W-10-7.5, second try ($K_{\theta} = 2654$ kip-ft/rad) is attributed to prior loading history rather than any fundamental difference in connection behavior, consistent with the interpretation applied to W-10-15 degradation as well.

Comparing W-10-7.5R ($K_{\theta} = 7869$ kip-ft/rad) to W-10-10 ($K_{\theta} = 9579$ kip-ft/rad), reducing embedment from 10 to 7.5 inches produces a 17% drop in rotational stiffness. This contrasts with the substantially larger reduction observed between W-10-15 and W-

10-10, suggesting that the relationship between rotational stiffness and embedment depth is nonlinear in this range.

The linear elastic region of W-10-7.5R ends at approximately 68 kip-ft, which is notably earlier than the approximately 85 kip-ft observed in specimens W-10-15 and W-10-10. In these specimens with deeper embedment, the 85 kip-ft limit correlates with the onset of steel yielding, approaching the theoretical flange local buckling moment of 92 kip-ft for the HP10x42 section. W-10-7.5R, on the other hand, failed due to concrete bearing, degradation of the connection concrete that occurred prior to reaching the steel yielding threshold, resulting in an earlier deviation from linear behavior and providing additional evidence of the failure mode transition discussed in Chapter 5.

1.5. Specimen W-10-5

Figure 188 shows the moment-rotation curve for the specimen W-10-5 load test.

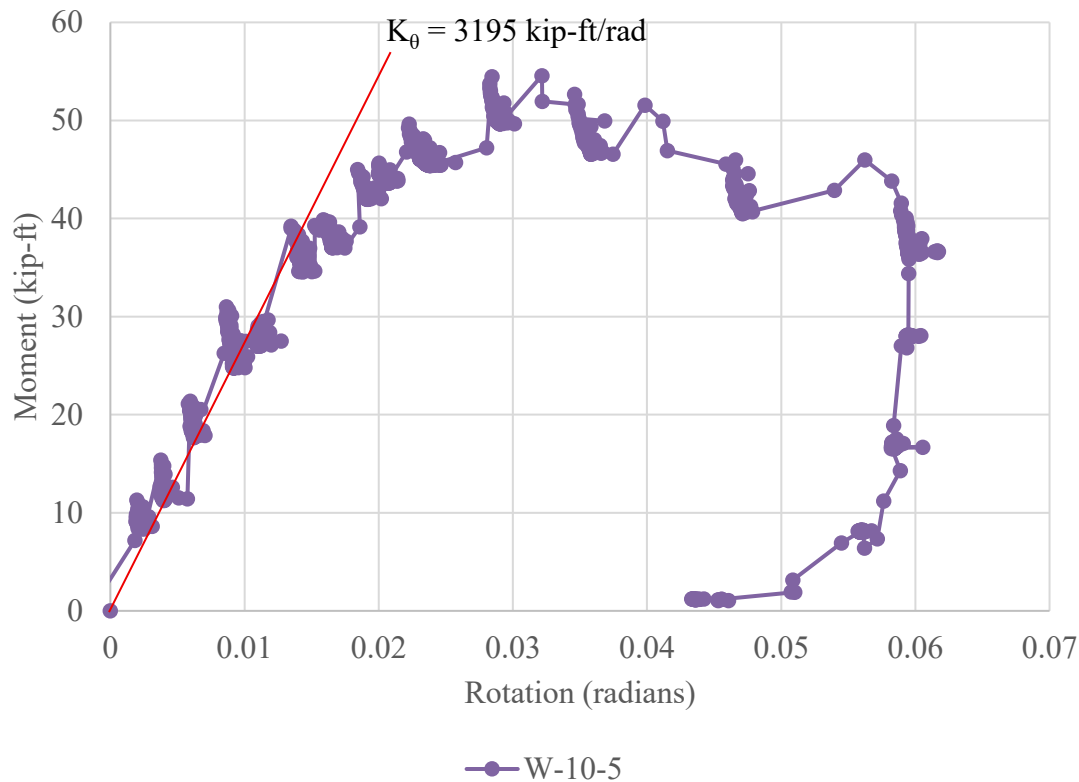


Figure 188. Moment-rotation plot of specimen W-10-5

As observed, the linear elastic region increase from the origin up to approximately 36 kip-ft (horizontal load 5.1 kips, rotation 0.014 radians or 0.80°), yielding a rotational stiffness of approximately $K_{\theta} = 3195 \text{ kip-ft/rad}$. Beyond the linear region, the curve transitioned from elastic to plastic behavior and reached a maximum moment of approximately 50 kip-ft (horizontal load 7.1 kips) at a rotation of 0.03 radians (1.71°). In post-failure, the plateau was not sustained and the load dropped quickly after the peak, consistent with the brittle concrete bearing failure described in Chapter 5.

The rotational stiffness of W-10-5 ($K_{\theta} = 3195 \text{ kip-ft/rad}$) is approximately 59% lower than that of W-10-7.5R ($K_{\theta} = 3233 \text{ kip-ft/rad}$). This is a substantially larger drop than the 17% reduction observed between W-10-10 and W-10-7.5R, despite the same 2.5-inch reduction in embedment depth in both cases. The marked increase in stiffness sensitivity below 7.5 inches is consistent with the failure mode transition identified in Chapter 5: once

embedment drops below the threshold where concrete bearing governs, the connection loses rotational restraint much more rapidly with each reduction in embedment depth.

The linear elastic region of W-10-5 also ends at a notably lower moment level than the previous specimens, 36 kip-ft, compared to 68 kip-ft for W-10-7.5R and 85 kip-ft for W-10-10 and W-10-15. This progressive earlier departure from linearity with decreasing embedment reflects the increasing dominance of concrete bearing as the governing mechanism: the shallower the embedment, the sooner the concrete begins to degrade under the applied moment, and the earlier the connection departs from elastic behavior.

1.6. Specimen W-12-12

Figure 189 shows the moment-rotation curve for the specimen W-12-12 load test.

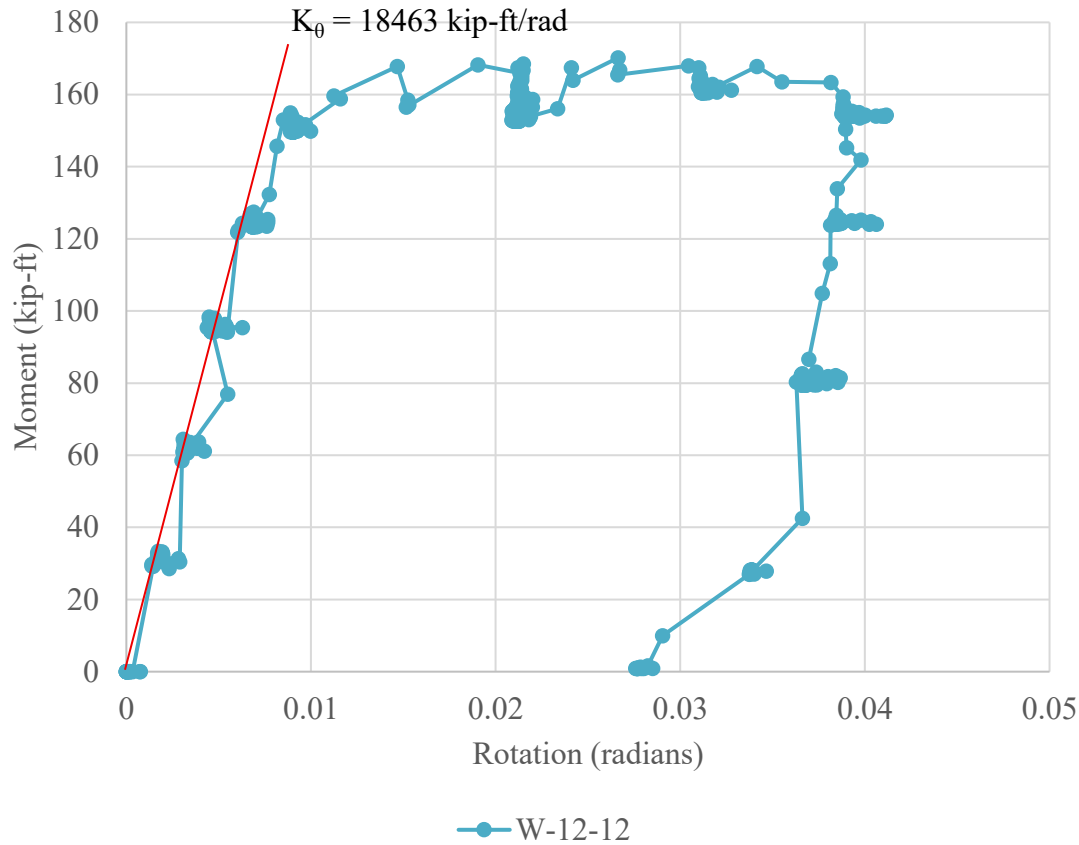


Figure 189. Moment-rotation plot of specimen W-12-12

As observed, the linear elastic region was identified visually from the origin up to approximately 125 kip-ft (horizontal load 17.8 kips, rotation 0.007 radians or 0.40°), yielding a rotational stiffness of approximately $K_0 = 18463$ kip-ft/rad. The linear region extends to a moment level slightly above the flange buckling moment of the HP12x53 section (118 kip-ft), at which point the curve slightly changed its slope then transitioned from linear elastic to plastic behavior. The maximum achieved moment was approximately 160 kip-ft (horizontal load 23 kips) at a rotation of 0.015 radians (0.86°). This moment level is slightly above the pile's full plastic moment (146 kip-ft). In post-failure, the plateau was sustained until reaching a rotation of 0.039 radians (2.23°).

Specimen W-12-12 and specimen W-10-10 share the same embedment-to-pile-depth ratio of 1.0, making them directly comparable despite the difference in pile section. The rotational stiffness of W-10-10 ($K_0 = 9579$ kip-ft/rad) is approximately 52% of that of

W-12-12 ($K_{\theta} = 18463$ kip-ft/rad). Notably, the ratio of the weak-axis moments of inertia of the HP10x42 to the HP12x53 is approximately 0.56 - very similar to the observed stiffness ratio of 0.52. This suggests that, for the same embedment-to-pile-depth ratio, the rotational stiffness of the connection scales approximately with the pile's own bending stiffness (EI). A more systematic evaluation of this relationship across all specimens is presented in Section 2.

1.7. Specimen W-12-6

Figure 190 shows the moment-rotation curve for the specimen W-12-6 load test.

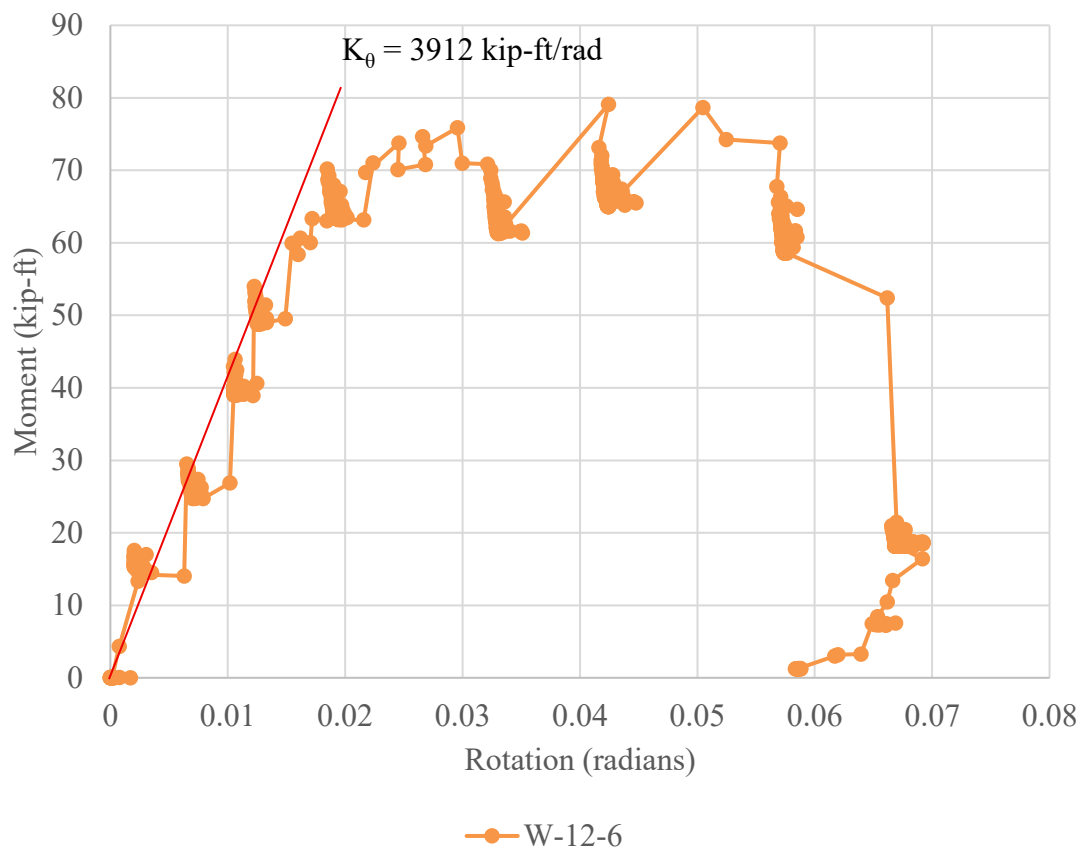


Figure 190. Moment-rotation plot of specimen W-12-6

As observed the linear elastic region was identified visually from the origin up to approximately 55 kip-ft (horizontal load 7.9 kips, rotation 0.015 radians or 0.86°), yielding

a rotational stiffness of approximately $K_{\theta} = 3912$ kip-ft/rad. Beyond the linear region, the curve transitioned from elastic to plastic behavior as the concrete bearing capacity was reached. The maximum achieved moment was approximately 70 kip-ft (horizontal load 10 kips) at a rotation of 0.032 radians (1.83°). In post-failure, the plateau was sustained briefly before dropping upon reaching a rotation of 0.057 radians (3.26°).

Specimen W-12-6 and specimen W-10-5 share the same embedment-to-pile-depth ratio of 0.5, enabling a direct comparison. The rotational stiffness of W-10-5 ($K_{\theta} = 3195$ kip-ft/rad) is approximately 81% of that of W-12-6 ($K_{\theta} = 3912$ kip-ft/rad). The weak-axis moment of inertia ratio of the HP10x42 to the HP12x53 is approximately 0.56, suggesting the proportionality observed between W-10-10 and W-12-12, no longer applies here. The linear elastic region of W-12-6 terminates at 55 kip-ft, higher than the 36 kip-ft observed in W-10-5 but still well below the steel yielding threshold of the HP12x53 section, consistent with concrete bearing governing the connection at this shallow embedment depth in both pile series.

1.8. Specimen S-10-10

Figure 191 shows the moment-rotation curve for the specimen S-10-10 load test.

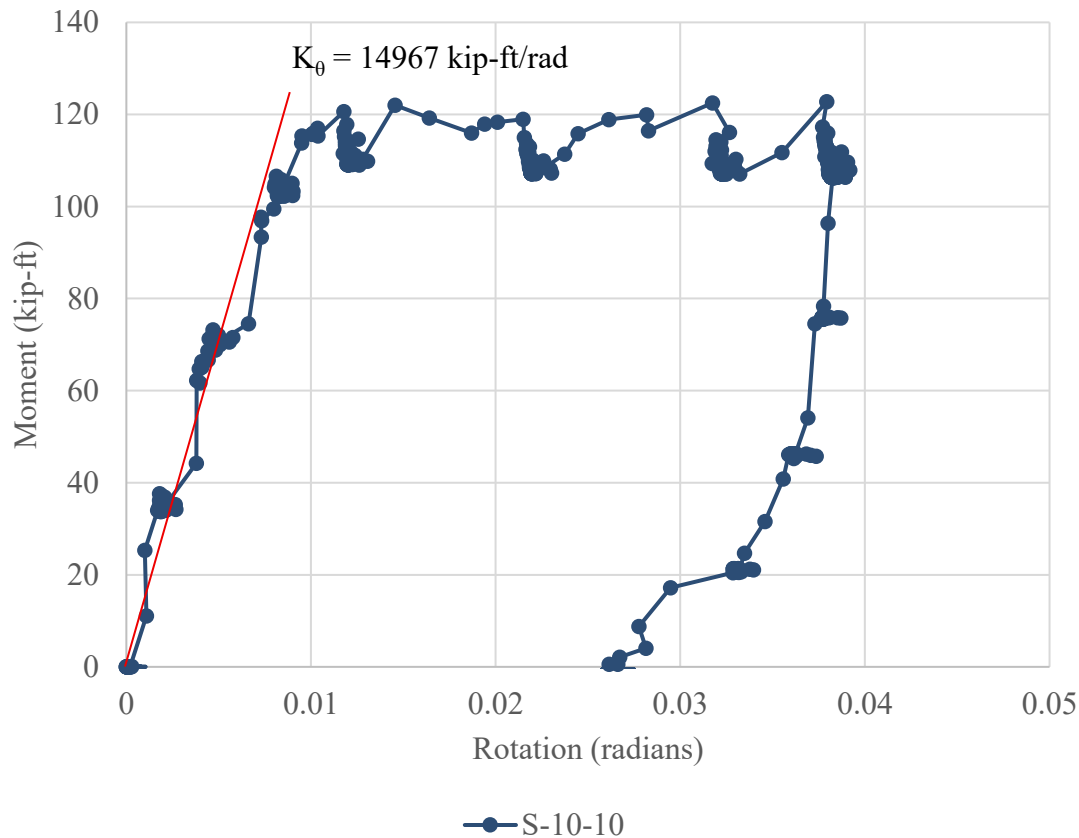


Figure 191. Moment-rotation plot of specimen S-10-10

As observed, the linear elastic region was identified visually from the origin up to approximately 105 kip-ft (horizontal load 15 kips, rotation 0.008 radians or 0.46°), yielding a rotational stiffness of approximately $K_{\theta} = 14967$ kip-ft/rad. Beyond the linear region, the curve quickly transitioned from elastic to plastic behavior as the concrete bearing capacity was reached. The maximum achieved moment was approximately 115 kip-ft (horizontal load 16.4 kips) at a rotation of 0.012 radians (0.69°). In post-failure, the plateau was maintained until reaching a rotation of 0.038 radians (2.17°).

The rotational stiffness of S-10-10 ($K_{\theta} = 14967$ kip-ft/rad) is approximately 56% higher than that of W-10-10 ($K_{\theta} = 9579$ kip-ft/rad), despite both specimens sharing the same pile section and embedment depth. This difference is attributed to the change in bearing geometry between the two bending orientations. In weak-axis bending (W-10-10), the pile flanges project parallel to the load direction and the embedded web face

engages the concrete in bearing, with resistance concentrated near the flange tips. In strong-axis bending (S-10-10), both full flange faces engage the concrete simultaneously across their entire width, producing a substantially larger and more distributed bearing area. This increased engagement stiffens the connection under rotation.

1.9. Summary

Table 22 summarizes the obtained rotational stiffness from each load test.

Table 22. Summary of Rotational Stiffness results from the test specimens

Specimen	Specimen condition	Rotational stiffness (kip-ft/rad)
W-10-15 1st try	Undamaged	26056*
W-10-15 2nd try	Pre-damaged	20467*
W-10-15 3rd try	Pre-damaged	10350
W-10-10	Undamaged	9579
W-10-7.5 1st try	Undamaged	7376*
W-10-7.5 2nd try	Pre-damaged	2654
W-10-7.5R	Undamaged	7869
W-10-5	Undamaged	3195
W-12-12	Undamaged	18463
W-12-6	Undamaged	3912
S-10-10	Undamaged	14967

*Incomplete or inappropriate load test.

Rotational stiffness values span from 2654 to 26056 kip-ft/rad across the test matrix, reflecting the combined influence of embedment depth, pile section, and specimen condition. Specimens subjected to multiple loading cycles (W-10-15 and W-10-7.5) exhibit a progressive reduction in K_θ with each successive test. Specimen S-10-10, tested in strong-axis bending with the same pile section and embedment depth as W-10-10,

yields a rotational stiffness 56% higher, indicating that bending axis is a meaningful variable within this dataset.

A few notes to consider:

- W-10-15 1st try: The load test achieved small loads and had a load imbalance due to the hydraulic jacks. Therefore, the rotational stiffness result must be carefully considered.
- W-10-15 2nd try: Again, the load test achieved small loads and had a load imbalance. Therefore, the rotational stiffness result must be carefully considered.
- W-10-7.5 1st try: The load test was not successful due to the poor behavior of the reaction frame. However, the rotational stiffness result may still correspond to reality, as it matches well with the result from W-10-7.5R.

2. Discussion

The moment-rotation results presented in the preceding sections are examined here collectively, with attention to two primary analytical aspects: the dependence of rotational stiffness on embedment depth within each pile series, and the role of pile cross-sectional properties and bending axis in governing stiffness magnitude. The W-10 and W-12 series are discussed separately before being compared directly, followed by an examination of the strong-axis specimen S-10-10.

2.1. W-10 Series

Figure 192 presents the moment-rotation curves for the undamaged W-10 series specimens across the full range of embedment depths tested. The general trend is clear: rotational stiffness decreases as embedment depth decreases. Across the undamaged specimens - from W-10-15 ($L_e/d = 1.5$) to W-10-5 ($L_e/d = 0.5$) - K_θ ranges from 26056 to 2654 kip-ft/rad. However, the decline is not uniform across this range. Moving from $L_e/d = 1.5$ to 1.0 produces a 64% reduction in K_θ (from 26056 to 9579 kip-ft/rad). Moving from $L_e/d = 1.0$ to 0.75 produces only a 20% reduction (from 9579 to 7622 kip-ft/rad). Moving from $L_e/d = 0.75$ to 0.5 again produces a 58% reduction (from 7622 to 3195 kip-ft/rad).

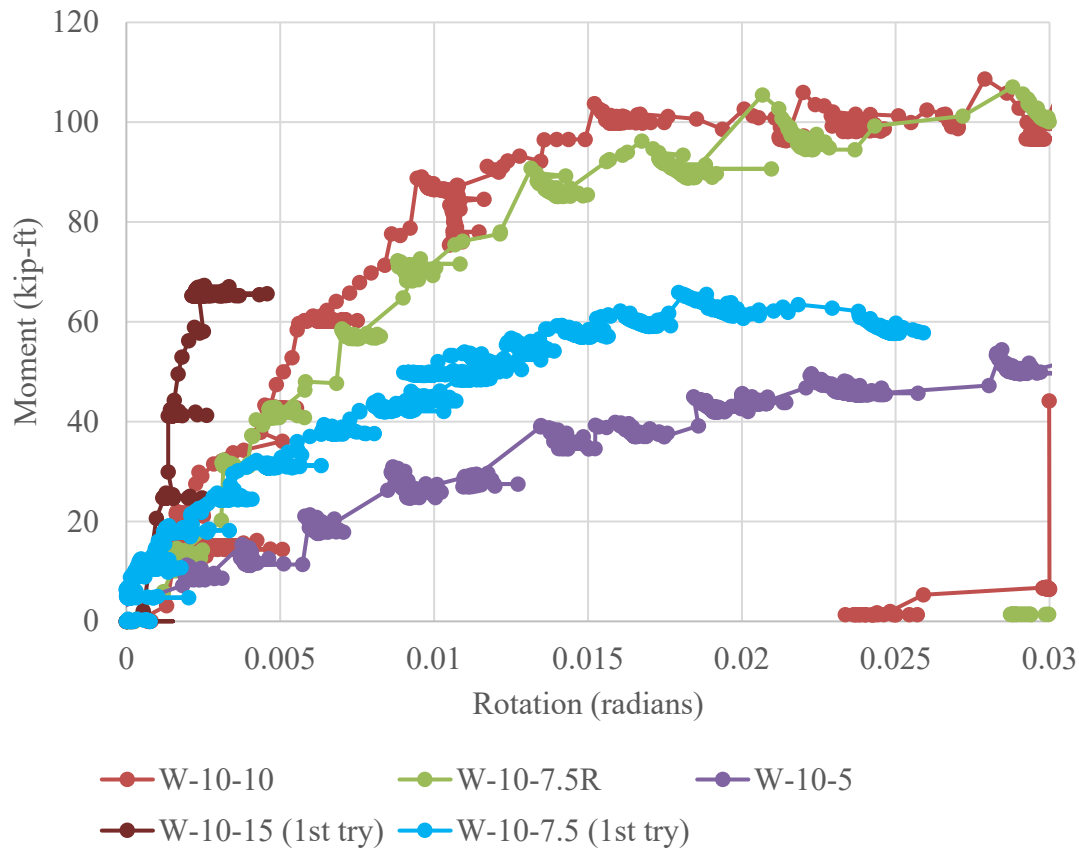


Figure 192. Moment-rotation comparison for load tests performed in undamaged specimens: W-10-15 1st try, W-10-10, W-10-7.5 1st try, W-10-7.5R and W-10-5

At $Le/d \geq 1.0$, the linear elastic region terminates at approximately 85 kip-ft in both W-10-15 and W-10-10, approaching the theoretical flange local buckling moment of 92 kip-ft for the HP10x42 section. The connection develops its full rotational resistance until the pile steel begins to control. At $Le/d = 0.75$ (W-10-7.5R), the linear region terminates at approximately 68 kip-ft, and at $Le/d = 0.5$ (W-10-5) at only 36 kip-ft, both well below the steel yielding threshold. Concrete bearing degradation controls the linear response before the pile section reaches its capacity.

The small decrease in stiffness between $Le/d = 1.0$ and 0.75 may therefore reflect a transition zone: embedment is shallow enough for concrete bearing to begin influencing capacity, but not yet shallow enough to severely reduce behavior and stiffness. Below this zone, the connection becomes increasingly concrete-governed and highly sensitive to further reductions in embedment.

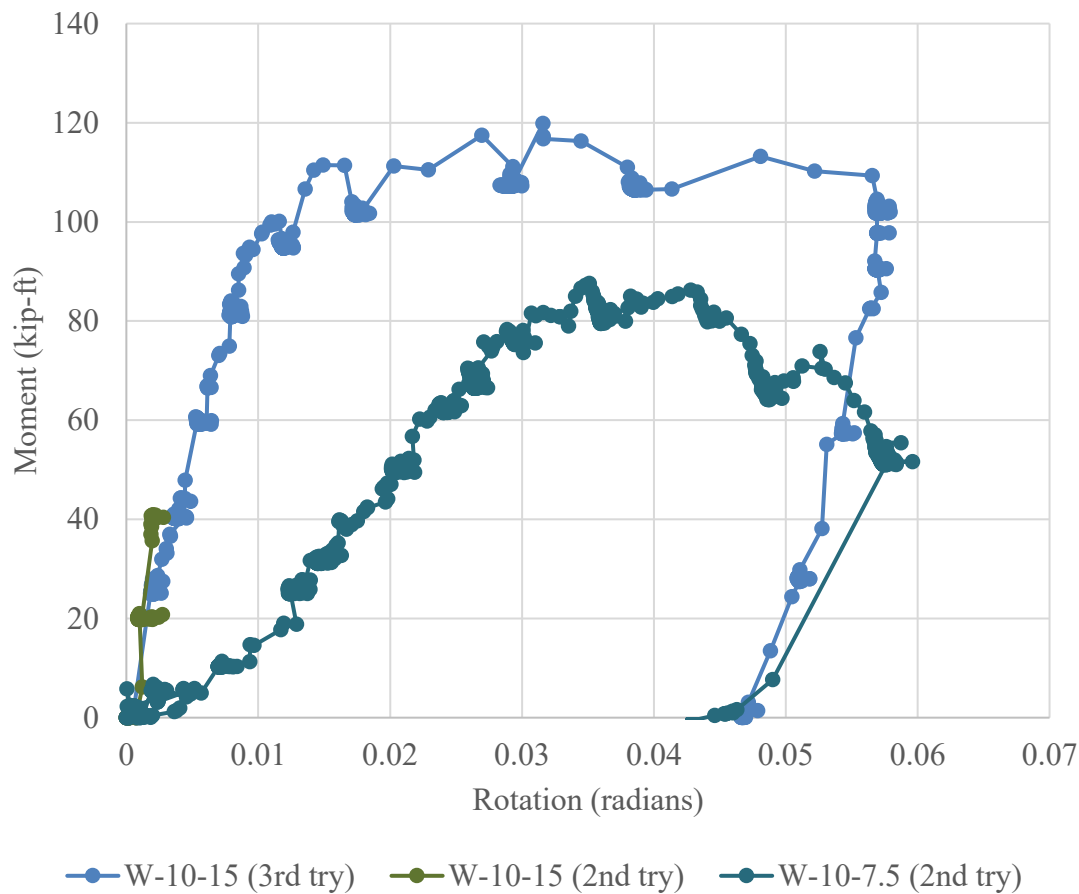


Figure 193. Moment-rotation comparison for load tests performed in pre-damaged specimens: W-10-15 2nd try, W-10-15 3rd try, and W-10-7.5 2nd try.

Figure 193 presents the pre-damaged specimens: W-10-15 2nd and 3rd tries and W-10-7.5 2nd try. The progressive reduction in K_{θ} across the W-10-15 loading sequence (26056 to 20467 to 10350 kip-ft/rad) and the more severe drop in W-10-7.5 (from approximately 7300-7800 to 2654 kip-ft/rad) both reflect cumulative damage to the connection concrete.

The magnitude of degradation differs notably between the two specimens. For W-10-15, successive events reduced K_{θ} by 21% and then 49%, for a total degradation of 60% over three load tests. For W-10-7.5, a single prior loading event reduced K_{θ} by approximately 64%. The more severe degradation in W-10-7.5, for two load tests only, is consistent with the larger rotations imposed during its first try: despite a lower applied

moment, the lower stiffness of the shallower connection produced substantially larger rotations, causing greater deformation of the concrete bearing zone.

A broader implication follows from this pattern: pre-loading to elastic moment levels, even without reaching apparent structural failure, is sufficient to meaningfully reduce connection stiffness. Load events that remain entirely within the linear range of the moment-rotation curve can initiate micro-cracking at the steel-concrete interface that, obviously, does not recover between loading cycles. This has practical relevance for the interpretation of field connections: a connection that has experienced prior loading from construction activities, traffic, or hydraulic storm events may exhibit lower initial stiffness than laboratory measurements on truly undamaged specimens would suggest.

2.2. W-12 Series

Figure 194 presents the moment-rotation curves for specimens W-12-12 ($L_e/d = 1.0$) and W-12-6 ($L_e/d = 0.5$).

With only two specimens in this series, the discussion is limited, but the trends parallel those observed in the W-10 series. Rotational stiffness decreases substantially with decreasing embedment: K_θ drops from 18463 kip-ft/rad at $L_e/d = 1.0$ to 3912 kip-ft/rad at $L_e/d = 0.5$, a reduction of approximately 78%.

The failure mode also differs between the two embedment depths. For W-12-12, the linear elastic region extends to approximately 125 kip-ft, slightly above the flange local buckling moment of the HP12x53 section (118 kip-ft), consistent with pile steel governing connection capacity. For W-12-6, the linear region terminates at 55 kip-ft, well below the steel yielding threshold, indicating that concrete bearing governs at this shallower embedment. This failure mode transition parallels the behavior observed in the W-10 series and suggests it is a general feature of plain pile embedment connections across HP pile sections, at least within the L_e/d range tested. With no intermediate embedment depths available in the W-12 series, it is not possible to assess whether the nonlinear stiffness sensitivity pattern observed in the W-10 series also manifests for the HP12x53, and additional specimens at intermediate L_e/d ratios would be required to resolve this.

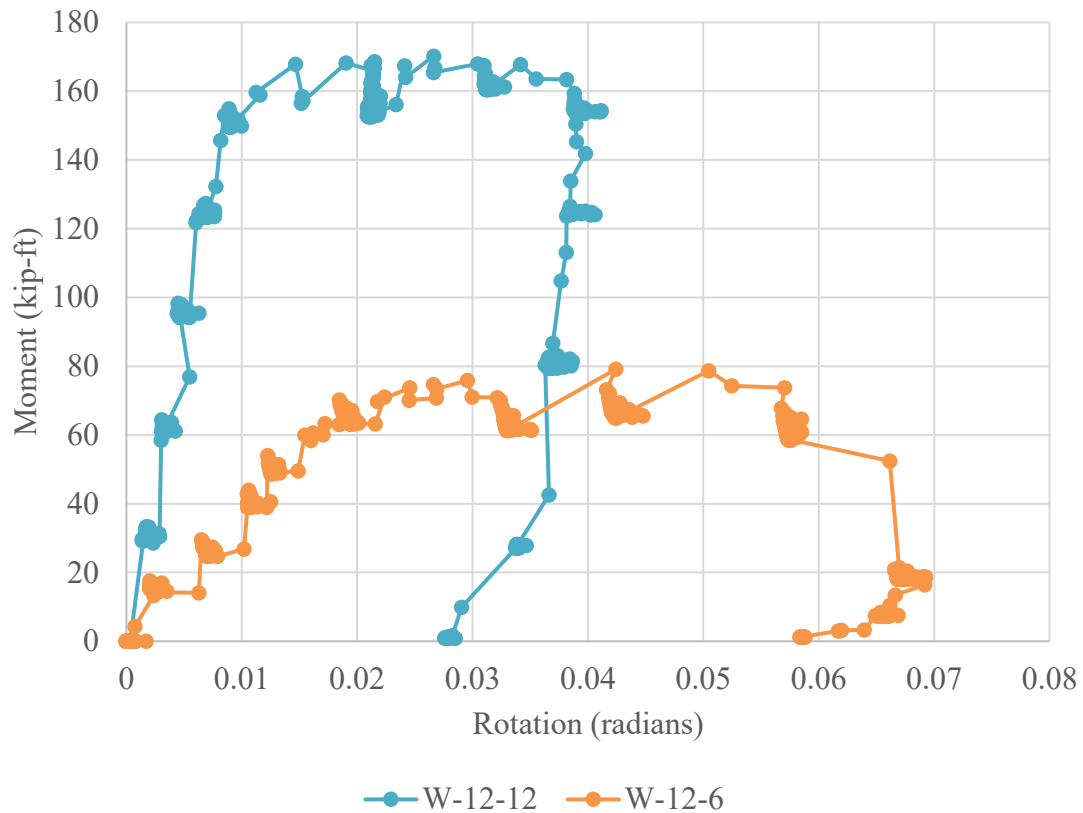


Figure 194. Moment-rotation comparison for load tests performed in specimens W-12-12 and W-12-6, both undamaged.

2.3. W-10 vs W-12 Comparison

Figures 195 and 196 compare specimens from the W-10 and W-12 series at matched embedment-to-pile-depth ratios: $L_e/d = 1.0$ (W-10-10 vs. W-12-12) and $L_e/d = 0.5$ (W-10-5 vs. W-12-6). This pairing isolates the effect of pile section from embedment geometry.

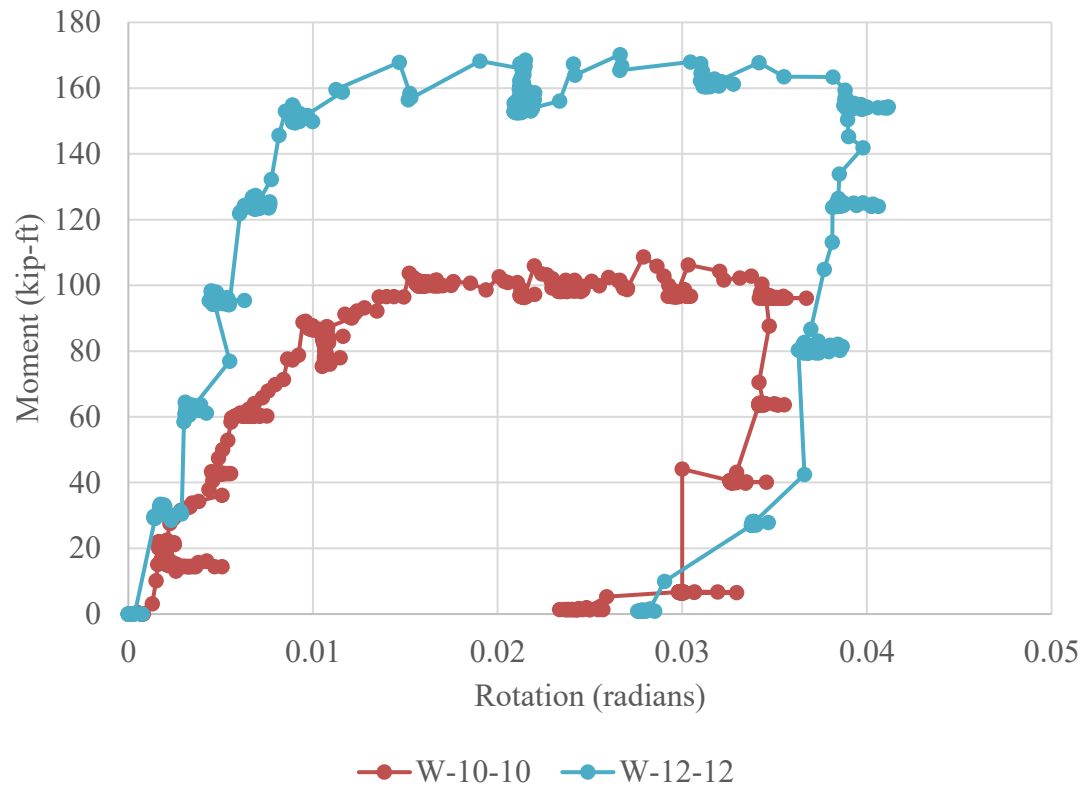


Figure 195. Moment-rotation comparison for specimens with $L_e/d = 1$ (W-12-12 and W-10-10).

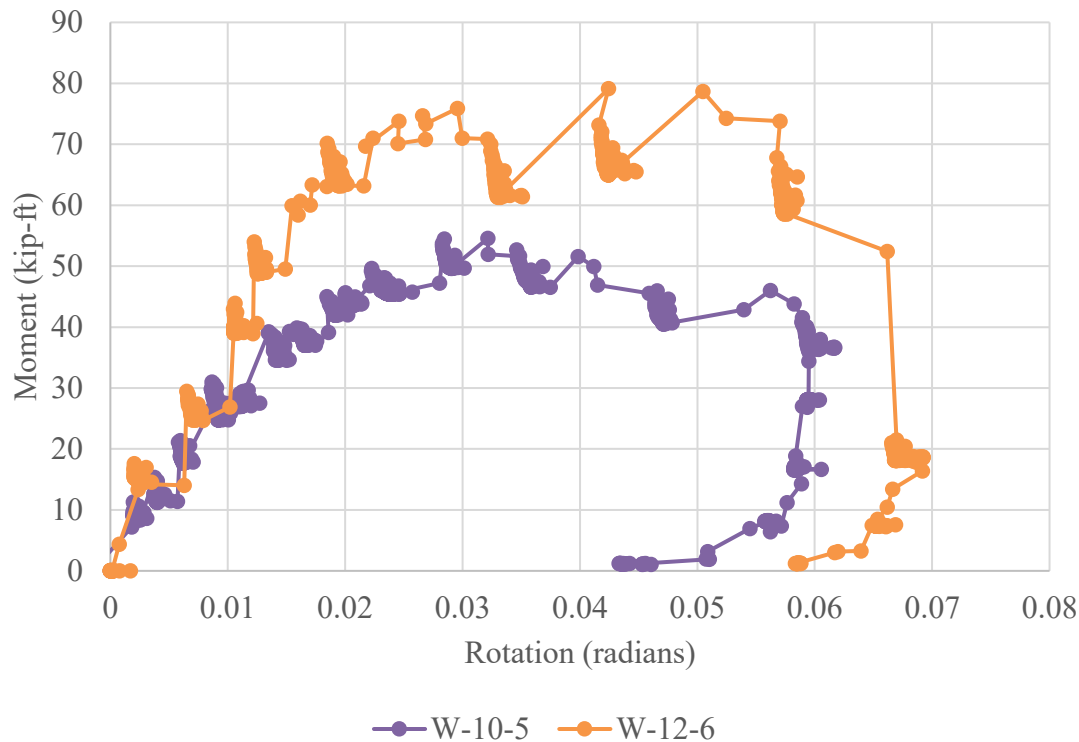


Figure 196. Moment-rotation comparison for specimens with $L_e/d = 0.5$ (W-12-6 and W-10-5).

At $L_e/d = 1.0$, the rotational stiffness of W-10-10 ($K_\theta = 9579$ kip-ft/rad) is 52% of that of W-12-12 ($K_\theta = 18463$ kip-ft/rad). The weak-axis moment of inertia ratio of the HP10x42 to the HP12x53 is approximately 0.56, similar to the observed stiffness ratio of 0.52. At $L_e/d = 0.5$, the K_θ ratio (W-10-5 to W-12-6) is $3195/3912 = 0.81$, compared to the same I_y ratio of 0.56. The agreement is not precise at shallow embedment.

The near-equality of the K_θ ratio and the I_y ratio at $L_e/d = 1.0$ motivates examining whether rotational stiffness scales with pile bending stiffness EI across pile sections at the same L_e/d . Since E is the same for both sections, I_y is the differentiating factor. To examine this directly, the normalized stiffness K_θ/EI is computed for each pair, using $EI = 14,440$ kip-ft² for the HP10x42 and $EI = 25,576$ kip-ft² for the HP12x53.

At $L_e/d = 1.0$, the normalized stiffnesses are 0.663 ft-rad⁻¹ for W-10-10 and 0.722 ft-rad⁻¹ for W-12-12, a difference of approximately 8%. At $L_e/d = 0.5$, they are 0.221 ft-rad⁻¹ for W-10-5 and 0.153 ft-rad⁻¹ for W-12-6, a difference of approximately 30%. This

shows that EI normalization brings two geometrically distinct pile sections to close proximity the hypothesis that rotational stiffness scales approximately with pile bending stiffness at the tested range of L_e/d ratio. However, it is clear that for shallow embedment, this normalization is less precise.

The 30% difference at $L_e/d = 0.5$ is therefore more plausibly attributed to pile cross-sectional geometry effects not fully captured by EI. At shallow embedment, pile dimensions such as flange width and web height directly influence the concrete bearing area and pressure distribution, and these differ between the HP10x42 and HP12x53 in ways that EI alone does not represent. However, EI-proportionality is a reasonable approximation at the tested range of L_e/d ratios. This normalized parameter, and its limitations, inform the regression analysis presented in Section 3.

2.4. S-10-10 and Comparison with Weak Axis

Figure 197 presents the moment-rotation curves for specimen S-10-10 alongside W-10-10 and W-12-12. The primary comparison in this section is between S-10-10 and W-10-10, which share the same pile section (HP10x42) and embedment depth ($L_e/d = 1.0$) and differ only in bending axis. Specimen W-12-12 is included as a contextual reference, as it also carries $L_e/d = 1.0$, allowing the magnitude of the axis effect to be calibrated against the effect of pile section.

The rotational stiffness of S-10-10 ($K_\theta = 14,967$ kip-ft/rad) is 56% higher than that of W-10-10 ($K_\theta = 9,579$ kip-ft/rad). As discussed in Section 1.8, this difference is attributed to the change in bearing geometry between the two orientations: in strong-axis bending, both full flange faces engage the concrete across their entire width, producing a larger and more distributed bearing area than the web-face-dominant contact in weak-axis bending. The increased concrete engagement likely stiffens the connection response under rotation.

The axis change also affects the limit of the linear elastic region. For W-10-10, the linear region ends at approximately 85 kip-ft, approaching the theoretical flange local buckling moment of 92 kip-ft for the HP10x42 section (the pile steel controls). For S-10-10, the linear region extends to approximately 105 kip-ft, a higher moment level, yet the specimen fails by concrete bearing rather than steel yielding. This represents a failure

mode inversion at the same embedment ratio: the configuration with higher rotational stiffness is also the one where concrete, not steel, becomes the limiting mechanism.

The reason for this is that the strong axis bending of the pile provides a structural capacity of 214 kip-ft (compression flange buckling), which is much higher than the connection concrete moment capacity.

The position of S-10-10 relative to W-12-12 in is also informative. S-10-10 ($K_{\theta} = 14,967$ kip-ft/rad) falls between W-10-10 ($K_{\theta} = 9,579$ kip-ft/rad) and W-12-12 ($K_{\theta} = 18,463$ kip-ft/rad), positioned closer to W-12-12. The 56% stiffness gain from the axis change is notably smaller than the 93% gain from upgrading pile section from HP10x42 to HP12x53 at the same L_e/d ratio. Bending axis is therefore a meaningful but secondary variable: it shifts connection stiffness significantly, but pile section remains the larger driver of rotational stiffness at a given embedment ratio.

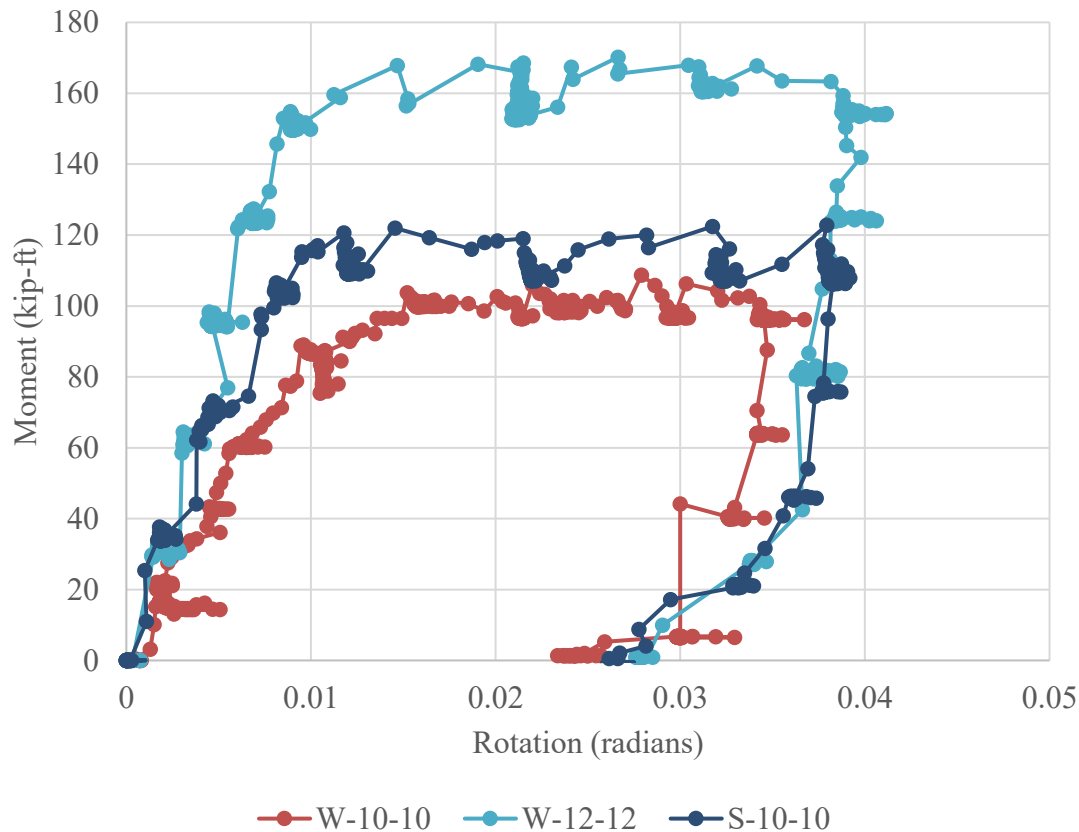


Figure 197. Moment-rotation comparison between S-10-10, W-10-10 and W-12-12.

3. Experimental Rotational Stiffness vs Embedment Relationship

The discussion in the previous section identified the embedment-to-pile-depth ratio L_e/d as the dominant variable governing rotational stiffness, with secondary contributions from pile section properties and bending axis orientation. This section consolidates the experimental observations into an empirical relationship between rotational stiffness and L_e/d that covers the full test matrix.

Figure 198 presents the rotational stiffness K_θ as a function of L_e/d for all load tests, organized into four series: W-10 undamaged, W-10 pre-damaged, W-12 undamaged, and S-10-10 undamaged.

In absolute terms, the W-12 undamaged series falls above the W-10 undamaged series at each comparable L_e/d level, reflecting the higher bending stiffness of the HP12x53 relative to the HP10x42. The pre-damaged W-10 specimens plot consistently

below their undamaged counterparts at both tested embedment depths, consistent with the cumulative stiffness degradation documented in Section 2.1. Specimen S-10-10 plots above the W-10 undamaged series at $Le/d = 1.0$, consistent with the 56% stiffness advantage of the strong-axis orientation established in Section 2.4.

While the general decrease of K_θ with decreasing Le/d is evident across all undamaged series, the separation between pile sections in absolute stiffness prevents drawing a unified relationship from the data.

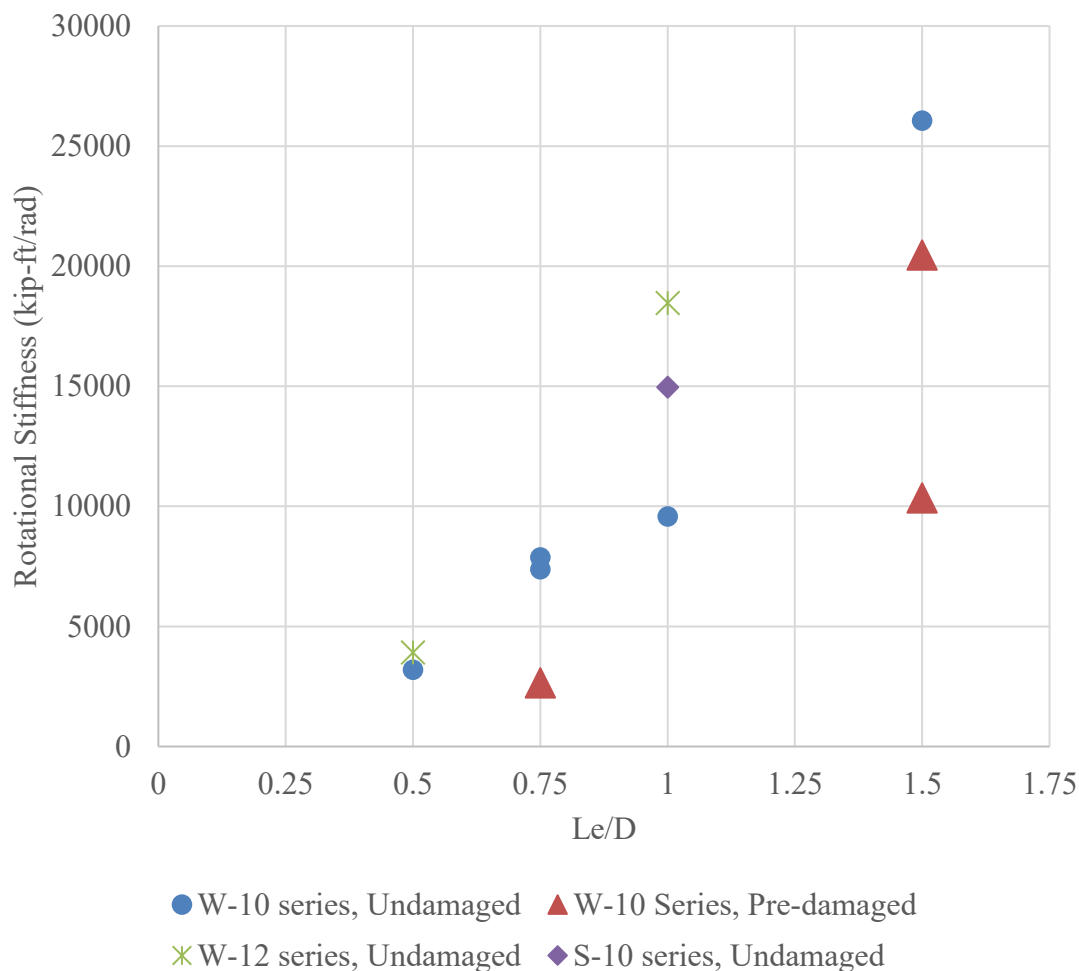


Figure 198. Rotational Stiffness vs Embedment/Pile Width ratio

Figure 199 presents the normalized rotational stiffness K_θ/EI as a function of Le/d , where EI is the bending stiffness of the respective pile section. Three series are shown: the combined W-10 and W-12 undamaged data, the W-10 pre-damaged data, and S-10-

10. The EI normalization substantially reduces the separation between the W-10 and W-12 undamaged series observed in Figure 198, collapsing them into a single band - consistent with the finding in Section 2.3 that rotational stiffness scales approximately with pile bending stiffness at the tested L_e/d range. The pre-damaged series plots below the combined undamaged band, and S-10-10 plots below it as well. The consolidation of the two pile series into a single normalized trend motivates fitting a regression to their combined undamaged data.

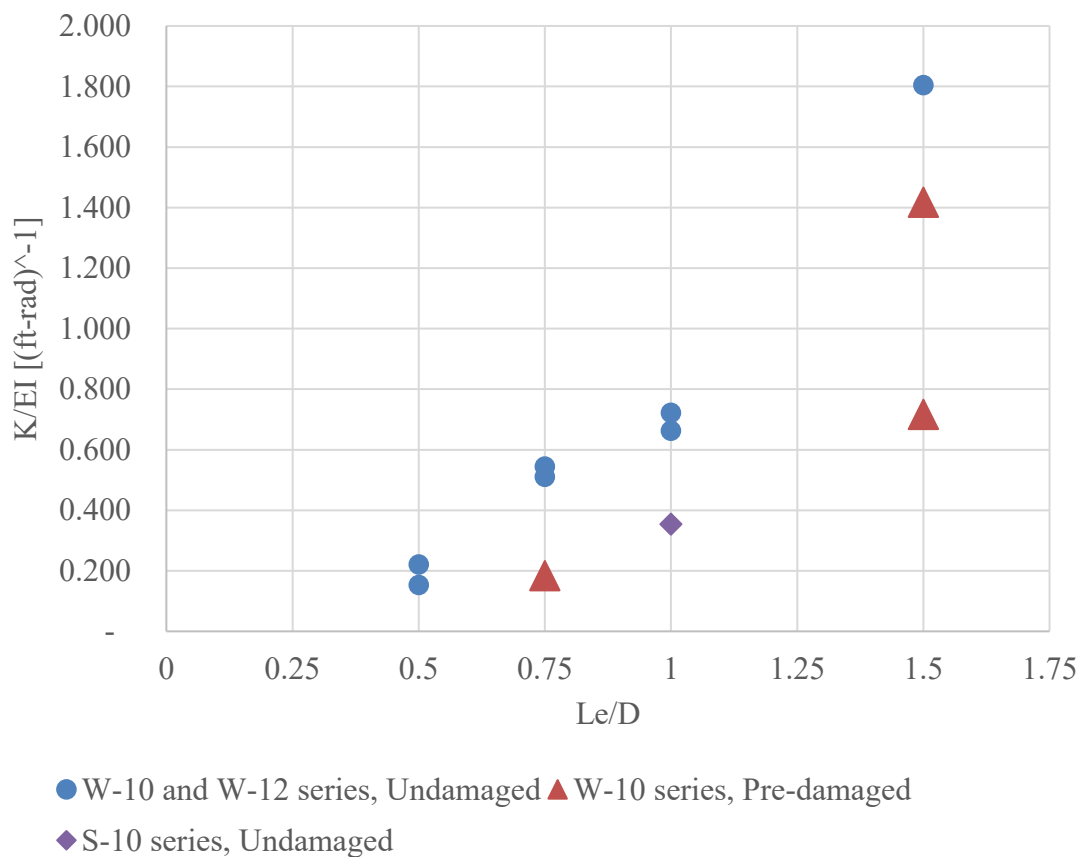


Figure 199. Normalized Rotational Stiffness vs Embedment/Pile Width ratio

Finally, Figure 200 presents the same normalized data as Figure 199 with a regression curve superimposed on the combined W-10 and W-12 undamaged series. A power-law model of the form $y = a \cdot x^n$ was fit to the seven undamaged data points from the W-10 and W-12 series. The resulting regression is:

$$\frac{K_{\theta}}{EI} = 0.78 (\text{ft} * \text{rad})^{-1} \left(\frac{L_e}{D} \right)^2 \quad (45)$$

Where:

K_{θ} = Rotational stiffness (kip-ft/rad)

E = Pile modulus of elasticity (kips/ft²)

I = Pile moment of inertia (ft⁴)

L_e = pile embedment into the concrete cap (in).

D = Pile depth or width (in)

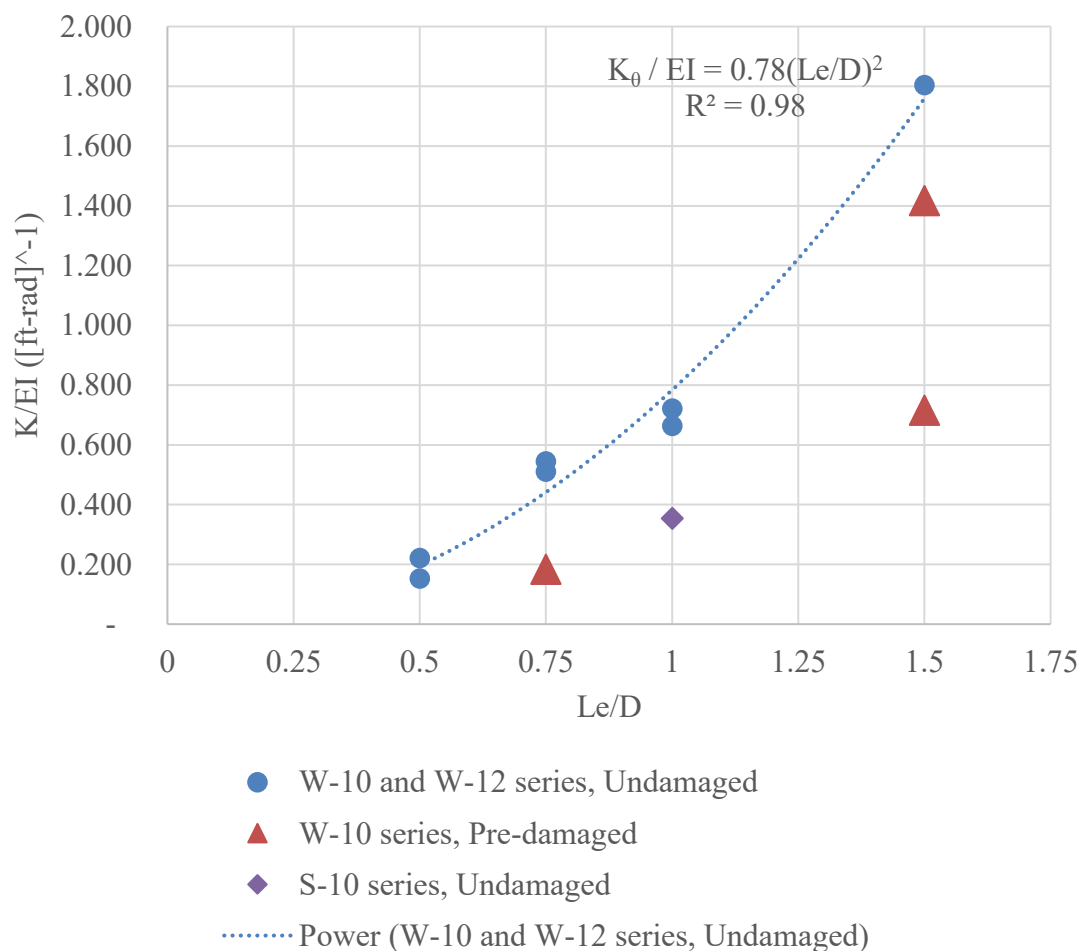


Figure 200. Regression curve on Normalized Rotational Stiffness vs Embedment/Pile Width ratio

The power law form was chosen simply because of a physical reason: as embedment length approaches zero, rotational stiffness theoretically must also approach zero. The power law form reflects that, unlike a polynomial or linear form.

The exponent of 2 suggests that normalized rotational stiffness scales with the square of the embedment ratio. This result has physical intuition: if rotational resistance in the connection is conceptualized as a concrete bearing force couple, both the bearing area and the internal moment arm increase approximately linearly with embedment depth, producing a combined squared dependence on L_e . The relationship is applicable within the tested range of L_e/d from 0.5 to 1.5 and should not be extrapolated beyond these bounds without further experimental validation.

This regression yields a coefficient of determination $R^2 = 0.98$. This means that approximately 98% of the variance in the K_θ/EI variable is accounted for by the chosen regression model. The remaining 2% is attributed to unmodeled scatter due to specimen-to-specimen variability, measurement uncertainty, and geometry effects.

R^2 should be interpreted with caution given the small dataset. In a dataset with 100 points, for example, getting $R^2 = 0.98$ means that the model is genuinely tracking something real and doing it precisely. With only 7 points, $R^2 = 0.98$ is expected whenever the data has any consistent trend at all. Therefore, the high R^2 is likely reflecting that K_θ/EI really does vary strongly and consistently with L_e/D , suggesting a strong and dominant trend, and that EI is a reasonable normalizing parameter. However, it does not indicate whether the chosen power-law form is superior to any other alternative (linear, polynomial, etc.).

Table 23 below shows the comparison of the datapoints with the results from the regression curve.

Table 23. Comparison between measured and fitted K_θ/EI data

Specimen	L_e / D	Measured K_θ/EI [(ft-rad) ⁻¹]	Predicted K_θ/EI [(ft-rad) ⁻¹]	Measured / Predicted
W-10-15 1 st try	1.5	1.804	1.748	1.03
W-10-10	1.0	0.663	0.780	0.85

W-12-12	1.0	0.722	0.780	0.93
W-10-7.5 1 st try	0.75	0.511	0.440	1.16
W-10-7.5R	0.75	0.545	0.440	1.24
W-10-5	0.50	0.221	0.196	1.13
W-12-6	0.50	0.153	0.196	0.78

The table shows that the ratio of measured/predicted data of the undamaged W-10 and W-12 load tests about the regression curve ranges from approximately 0.78 to 1.24. The largest deviation occurs at $L_e/d = 0.5$, where W-10-5 and W-12-6 fall on opposite sides of the regression line.

The three pre-damaged data points (W-10-15 2nd try, W-10-15 3rd try, and W-10-7.5 2nd try) plot below the regression curve in Figure 200. At $L_e/d = 1.5$, the W-10-15 2nd try, which experienced one prior loading event terminated at 67 kip-ft, falls approximately 19% below the regression prediction. The W-10-15 3rd try, following two prior loading events, falls approximately 59% below. At $L_e/d = 0.75$, the W-10-7.5 2nd try falls approximately 58% below the regression prediction, despite having experienced only a single prior loading event, this being consistent the larger rotations imposed during that first try, which caused more severe deformation of the concrete bearing zone than the load level alone would suggest. Across the pre-damaged series, deviations from the undamaged regression range from approximately 19% to 60%, with the magnitude of degradation correlating more closely with the peak rotation reached in prior loading events than with the peak moment.

For the undamaged specimen S-10-10, its normalized rotational stiffness is approximately 55% below the undamaged weak-axis regression curve. Although the rotational stiffness for strong-axis bending is higher than for weak-axis bending, its normalized parameter falls below the weak-axis regression because the higher strong-axis bending stiffness yields a smaller normalized value.

In summary, the presented regression applies only to weak-axis bending and, without further experimental results, should not be extrapolated beyond the tested range of L_e/D from 0.5 to 1.5, or applied for piles other than HP10 and HP12. Additionally, for strong-axis bending, no regression is proposed, only a general recommendation to

consider a 50% reduction of the regression curve result. Finally, for pre-loaded (or pre-damaged) connections, it is generally recommended to reduce the regression curve result by 19% to 59%, depending on the amount of moment previously imposed to the connection.

4. Comparison between Steel H Piles and Squared Prestressed Concrete Piles (Rakestraw et al, 2024)

As seen in the literature review, Rakestraw et al. (2024) performed experimental testing on ten squared prestressed concrete pile specimens, embedded into cast-in-place concrete pile caps. The objective of their work was to determine the moment capacity of the connection at failure. However, their published data also allows for a rotational stiffness analysis of the experiments.

The authors used 18 in and 30 in prestressed concrete piles, as commonly used by the FDOT. Figure 201 shows the basic cross section of the piles and Table 24 and Figure 202 show the rotational stiffness results for the specimens without any axial load or connection interface reinforcement.

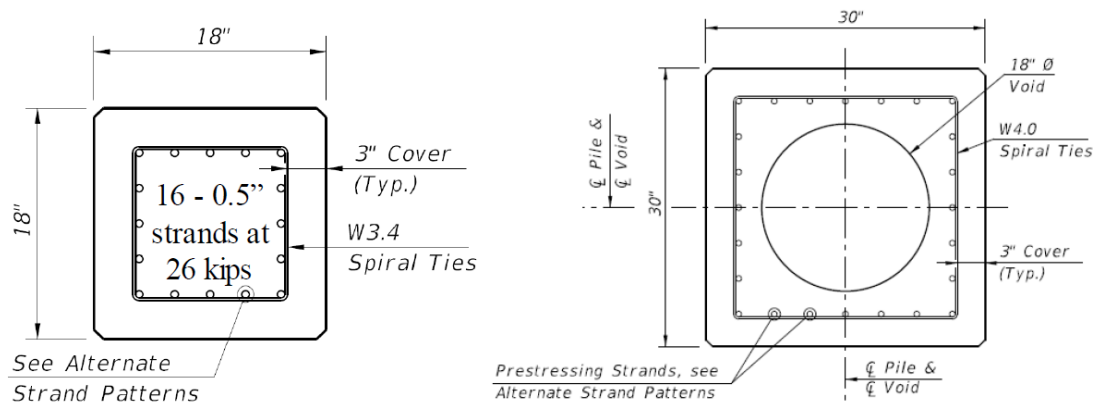


Figure 201. Typical details for 18-in and 30-in piles (Rakestraw et al, 2024)

Table 24. Rakestraw et al. (2024) results

Specimen	Pile Width (in)	Embedment length (in)	Le/D	Failure load	Moment developed (kip-ft)	Rotational Stiffness (kip-ft/rad)
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SP-01	18	6	0.33	9.5	114.12	28,540
SP-04	18	9	0.50	13.6	122.8	60,718
SP-06	18	12	0.67	17.7	159.4	43,117
SP-07	18	18	1.00	33.6	201.4	60,839
SP-08	18	27	1.50	44.6	267.6	131,598
SP-09	30	12	0.40	63.84	574.6	220,949
SP-10	30	30	1.00	96.45	868.1	491,537

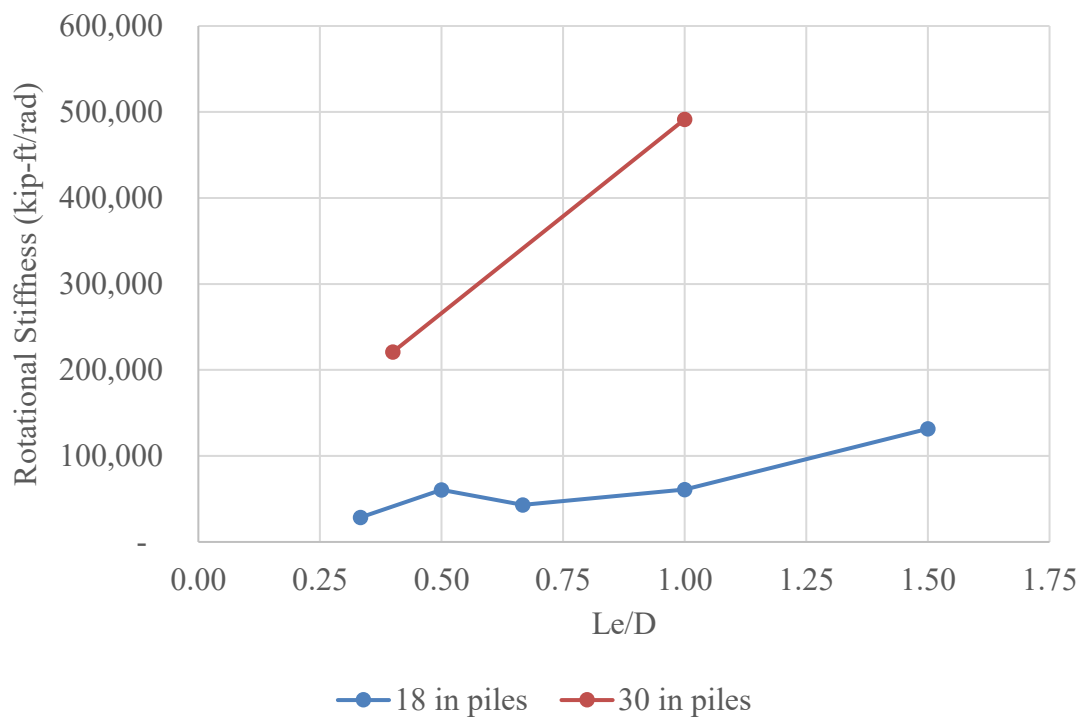


Figure 202. Rotational Stiffness vs Embedment/Pile Width ratio

Following the same process performed in section 3, Table 25 and Figure 203 show the results for the normalized rotational stiffness for the prestressed concrete piles. Figure 204 shows the regression curve used for this type of pile-to-cap connection.

Table 25. Normalized Rotational Stiffness results, deduced from Rakestraw et al. (2024)

Specimen	Pile Width (in)	Embedment length (in)	Le/D	Rot Stiff (kip-ft/rad)	Concrete f'_c (ksi)	E (kips/in ²)	I (in ⁴)	K/EI (ft-rad) ⁻¹
SP-01	18	6	0.33	28,540	11.9	5363	8971	0.085
SP-04	18	9	0.50	60,718	11.74	5334	8973	0.183
SP-06	18	12	0.67	43,117	11.74	5334	8973	0.130
SP-07	18	18	1.00	60,839	11.74	5334	8973	0.183
SP-08	18	27	1.50	131,598	11.74	5334	8973	0.396
SP-09	30	12	0.40	220,949	13.13	5583	63700	0.089
SP-10	30	30	1.00	491,537	13.82	5702	63665	0.195

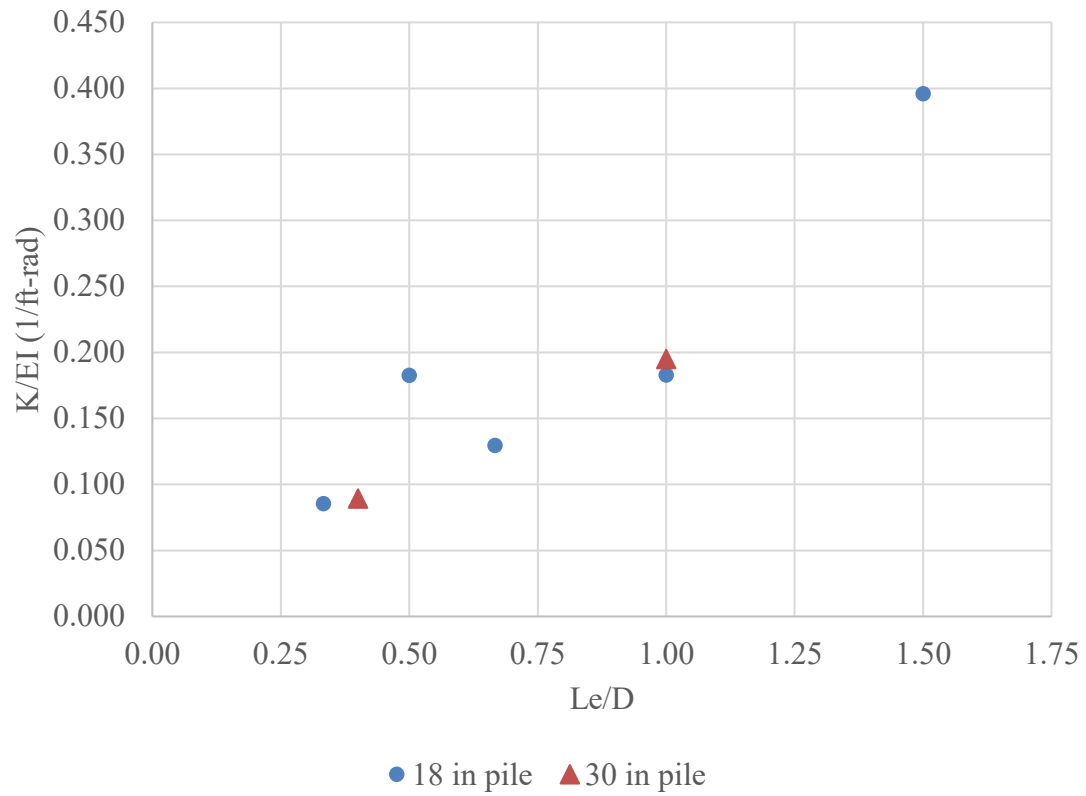


Figure 203. Normalized Rotational Stiffness vs Embedment/Pile Width ratio

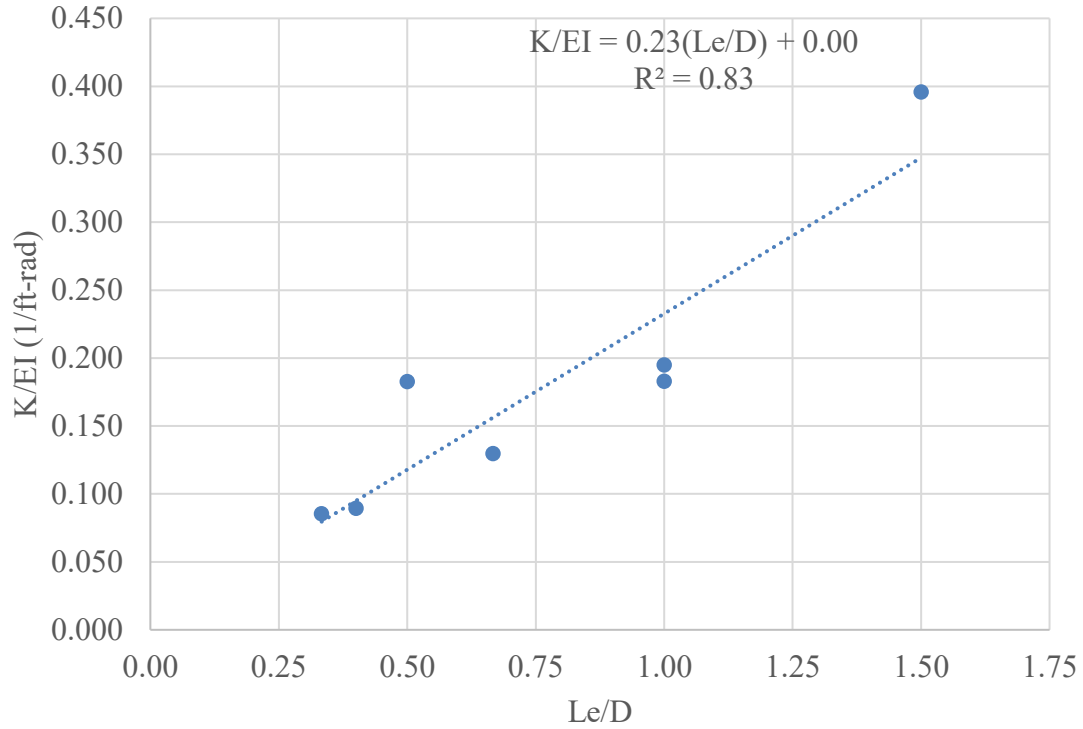


Figure 204. Regression curve on Normalized Rotational Stiffness vs Embedment/Pile Width ratio

For the prestressed concrete piles tested by Rakestraw et al. (2024), the relationship between normalized rotational stiffness and ratio of embedment length to pile width follows a more linear form:

$$\frac{K_{\theta}}{EI} = 0.23 (\text{ft} * \text{rad})^{-1} \left(\frac{L_e}{D} \right) \quad (46)$$

A direct comparison between the two regression relationships reveals meaningful differences in both the form and magnitude of the normalized rotational stiffness, as seen in Figure 205.

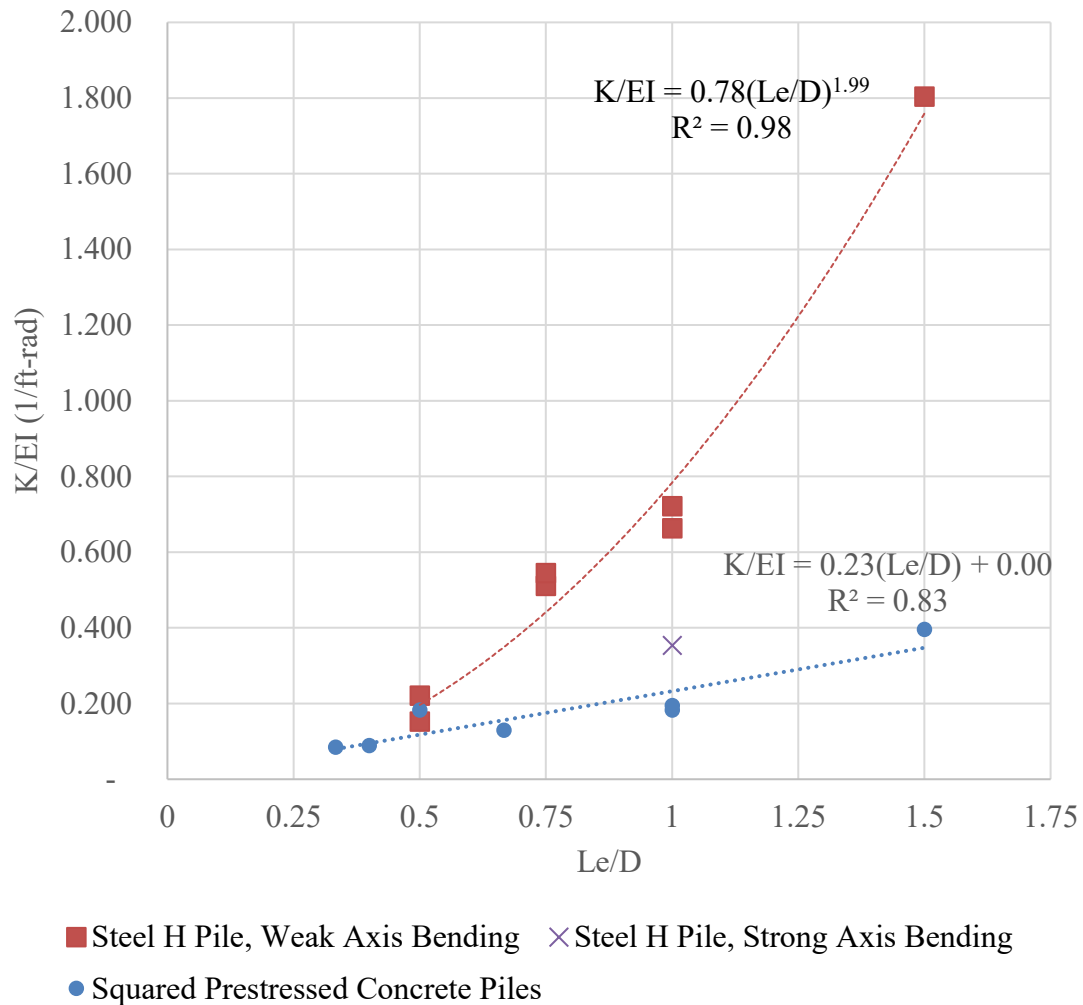


Figure 205. Relationship between K/EI vs Le/D : comparison between Steel H Pile and Squared Prestressed Concrete piles.

The steel H-pile relationship follows a power law dependence on Le/d , while the prestressed concrete pile relationship is approximately linear. This difference in functional form implies that the rate at which normalized stiffness increases with embedment depth is fundamentally different between the two connection types: for H-piles, rotational stiffness per unit pile bending stiffness grows rapidly with increasing embedment, while for the prestressed concrete pile connections, it grows more gradually.

The K/EI variable is substantially higher for steel H-pile than prestressed concrete piles across the entire comparable range. At $Le/d = 0.5$, the H-pile regression predicts a normalized stiffness approximately 70% higher than the prestressed concrete pile

regression. This difference grows to approximately 240% at $L_e/d = 1.0$ and to approximately 410% at $L_e/d = 1.5$. However, in absolute terms, prestressed concrete pile rotational stiffness is a lot higher than H-piles, in any tested range.

One important note is that one of the tested specimens (SP-04) yielded an anomalous result of rotational stiffness (60718 kip-ft/rad). Despite having only a 9-in embedment into the cap, it showed the same rotational stiffness as specimen SP-07 with an 18-in embedment (60839 kip-ft/rad). The authors briefly stated that there was a mix-up during specimen construction, but they did not discuss the matter further.

Both relationships require additional experiments to validate them. Despite the limited number of experiments on both experimental programs, both pile types confirm L_e/D as a meaningful parameter for normalized connection stiffness, with monotonically increasing K_θ/EI as L_e/d increases. The relationships developed in this chapter provide a basis for comparing connection fixity across different pile systems using a common dimensionless framework.

5. Evaluation of Analytical Methods

In this section two analytical methods are evaluated against the experimental rotational stiffness results obtained in this work: the moment-rotation methodology by Kappes et al. (2021) and the four methods proposed by Hammett (2017).

The Kappes et al. (2021) methodology was developed and calibrated exclusively for concrete-filled steel tube (CFST) piles embedded in reinforced concrete pile caps. The pile cross-section in that study was a circular steel tube (219 mm outer diameter) filled with concrete, and the method was validated against nine such specimens. The key model parameter that controls predicted stiffness is the effective length factor L_{eff} , which represents the strain penetration depth in the surrounding concrete and was calibrated against CFST test data to a value of 50.8 cm (20 in). The geometric and mechanical idealization at the core of the method (a cylindrical embedded body rotating as a rigid solid against uniformly discretized radial concrete fibers) is physically appropriate for a filled-tube section. Its extension to HP steel piles, which have an open wide-flange cross-section with fundamentally different bearing geometry, is therefore an extrapolation beyond the method's validated domain.

Hammett (2017), on the other hand, specifically targets HP steel piles embedded in concrete bent caps, and the pile stress transformation block (PSTB) concept is built around the rectangular bearing surface of an H-pile section in bending. Four analytical methods are proposed, differing in the assumed traction force distribution in the link elements that couple the PSTB to the surrounding concrete: Method 1 (linear traction, peak at pile face), Method 2 (uniform traction), Method 3 (linear traction, peak at idealized beam section), and Method 4 (rigid body behavior, strain density approach). Hammett validated his methods against a limited dataset from Rodas et al. (2017), finding that Method 3 overestimated connection stiffness by 11% and 36% for the two available full-scale test results, while Methods 1, 2, and 4 were substantially more unconservative (i.e., predicted even higher stiffness). Based on this, Method 3 was recommended as the best predictor of rotational stiffness and Method 4 was recommended for moment capacity.

For this evaluation, the Kappes et al. (2021) methodology was applied with a friction coefficient $\mu = 0.6$, consistent with the AASHTO-recommended value for steel-concrete interfaces. The effective length factor L_{eff} was calibrated against the specimens in this program that failed by concrete bearing (W-10-5, W-10-7.5, W-12-6 and S-10-10) because only these connections provide simultaneously applicable data for both stiffness and concrete-bearing-governed capacity. The deeper-embedment specimens ($L_e/d \geq 1.0$) failed by steel yielding rather than concrete bearing, so their capacity is controlled by the pile section and not by the connection mechanics that Kappes et al. (2021) models.

The calibration yielded a dimensionally consistent rule: $L_{eff} = 2.0 \times$ pile width for weak-axis bending, and $L_{eff} = 2.5 \times$ pile width for strong-axis bending. This produces $L_{eff} = 20$ in for the HP10×42 (weak axis), $L_{eff} = 24$ in for the HP12×53 (weak axis), and $L_{eff} = 25$ in for S-10-10 (HP10×42, strong axis). Once established, these values were kept fixed and applied to all specimens without further adjustment.

The Kappes et al. (2021) methodology was applied through a spreadsheet provided by the authors, with a few adaptations, such as altering the shape function from “ $1.5 \sin \theta$ ” to simply “1”. This is simply a geometric projection factor that defines how the pile’s lateral displacement translates into bearing pressure on the pile surface, as a function of geometry at the contact interface. With an approximation of considering the H-pile as a flat rectangular plate, its outward normal is always parallel to the direction of pile

movement. Every point on the face will push uniformly into the concrete with full displacement, without angular variation, as it would happen in a circular tube.

The Hammett (2017) methods were applied through the MATLAB implementation described in Appendix C, using the actual measured material properties of each specimen.

The results are organized into three subsections: undamaged weak-axis specimens (the primary dataset), pre-damaged weak-axis specimens, and the strong-axis specimen S-10-10.

5.1. Undamaged Weak-Axis Specimens

Table 26 summarizes the predicted rotational stiffness from each method and the ratio of predicted to measured stiffness for the seven undamaged weak-axis specimens. Figure 206 shows the predicted versus measured stiffness on a log-log scale, with the 1:1 line and $\pm 50\%$ bounds indicated.

Table 26. Predicted and measured rotational stiffness for undamaged weak-axis load tests.

Specimen	Le/d	Measured K_θ (kip-ft/rad)	Kappes ratio	M1 ratio	M2 ratio	M3 ratio	M4 ratio
W-10-15 1st *	1.5	26056	1.76	4.28	3.01	2.3	5.89
W-10-10	1	9579	1.85	5.43	3.62	2.55	5.07
W-10-7.5 1st *	0.75	7376	1.3	3.28	2.12	1.42	2.33
W-10- 7.5R	0.75	7869	0.98	3.08	2	1.34	2.19
W-10-5	0.5	3195	0.67	2.3	1.45	1	1.44
W-12-12	1	18463	1.7	5.56	3.75	2.75	5.94
W-12-6	0.5	3912	0.99	4.21	2.65	1.82	2.81

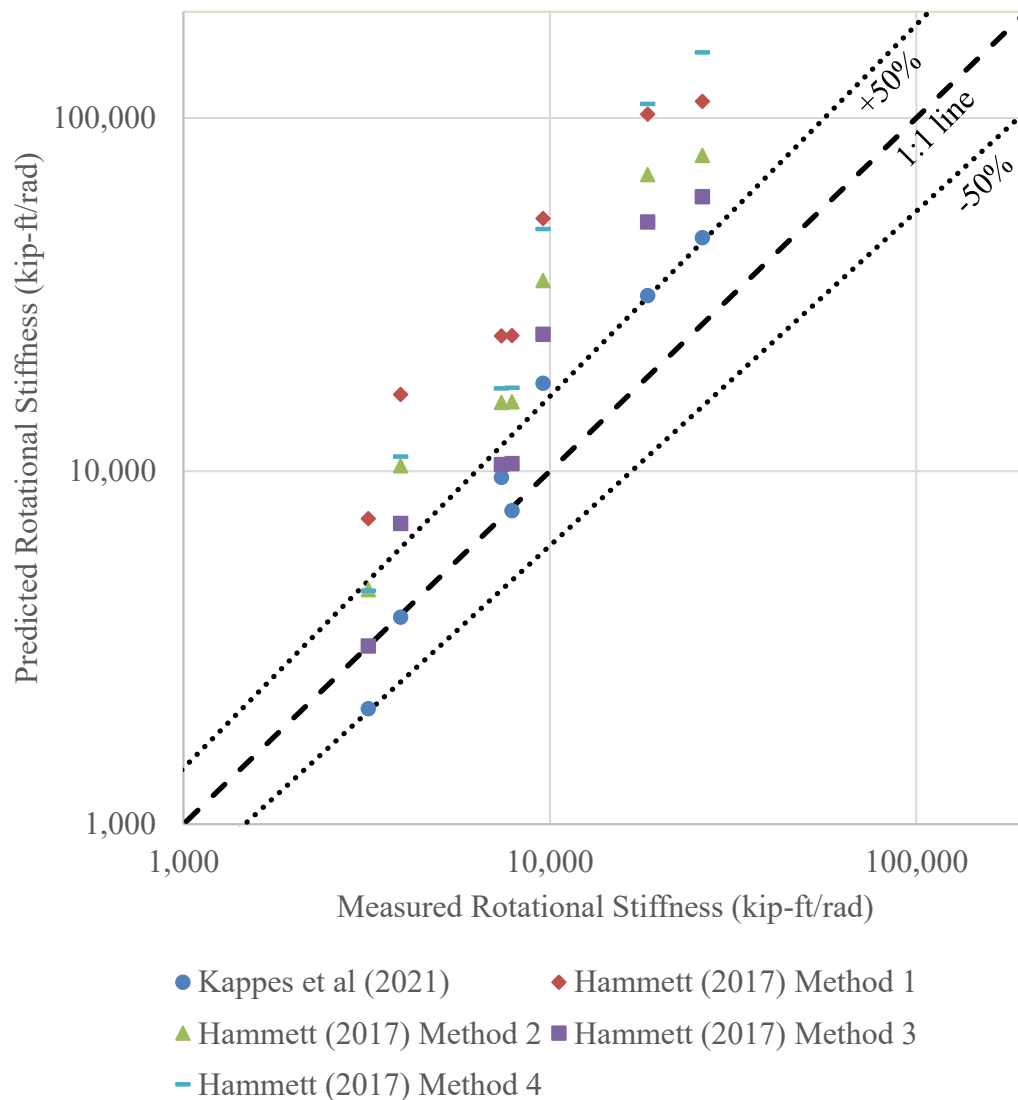


Figure 206. Predicted vs. Measured Rotational Stiffness for undamaged weak-axis load tests.

As observed, all methods proposed by Hammett (2017) consistently over-predict rotational stiffness for every undamaged weak-axis specimen, with P/M ratios ranging from 1.00 to 5.94. Kappes et al. (2021) over-predicts at deeper embedments but under-predicts at shallow embedment, crossing below 1.0 for the two $L_e/d = 0.5$ specimens. Among all methods, Hammett (2017) method 3 and the Kappes et al. (2021) methodology

produce the predictions closest to measured values. Additionally, although Kappes et al. (2021) methodology over-predicted for deeper embedments, its results still fall very close to the $\pm 50\%$ bounds.

Table 27 presents the mean, standard deviation, and coefficient of variation of the P/M ratio across the seven undamaged weak-axis specimens for each method.

Table 27. Statistical summary of predicted/measured ratios for undamaged weak-axis load tests

Method	Mean P/M	SD	CV (%)	Mean abs. deviation (%)
Kappes	1.32	0.46	34.9	42.6
Hammett M1	4.02	1.21	30.2	301.9
Hammett M2	2.66	0.86	32.4	165.6
Hammett M3	1.88	0.67	35.4	88.2
Hammett M4	3.67	1.9	51.9	266.5

Kappes et al. (2021) produces the lowest mean P/M ratio (1.32) and the lowest mean absolute percent deviation (42.6%), with four of seven specimens falling within $\pm 50\%$ of the measured value. Hammett (2017) method 3 produces a mean P/M of 1.88 and mean absolute percent deviation of 88.2%, with only three of seven specimens within $\pm 50\%$. Kappes et al. (2021) is closer to the measured value than Hammett (2017) method 3 for six of the seven undamaged weak-axis specimens. The exception is W-10-5 (Le/d

= 0.5), where Hammett (2017) method 3 achieves a near-perfect P/M of 1.00 while Kappes et al. (2021) significantly underestimates at P/M = 0.67.

All Hammett (2017) methods are consistently unconservative ($P/M > 1$ for all seven undamaged weak-axis load tests) as they predict a stiffer connection than reality. Kappes et al. (2021) is also unconservative for $Le/d > 0.75$. At $Le/d = 0.75$, Kappes et al. (2021) transition to more accurate or conservative behavior ($P/M \leq 1$), where it predicts a more flexible connection than reality. Hammett (2017) method 3 is uniformly unconservative but never severely so within the undamaged test specimens, with P/M ranging from 1 to 2.75.

Figure 207 and Table 28 show the P/M ratio as a function of Le/d for each method.

Table 28. Average P/M ratio by Le/d group, for undamaged weak-axis load tests.

Le/d	n	Kappes	M1	M2	M3	M4
1.5	1	1.76	4.28	3.01	2.3	5.89
1	2	1.78	5.49	3.69	2.65	5.5
0.75	2	1.14	3.18	2.06	1.38	2.26
0.5	2	0.83	3.26	2.05	1.41	2.13

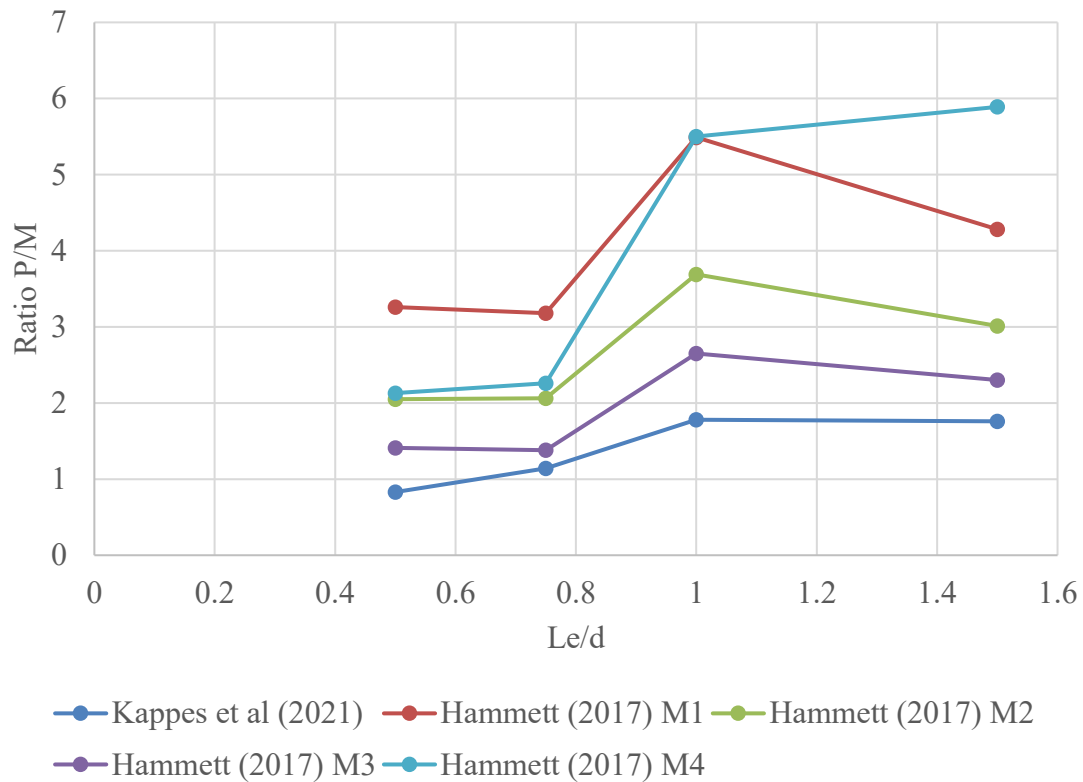


Figure 207. P/M Ratio vs. Le/d for undamaged weak-axis load tests.

As observed, a decreasing trend is apparent in both Kappes et al. (2021) and all Hammett (2017) methods: the ratio is largest at $Le/d = 1.0$ to 1.5 and smallest at $Le/d = 0.5$.

This trend suggests a systematic structural problem in both methodologies when applied to plain HP-pile embedment connections: both models over-amplify the stiffness gain from increasing embedment depth relative to what the experiments show. The experimental data in Section 3 demonstrated that K_{θ}/EI scales approximately with $(Le/d)^2$, a squared relationship. If the analytical methods were perfectly calibrated to the correct Le/d sensitivity, the P/M ratio would remain constant across embedment depths. The decreasing trend indicates that both methods predict a stiffness that grows faster with Le/d than the experiments show, meaning they are close at shallow embedment but become progressively more unconservative as embedment depth increases.

Kappes et al. (2021) methodology is highly sensitive to L_{eff} parameter, becoming a limitation to the method. The L_{eff} value is geometry dependent and must be calibrated for each new pile type and embedment range.

Considering specimens W-10-5 vs W-12-6, both of them share $L_e/d = 0.5$ and fail by concrete bearing, but the predictions diverge for the two pile sections. For Kappes et al. (2021), $P/M = 0.67$ for W-10-5 vs 0.99 for W-12-6. For Hammett (2017) method 3, $P/M = 1.00$ for W-10-5 vs 1.82 for W-12-6. The two specimens have relatively similar measured stiffness ($3,195$ vs $3,912$ kip-ft/rad), yet the predicted stiffness differs substantially, primarily because both methods scale with pile section properties (pile bending stiffness EI for Hammett, 2017), and pile depth and surface area for Kappes et al, 2021). These pile-size corrections work well at deeper embedment where the stiffness is dominated by concrete bearing over a large area, but at very shallow embedment the pile geometry (flange width, web height) likely drives the response in ways not fully captured by either 2D model. The 30% difference in K_θ/EI between W-10-5 and W-12-6 at $L_e/d = 0.5$ (noted in Section 3) is the experimental evidence that cross-sectional geometry effects beyond EI are non-negligible at shallow embedment, a regime that neither method handles with precision.

The systematic overestimation by all Hammett (2017) methods, and the partial overestimation by Kappes et al. (2021) likely have a few contributing factors:

- Both models assume an idealized concrete bearing surface in full contact with the pile face from the beginning of loading. In practice, there is a finite gap tolerance at the steel-concrete interface resulting from placement and concrete shrinkage. Even if small, this gap delays the start of concrete bearing engagement and reduces measured initial stiffness relative to the model. This effect would be most pronounced at small rotations, precisely in the linear elastic range where the experimental stiffness is measured.
- Both models initially use the elastic (initial tangent) modulus of the concrete in their spring stiffness calculations. As iterations proceed, the tangent modulus is updated to the value corresponding to the current stress level. At the low stress levels characteristic of the linear elastic range, this tangent modulus

remains close to E_c . However, any overestimation of E_c can propagate directly into an overestimation of K_θ .

5.2. Pre-Damaged Specimens

The pre-damaged specimens (W-10-15 2nd and 3rd tries, and W-10-7.5 2nd try) represent connections that experienced one or more prior loading events before the test being evaluated.

Neither the Kappes et al. (2021) nor the Hammett (2017) methods include any mechanism to model stiffness degradation from prior loading, both assuming an undamaged concrete bearing zone.

Therefore, applying either method to pre-damaged specimens is formally outside the intended use of the methods, and the predicted values correspond to the expected undamaged stiffness, not the actual degraded stiffness. The comparison is nevertheless informative, since it quantifies the magnitude of the mismatch that would arise if either method were applied without knowledge of loading history.

Table 29 summarizes the predicted-to-measured ratios for the three pre-damaged specimens.

Table 29. Predicted/measured ratios for pre-damaged weak-axis load tests.

Specimen	Le/d	Measured K_θ (kip-ft/rad)	Kappes ratio	M1 ratio	M2 ratio	M3 ratio	M4 ratio
W-10-15 2 nd	1.5	20467	2.24	5.45	3.83	2.93	7.49
W-10-15 3 rd	1.5	10350	4.43	10.77	7.58	5.79	14.82
W-10-7.5 2 nd	0.75	2654	3.46	9.56	6.1	4.06	6.75

As expected, the P/M ratios for pre-damaged specimens are uniformly and substantially higher than those for undamaged specimens. Across all methods and

specimens, every pre-damaged P/M ratio exceeds the corresponding undamaged ratio for the same embedment depth, and by a margin that increases with the severity of the prior damage. For specimen W-10-15, the predicted-to-measured ratio for Hammett (2017) method 3 increases from 2.30 (1st try, undamaged), to 2.93 (2nd try, one prior event), to 5.79 (3rd try, two prior events). The corresponding Kappes et al. (2021) ratios are 1.76, 2.24, and 4.43. For W-10-7.5, the single prior loading event raised the Hammett (2017) method 3 P/M ratio from 1.42 (undamaged, 1st try) to 4.06 (pre-damaged, 2nd try). For Kappes et al. (2021), the ratios went from 1.30 to 3.46. These escalating ratios reflect the progressive stiffness degradation of the connection concrete, which neither method can account for.

The magnitude of mismatch for the 3rd try of W-10-15, where two prior loading events reduced the measured stiffness from 26,000 kip-ft/rad to 10,350 kip-ft/rad, is notable. Hammett (2017) method 3 predicts 59,966 kip-ft/rad, nearly 6 times the measured value. Kappes et al. (2021) predicts 45,898 kip-ft/rad, more than 4 times the measured value. This level of discrepancy would make either prediction meaningless for a pre-loaded connection.

This finding has a practical implication. In field applications, a bridge bent that has been subjected to prior lateral loading from construction activities, traffic, or hydraulic events will have a degraded connection stiffness that neither method will predict accurately if the loading history is unknown. The degradation factors noted in Section 2.1 (stiffness reductions of 19% to 60% for a single prior loading event) suggest that applying the undamaged predictions with an explicit degradation factor may be the most practical approach for pre-loaded connections.

Based on the experimental data in Section 3, a reduction of 19 to 65% to the undamaged prediction appears appropriate depending on the intensity and number of prior loading events, with the larger reduction applying to shallower embedments or higher prior rotation demands.

5.3. Strong-Axis Specimens

Table 30 presents the predicted and measured rotational stiffness for specimen S-10-10 under strong-axis bending.

Table 30. Predicted/measured ratio, S-10-10 (strong-axis bending, $L_e/d = 1.0$)

Method	Predicted K_θ (kip-ft/rad)	Measured K_θ (kip-ft/rad)	P/M Ratio
Kappes	13050	14967	0.87
M1	49644		3.32
M2	32207		2.15
M3	21041		1.41
M4	41858		2.8

The Kappes et al. (2021) methodology fits the experimental results notably better for S-10-10 than for any weak-axis specimen at comparable embedment depth, slightly underestimating the measured stiffness by 13% ($P/M = 0.87$). For comparison, at $L_e/d = 1.0$, the weak-axis Kappes et al. (2021) P/M ratios are 1.76 (W-10-15) and 1.85 (W-10-10) for the HP10×42, and 1.70 (W-12-12) for the HP12×53.

This methodology assumes a cylindrical steel pile rotating rigidly against surrounding concrete discretized with radial fibers over the full perimeter. In weak-axis bending of an HP pile, the actual bearing zone is concentrated on the web face, with an effective depth smaller than the actual pile depth, and localized stresses concentrated in the flange tips. These mechanisms are not accounted for in the methodology.

In strong-axis bending, however, both full flanges engage the concrete across their entire width simultaneously, with the leading flange engaging the concrete as a flat rectangular surface. This creates a more distributed bearing pattern that, although not calibrated by the methodology, still generates more consistent results. Also, the more distributed bearing in strong-axis bending also means the pile itself behaves more like a rigid body within the cap, which is a fundamental assumption of the Kappes rigid-rotation model. This geometry alignment, combined with the larger $L_{eff} = 25$ in used for the strong-axis prediction, leads to a prediction that slightly underestimates the measured value rather than overestimating it.

Among the Hammett (2017) methods, Method 3 produces $P/M = 1.41$ for S-10-10, which is actually closer to 1.0 than its weak-axis performance at $L_e/d = 1.0$ (P/M of 2.55 for W-10-10 and 2.75 for W-12-12). This is consistent with the larger pile bending stiffness in the strong axis ($EI_{\text{strong}} \approx 6,090,000 \text{ kip-in}^2$ vs $EI_{\text{weak}} \approx 2,079,300 \text{ kip-in}^2$ for the HP10×42), which makes the pile more rigid within the embedment zone, moving actual behavior closer to the rigid-body assumption that Hammett's framework is built on. Although the improvement is not substantial (1.41 vs 2.55), the direction is consistent with the hypothesis that the Hammett (2017) model performs better when the pile is more rigid.

However, the Hammett (2017) methodology was still claimed to be developed for weak-axis bending. Therefore, the closer prediction for strong-axis bending may just be a coincidence, requiring more experimental tests for verification.

6. Experimental Fixity Evaluation

In this subsection, the idealized analytical approach presented by Shishlov et al. (2022), and reviewed in the literature review, is used to demonstrate how far the experimental rotational stiffness values for steel H-piles obtained in this work are to theoretical fixity.

$$k_{fh} = \frac{2EI}{L\left(\frac{1}{\eta_{fh}} - 1\right)} \quad (47)$$

$$k_{\text{partial}} = \frac{2EI}{L\left(\frac{1}{\Lambda * \eta_{fh}} - 1\right)} \quad (48)$$

Initially, a theoretical fixed-head rotational stiffness was required to be adopted. Since this parameter tends to infinity in a pure fixed-head condition, it was estimated based on a simple hand calculation of a fixed-free column - with the same properties as the piles studied in this work - subjected to lateral load 7ft away from the base. The rotational stiffness approximation was based on the moment and lateral deflection

measured at 1 in above the base, simulating the C1 level in the tested specimens in this work. Figure 208 shows the model used.

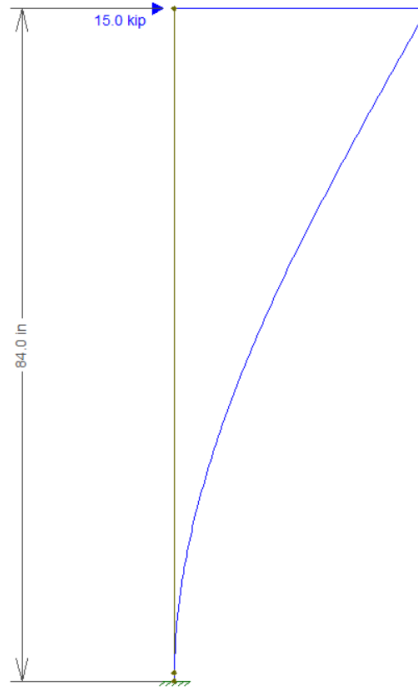


Figure 208. Fixed-free column deformation model for rotational stiffness calculation.

Table 31 shows the adopted fixed-head rotational stiffness for each experimental series.

Table 31. Adopted theoretical Fixed-head rotational stiffness

Specimen	Rotational Stiffness K_{fixed} (kip-ft/rad)
W-10 series	347,960
S-10 series	1,019,131
W-12 series	616,331

For application of the Shishlov et al. (2022) idealized approach, the unsupported equivalent length (L_e) was estimated as 13 ft using the theory proposed by Davisson and Robinson (1965), with a few assumptions for demonstration purposes.

Finally, the experimentally obtained rotational stiffness (K_{partial}) values for steel H-piles are input into Shishlov et al. (2022) idealized equation and the degree of fixity (Λ) for each specimen is obtained. Figures 209, 210 and 211 show the results for the ratio $K_{\text{partial}}/K_{\text{fixed}}$ vs degree of fixity.

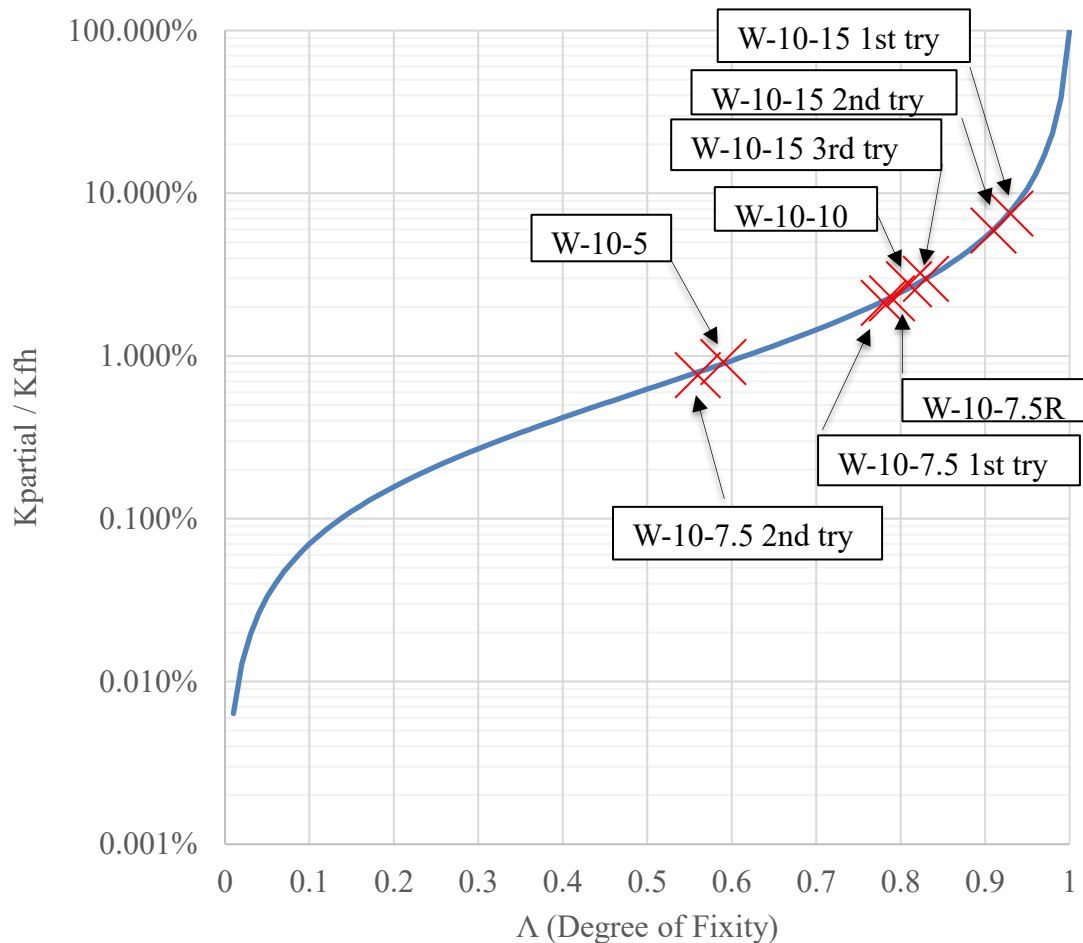


Figure 209. Degree of fixity estimates for W-10 series

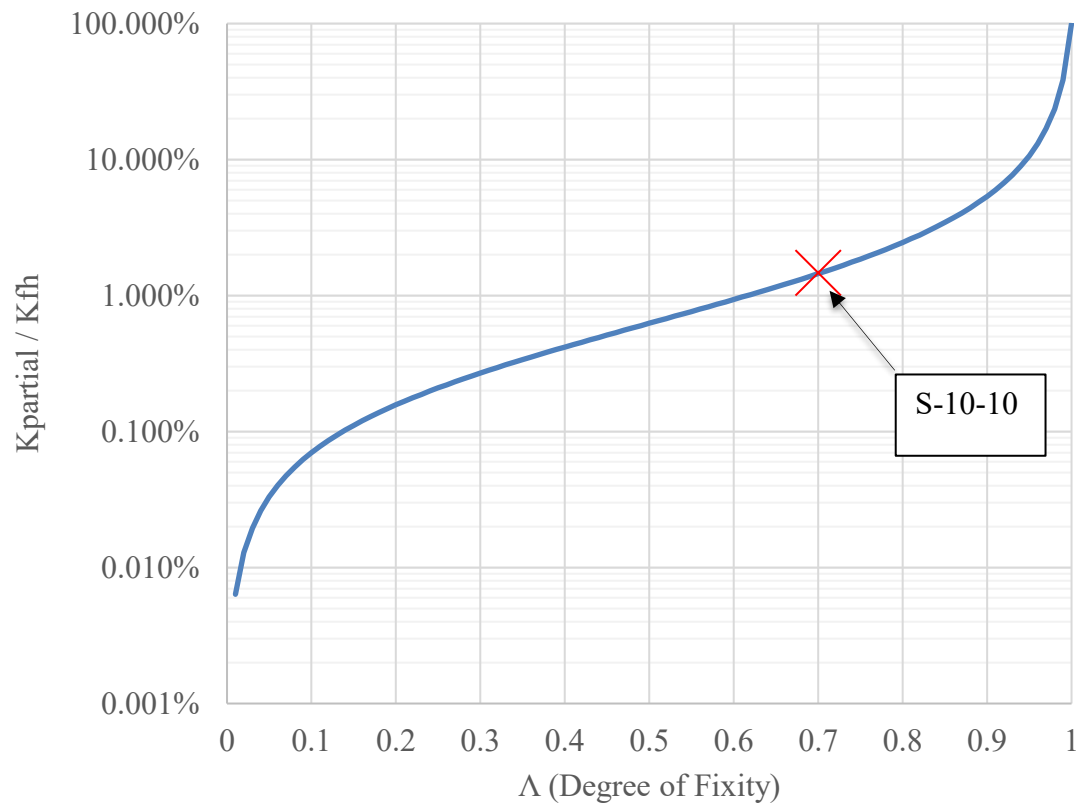


Figure 210. Degree of fixity estimates for S-10 series

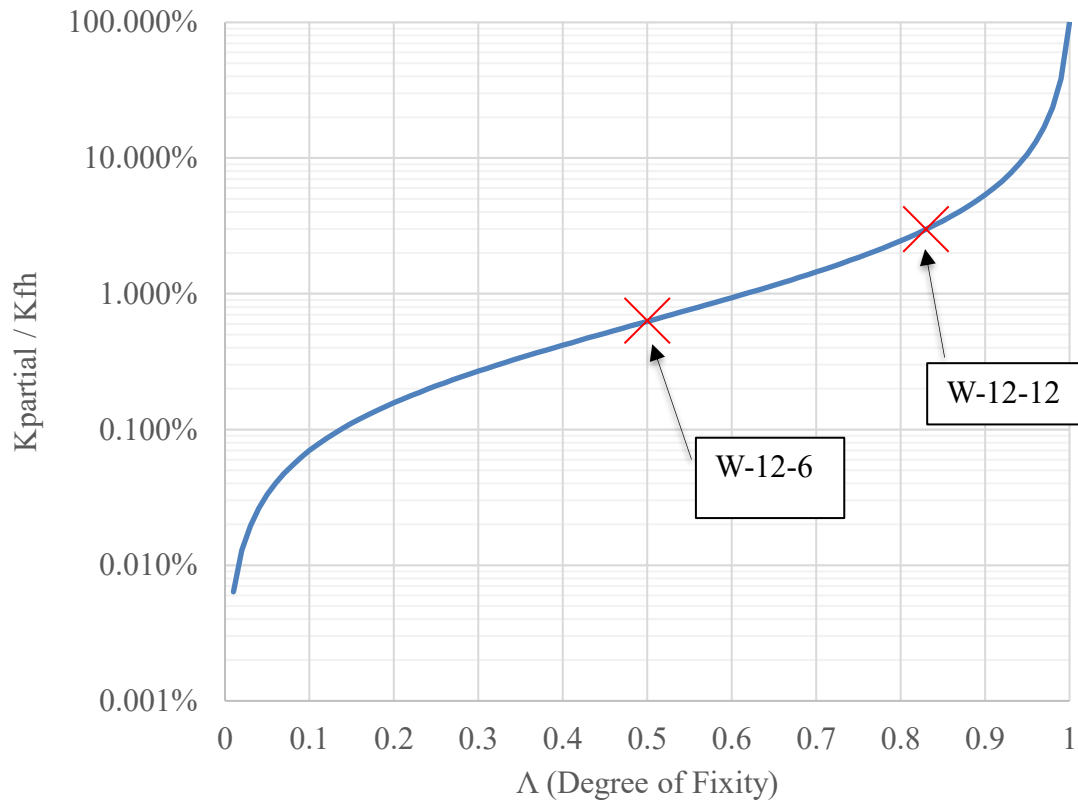


Figure 211. Degree of fixity estimates for W-12 series

The most interesting result from the above figures is the contrast between the $K_{\text{partial}}/K_{\text{fixed}}$ and Λ . Across all specimens, the experimental stiffness represents less than 7.5% of the theoretical fixed-head reference. However, the corresponding degree of fixity ranges from 50% to 93%. This apparent paradox is a direct consequence of the nonlinear shape of the Λ vs. $K_{\text{partial}}/K_{\text{fixed}}$ curve. The curve rises steeply at low $K_{\text{partial}}/K_{\text{fixed}}$ values, meaning that even a modest rotational stiffness captures a large portion of the theoretical fixity spectrum. Conversely, the final portion of the curve (approaching $\Lambda = 1.0$) is asymptotically steep, requiring arbitrarily large stiffness increments to achieve diminishing fixity gains. In practice, this region of the curve is physically unattainable. The implication is that reporting Λ without also reporting $K_{\text{partial}}/K_{\text{fixed}}$ can be misleading: a connection described as "93% fixed" is still operating at only 7.5% of the theoretical fixed-head stiffness.

Despite the absolute magnitudes being small, the degree of fixity follows a clear and consistent trend with L_e/d across the undamaged specimens. Deeper embedment produces higher Λ , as expected from the rotational stiffness results in Section 3. For the W-10 series, Λ ranges from 59% at $L_e/d = 0.5$ to 93% at $L_e/d = 1.5$, a 34% spread driven primarily by embedment geometry. The W-12 series shows the same direction, from 50% at $L_e/d = 0.5$ to 83% at $L_e/d = 1.0$. Notably, the fixity gains are larger in the range $L_e/d = 0.5$ to 1.0 than above $L_e/d = 1.0$.

Damage from prior loading events reduces Λ measurably but not uniformly. For the W-10-15 sequence, successive loading events reduced Λ from 93% to 91% to 83%, moderate absolute reductions despite the large percentage drops in K_θ as seen in section 2. The nonlinear fixity curve partially diminishes the effect of stiffness loss at high Λ values: a 60% drop in K_θ (from try 1 to try 3) translates to only a 10% drop in Λ . For W-10-7.5, the effect is more severe as it falls in the more linear portion of the fixity curve: a 65% drop in K_θ (from try 1 to 2) generated a 22% drop in Λ .

Specimen S-10-10 presents an instructive contrast with W-10-10. Despite having a 56% higher absolute rotational stiffness (14,967 vs. 9,579 kip-ft/rad), S-10-10 yields a lower degree of fixity (70% versus 81%). The explanation lies in the adopted theoretical fixed-head rotational stiffness for strong-axis bending ($K_{\text{fixed}} = 1,019,131$ kip-ft/rad), which is approximately three times that for weak-axis bending (347,960 kip-ft/rad). This result highlights a limitation of the degree of fixity metric when applied across different bending axes: a connection that is physically stiffer may appear "less fixed" simply because the theoretical reference scales with EI and, therefore, is much larger for strong-axis bending.

None of the tested connections behaved as pinned, and none reached full theoretical fixity. The lowest observed Λ is 50% (W-12-6, $L_e/d = 0.5$), and the highest is 93% (W-10-15 1st try, $L_e/d = 1.5$). Design approaches that assign a binary pinned or fixed boundary condition to these connections will misrepresent their actual behavior in both cases. The pinned assumption overestimates deformation and underestimates moment load at the connection, while the fixed assumption may overestimate the connection's rotational restraint contribution to the structural system. The partial fixity framework, though dependent on assumptions for K_{fixed} and pile length, at least provides a rational

basis for situating experimental connections within the pinned-to-fixed spectrum using measured data.

The degree of fixity results in this section were calculated and presented for demonstration purposes. The unsupported pile length, for example, was assumed for illustration purposes and would vary with site-specific soil conditions. More critically, the results are highly sensitive to the assumed theoretical fixed-head stiffness K_{fixed} : small changes in K_{fixed} can produce disproportionately large changes in Λ . The framework is therefore more meaningful for comparing specimens relative to each other than for assigning absolute fixity values to any specific field connection.

7. Translation Effects

The rigid body analysis presented in Section 1.1 confirmed, for all specimens, that the pile deflection profile is only consistent with rigid body rotation at the connection level (POTs C1 and C6), as they produce collinear deflection profiles throughout loading. While this validates the rotation measurement methodology, it does not eliminate the simultaneous presence of a translational degree of freedom at the connection, due to concrete deformation - bearing zone compression on the leading side, and gap formation on the trailing side. A connection exhibiting both rotational and translational flexibility is not fully characterized by K_{θ} alone. This section examines the translational component of cap displacement across all eleven load tests.

Lateral translation at the cap face is computed by decomposing the POT C1 reading into its rotational and translational components. The total displacement measured by POT C1 is:

$$POT_C1 = u + \theta * L \quad (49)$$

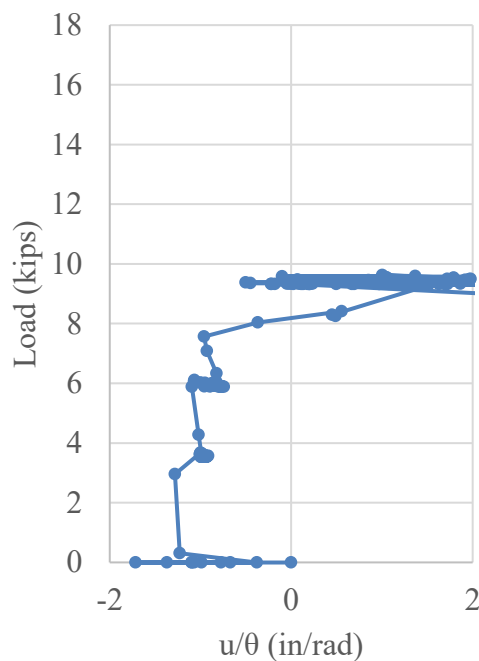
Where:

u = pure lateral translation

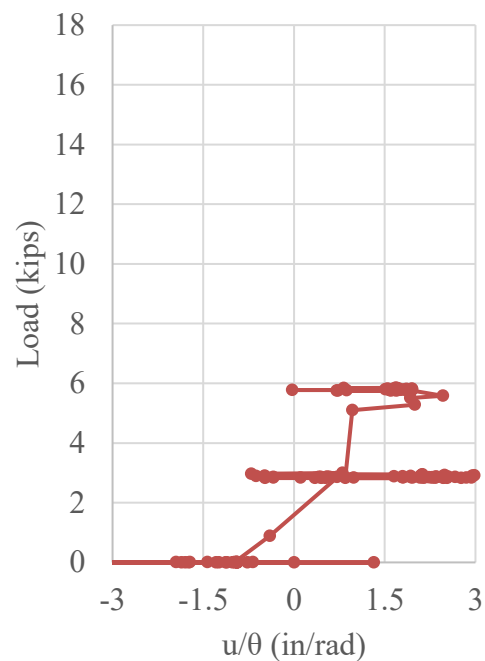
θ = rotation measured from inclinometer at C1 (TILT_C1) corrected by the cap rotation itself (TILT_CAP).

L = distance from the inclinometer mounting point to the cap surface. For TILT_C1, $L = 1$ in

After solving for u , the ratio u/θ becomes the quantity of interest. It expresses how much translation (in inches) accompanies unit rotation (in radians), providing a dimensionally consistent measure of the relative magnitude of the translational and rotational components. Figures 212, 213 and 214 show u/θ as a function of applied load for all eleven load tests.



W-10-15 1st try



W-10-15 2nd try

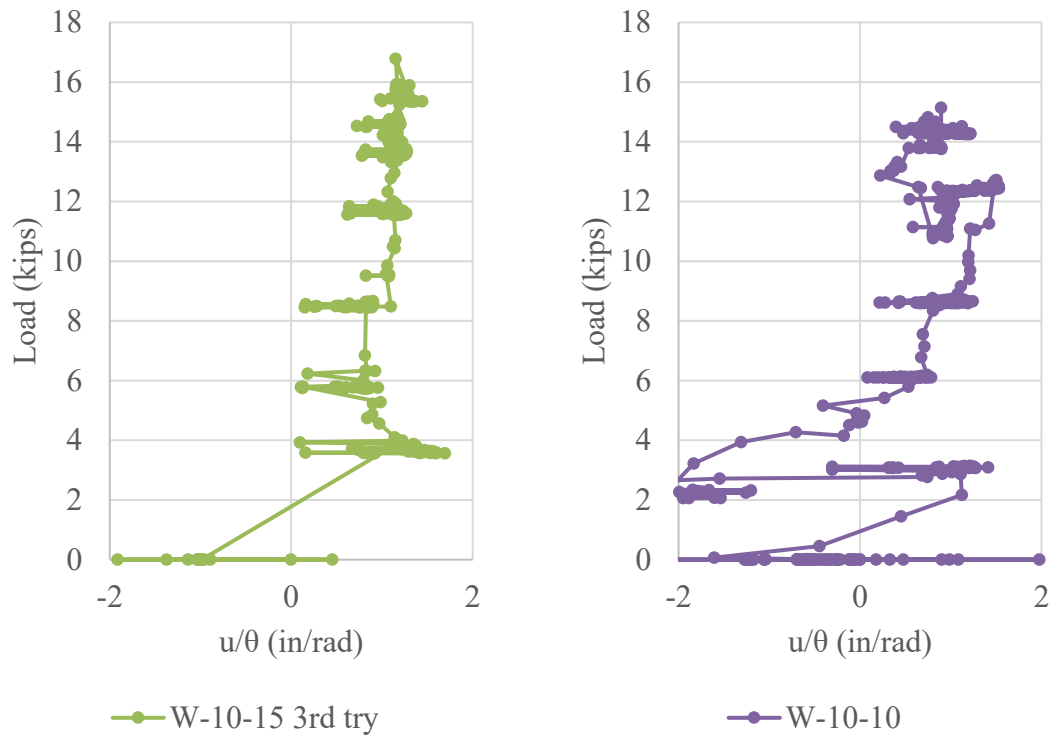
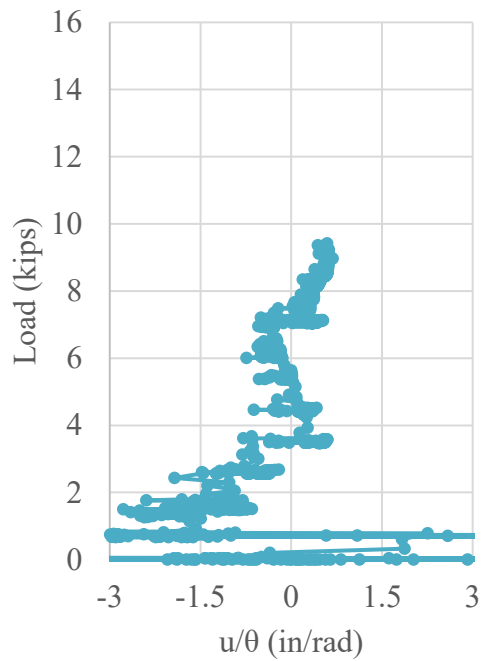
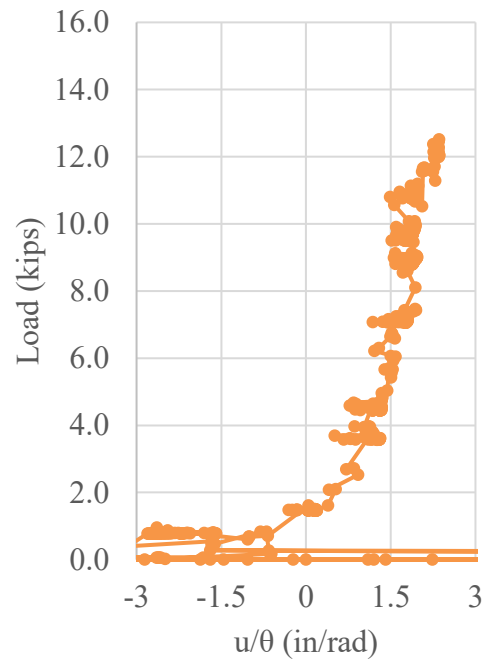


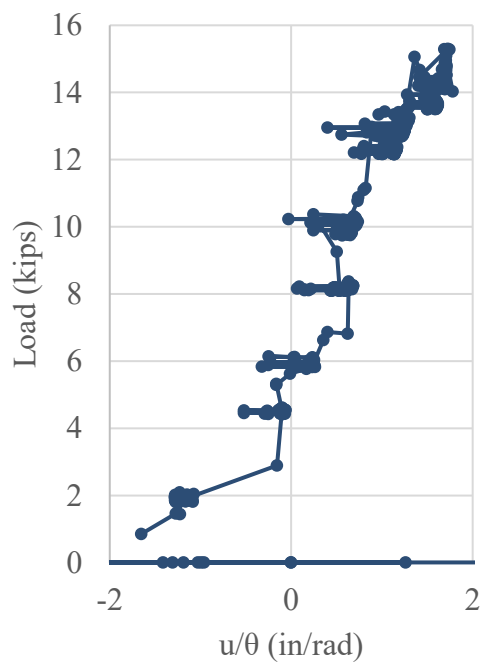
Figure 212. Ratio of Translation to rotation for specimens W-10-15 1st, 2nd and 3rd try, and W-10-10 (applied load at 84 in level).



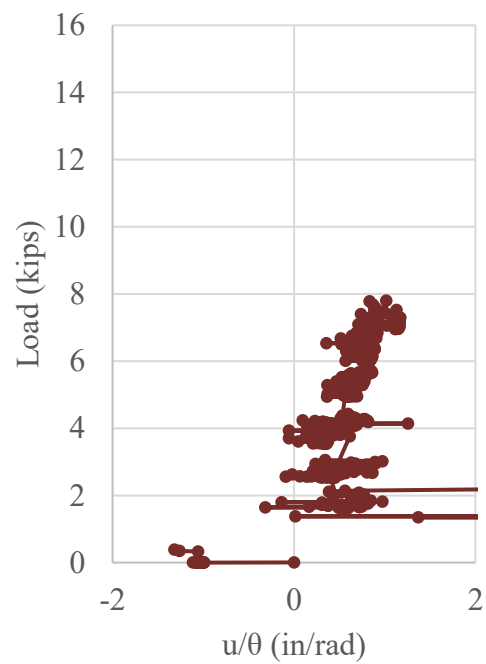
W-10-7.5 1st try



W-10-7.5 2nd try

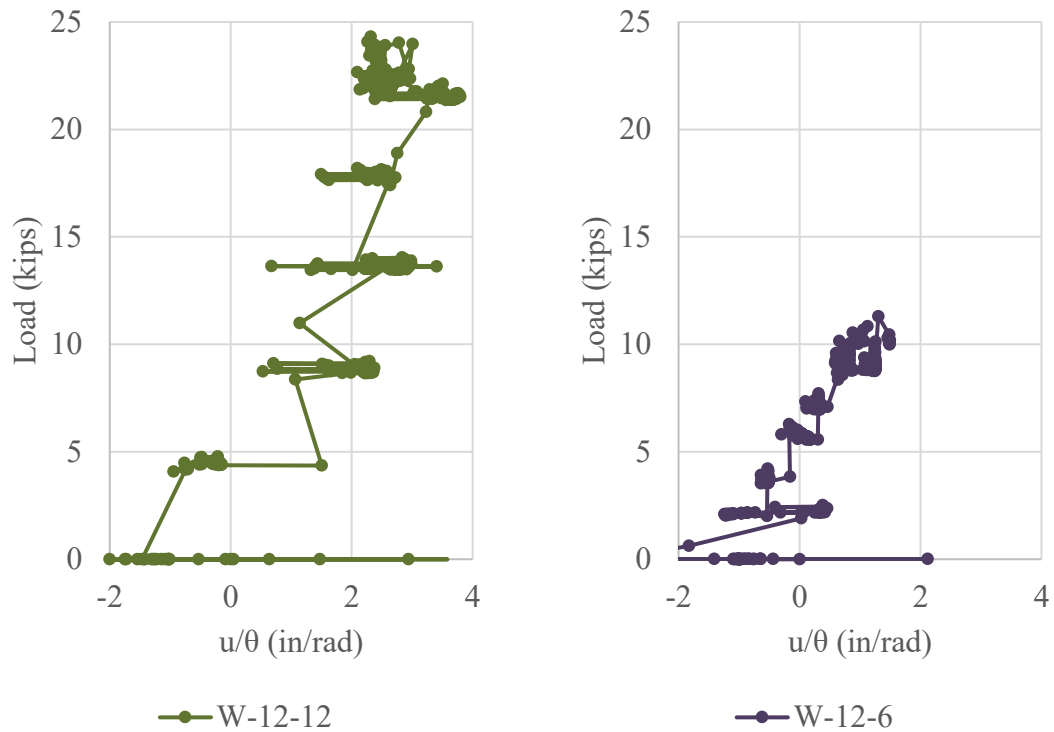


W-10-7.5R



W-10-5

Figure 213. Ratio of Translation to rotation for specimens W-10-7.5 1st and 2nd tries, W-10-7.5R and W-10-5 (applied load at 84 in level).



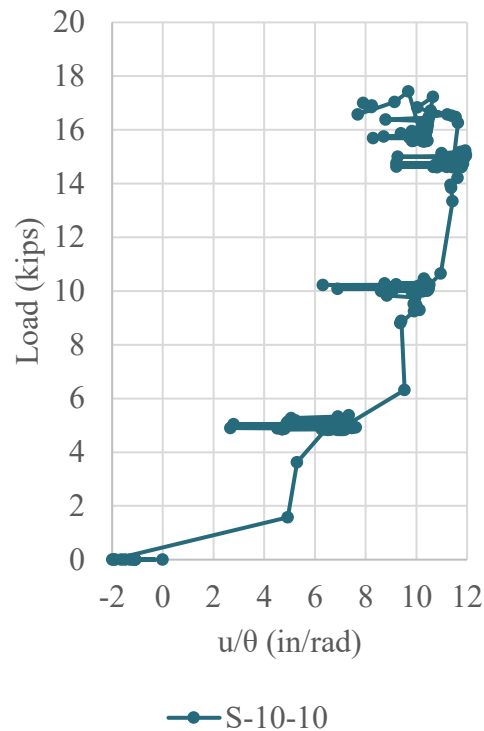


Figure 214. Ratio of Translation to rotation for specimens W-12-12, W-12-6 and S-10-10

A few caveats from the plots:

- The extreme measurements (both positive and negative) concentrated at the beginning of each test, when applied load is near zero or low, are simply random noise. When the applied load is still near zero, random noise in the ratio numerator (u) can produce arbitrarily large numbers due to the fact that rotation is also near zero.
- Throughout the loading range, there are isolated spikes which are single data points that shoot far to one side before immediately returning to the surrounding trend. These correspond to transient inclinometer readings at the instant of each load increment application. When a load step is applied, the connection responds with an instantaneous rotation impulse that the inclinometer registers before the structure reaches equilibrium.
- In the plots, there is a consistent band of negative u/θ values observed during the low-to-moderate load range for most specimens, generally between -

0.5 and -1.5 in/rad. This is an algebraic consequence of the decomposition formula. In the elastic range, where the connection is rotating with negligible translation, the ideal result is $u = 0$ and $u/\theta = 0$. However, when POT_C1 reading is small, any measured rotation can result in a negative translation u . Any lag between the inclinometer reading and the potentiometer reading during the load application will also push u/θ to a negative value.

For the W-10 series, the u/θ ratio at sustained high load is small, generally within the range of approximately -1 to $+1.5$ in/rad and roughly constant, mainly for deeper embedments (15 in). For intermediate and shallow embedments (10 in, 7.5 in and 5 in), the plots clearly show a growing trend of u/θ ratio with increasing load indicating that connection translation is becoming larger as increasing moment is applied to the connection. Regarding the pre-damaged specimens (W-10-15 2nd and 3rd tries, W-10-7.5 2nd try), the plots show an increase in u/θ relative to their undamaged counterparts, suggesting that the stiffness degradation from prior loading also affects the translational resistance of the connection, not only the rotational resistance.

For specimen W-12-12, the u/θ ratio at high load is visibly larger than for the W-10 series, settling into a positive range of approximately 2-3 in/rad at sustained peak load. Also, unlike W-10-10, the u/θ ratio grows with growing load. In absolute terms, considering the typical peak rotation for W-12-12, this still corresponds to a small physical translation.

Specimen W-12-6 shows, on the other hand, shows behavior more similar to the specimen W-10-5, although with smaller u/θ ratio. The ratio remains close to zero or slightly negative through most of loading, turning positive at around 6 kips, with a growing trend with increasing load.

Specimen S-10-10 is clearly distinct from all the other specimens. The u/θ plot for S-10-10 shows a big increase at moderate-to-high loads, with values reaching approximately 7-12 in/rad (four to five times the magnitude observed in any weak-axis specimen). This S-10 specimen was loaded in the bent cap transversal direction. This means that, unlike the W-10 specimens, there is less cap cross-sectional moment of inertia to combat the rotational effect. Additionally, there is less concrete mass, yielding

less resistance against rotation, leading to more deformation, and thus, more horizontal translation.

In absolute terms, for specimen S-10-10, the maximum measured rotation upon reaching maximum load (or moment) was 0.012 rad. Considering that, at this stage, $u/\theta = 15$, this means that the specimen suffered a pure translation of 0.18 in (or 4.6 mm) at the C1 level.

8. Cap Concrete Compressive Strength Effects on Rotational Stiffness

Based on the moment-rotation methodology presented by Kappes et al. (2021) as well as the methods proposed by Hammett (2017), changes in concrete compressive strength produce a directly proportional change to rotational stiffness.

The regression analysis performed in this chapter identifies L_e/d as the primary governing variable for normalized rotational stiffness, with EI normalization accounting for pile section differences. However, the residual scatter in the regression (measured/predicted ratios ranging from 0.78 to 1.24 for undamaged specimens) leaves room for other possible variables. Concrete compressive strength f'_c of the cap is a physically plausible candidate: since rotational resistance in plain embedment connections is provided entirely by concrete bearing on the embedded pile surfaces, a stronger and stiffer concrete would be expected to produce a stiffer connection, all else being equal. This section examines whether f'_c is a meaningful secondary predictor within this dataset.

Table 32 summarizes the cap concrete strength and modulus of elasticity for all specimens.

Table 32. Specimens Cap concrete compressive strength and estimated elastic modulus

Specimen	Specimen condition	Rotational stiffness (kip-ft/rad)	L_e/d	f'_c (psi)	E_c (ksi)
W-10-15 1st try	Undamaged	26056	1.5	5,754	4,324

W-10-10	Undamaged	9579	1	7,619	4,975
W-10-7.5 1st try	Undamaged	7376	0.75	6,925	4,743
W-10-7.5R	Undamaged	7869	0.75	6,965	4,757
W-10-5	Undamaged	3195	0.5	5,849	4,359
W-12-12	Undamaged	18463	1	7,014	4,774
W-12-6	Undamaged	3912	0.5	6,192	4,485
S-10-10	Undamaged	14967	1	6,232	4,500
W-10-15 2nd try	Pre-damaged	20467	1.5	5,754	4,324
W-10-15 3rd try	Pre-damaged	10350	1.5	5,754	4,324
W-10-7.5 2nd try	Pre-damaged	2654	0.75	7,619	4,975

Across the undamaged test matrix, cap concrete strength ranges from 5,754 psi (W-10-15) to 7,619 psi (W-10-10), a 32% spread. The estimated modulus of elasticity ranges from 4,324 to 4,975 ksi, a 15% spread. The variation in rotational stiffness (2654 to 26056 kip-ft/rad) represents more than 800% variation. Therefore, even under the assumption that rotational stiffness is proportional to E_c , the f'_c variation in this dataset would account for at most 15% of the range of rotational stiffness, a number that could fall well within noise attributed to specimen geometry and variability.

Comparing specimens at the same L_e/d , f'_c shows no evidence at any embedment level. At $L_e/d = 1.0$, both specimens fail by steel yielding, making f'_c mechanistically irrelevant, as concrete strength does not control when the pile steel yields. At $L_e/d = 0.5$,

where both specimens fail by concrete bearing and f'_c is physically relevant, the result runs contrary to expectation: W-10-5 ($f'_c = 5,849$ psi) exhibits lower stiffness than W-12-6 ($f'_c = 6,192$ psi), but the explanation is likely due to pile geometry. At $L_e/d = 0.75$, the two undamaged specimens have nearly identical f'_c values (6,925 and 6,965 psi), so no sensitivity can be extracted. For the pre-damaged specimens, the W-10-7.5 2nd try was conducted on concrete that had gained strength over time relative to the 1st try (6,925 and 7,619 psi), yet K_θ dropped by 64%, suggesting that loading-induced damage controls stiffness reduction, not concrete strength.

The main takeaway is that concrete compressive strength is not a meaningful secondary predictor of rotational stiffness in this dataset. The f'_c variation present is too small to produce a detectable effect. More experiments are required with repeated specimens and varying concrete compressive strength to determine whether this parameter has an effect.

Chapter 8:

Numerical Analysis

1. Introduction

This chapter presents a numerical analysis of steel H-pile to bent cap connections, using the experimental results from the previous chapters as the basis for model calibration and validation. The primary objective is to demonstrate how the rotational stiffness relationship developed in Chapter 7 can be implemented in an analysis framework to produce load-displacement predictions consistent with experimental observations.

All analyses were carried out using FB-MultiPier, a 3D non-linear finite element analysis (FEA) software used by structural and geotechnical engineers to analyze and design bridge piers and foundation systems. It was developed by the Bridge Software Institute (BSI) at the University of Florida.

FB-MultiPier was selected for its established use in practice and its direct applicability to the bridge bent configurations studied in Chapter 3. Two built-in connection features intended to represent pile-to-cap fixity were initially evaluated - the Pile Head Restraint Stiffness (PHRS) parameter and the pile head rigidization factor - and both were found unsuitable for representing the partial fixity conditions observed experimentally. The basis for this conclusion is explained in Section 2.

The adopted approach applies a simple discrete rotational spring at the pile head to represent the connection. Spring stiffness values are derived from the empirical regression developed in Chapter 7, which relates the normalized rotational stiffness K_θ/EI to the embedment-to-pile-depth ratio L_e/d . The regression provides the initial linear stiffness of the connection. Since the connection does not remain linear through the full loading range, a simple guideline for representing stiffness degradation (accounting for the onset of concrete bearing failure or steel yielding) is proposed in Section 4.

The chapter is organized as follows. Section 2 describes the modeling framework and the rationale for the adopted approach. Section 3 presents the application of the

rotational spring to the individual laboratory specimens, with comparison against the experimental moment-rotation curves. Section 4 formalizes the spring characterization guideline for general use. Section 5 applies the methodology to the full-scale experimental pile bents tested by Marshall et al. (2017) and Anderson et al. (2018), including a theoretical case at reduced embedment depth to understand the sensitivity of bent response to connection stiffness.

2. Connection Modeling Approach

Before defining the numerical model configurations, the available connection representation tools within FB-MultiPier were evaluated for their suitability in capturing the partial fixity behavior documented in the experimental program. Both tools - Pile Head Rotational Stiffness (PHRS) parameter and the Pile Top Rigidization feature - are built-in software mechanisms that are recommended by the software developers for representing a partially fixed pile-to-cap connection, and both are evaluated here. The evaluation revealed fundamental limitations in each, which motivated the adoption of a discrete rotational spring inserted externally between the cap and pile nodes.

2.1 Pile Head Rotational Stiffness (PHRS) Parameter

One of the features tried was the use of Pile Head Rotational Stiffness (PHRS) feature from FB-MultiPier. This parameter is a dimensionless scalar ranging from 0 to 1 that modifies the rotational spring stiffness at the pile head node in the pile-to-cap connection. A value of PHRS = 1 represents full rotational fixity (very large stiffness), while PHRS = 0 (zero stiffness) produces a pinned connection with no moment transfer. Intermediate values reduce the connection rotational stiffness proportionally relative to the fully fixed reference stiffness computed internally by the software from the pile section and element properties.

To evaluate whether PHRS could replicate the partial fixity behavior observed experimentally, three values were tested on specimen W-10-10 (HP10×42, $L_e = 10$ in, $L_e/d = 1.0$): PHRS = 1, 0.5, and 0.02.

The specimen was modeled as an isolated single pile with a lateral point load applied at the pile head, matching the laboratory setup. Figure 215 shows the resulting moment vs. rotation curves alongside the experimental result.

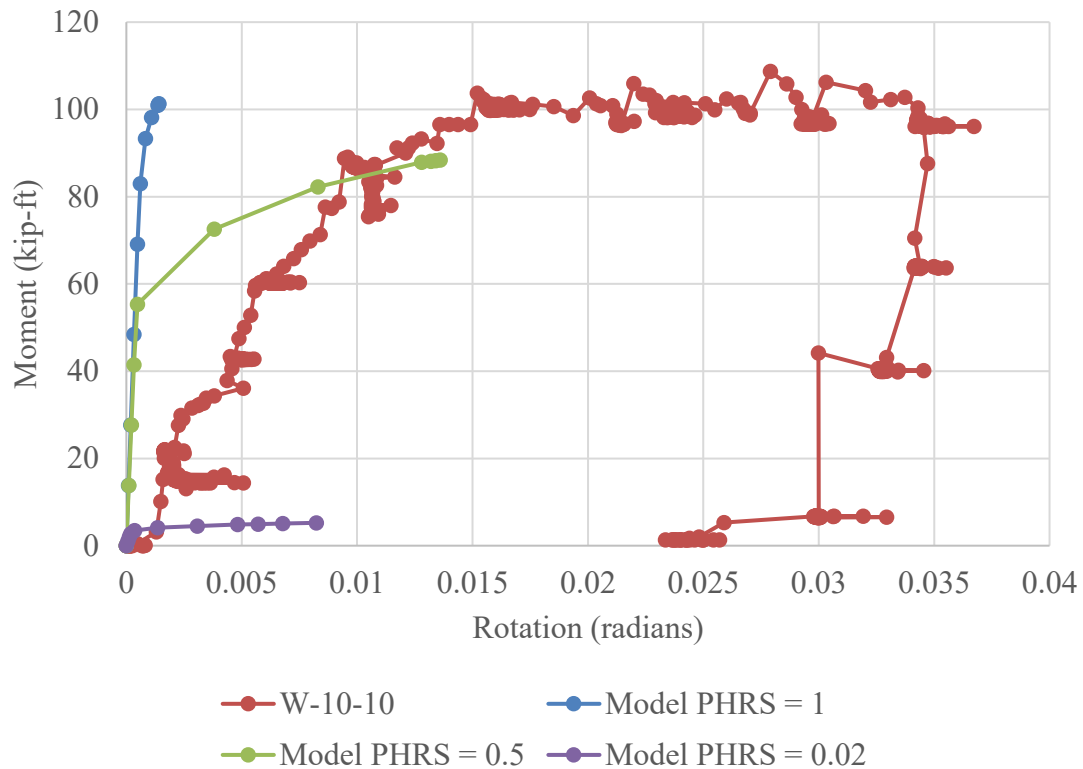


Figure 215. Moment vs. rotation curves for W-10-10, actual load test, and different model runs.

The PHRS = 1 and PHRS = 0.5 models produced initial rotational stiffness values of 148,332 kip-ft/rad and 125,644 kip-ft/rad, respectively, significantly stiffer than the experimentally measured value of 9,579 kip-ft/rad. The PHRS = 0.02 model yielded an initial rotational stiffness of approximately 15,292 kip-ft/rad, the closest among the three but still 60% above the measured value. Extrapolating from these results, a PHRS in the range of 0.01-0.02 would be required to approach the correct initial stiffness for this connection geometry.

However, calibrating PHRS to match initial stiffness exposes a more fundamental limitation of the parameter. Because PHRS is a single scalar, it simultaneously governs

both the initial rotational stiffness and the effective moment capacity at the connection. Reducing PHRS to approach the correct stiffness proportionally eliminates the connection's ability to carry moment, since it does not independently set a moment limit.

For PHRS = 0.02 on the isolated W-10-10 specimen, the model lost convergence at a lateral load of 0.76 kips, corresponding to a connection moment of only 5.24 kip-ft, less than 5% of the experimentally observed failure moment of approximately 100-108 kip-ft. The solver log recorded a catastrophic residual increase from 0.55 to 1,145 kips between iteration 34 and 35 at 0.77 kips load: with no soil and near-zero connection stiffness, the isolated pile is structurally near-unstable in theory, and the tangent stiffness matrix approaches singularity at arbitrarily small loads.

The root cause is that PHRS couples two physically independent properties through a single dimensionless scalar: initial rotational stiffness and moment capacity. In the actual embedded connection, these two quantities are governed by different mechanisms: initial stiffness is controlled by the bearing contact between pile flanges and web and the surrounding concrete, while moment capacity is controlled by the ultimate concrete bearing resistance. No single PHRS value can simultaneously reproduce both, because reducing PHRS to match K_{initial} eliminates the moment redistribution capacity that prevents a near-mechanism condition. Conversely, setting PHRS near 1 to preserve moment capacity produces a connection far stiffer than what the embedment actually provides.

2.2 Rigidization Length parameter

The Pile Top Rigidization feature in FB-MultiPier was also evaluated as a potential tool for representing pile embedment. The feature was designed for structures where a relatively thick or rigid pile cap partially encloses the pile head, and it was a natural candidate to consider: if the pile is embedded 10 inches into a concrete cap, rigidizing those 10 pile inches and making them behave as a rigid extension of the cap could in principle simulate the confinement provided by the surrounding concrete.

When a non-zero Rigidization Length is specified, all pile elements within that length below the cap midplane have their constitutive properties (moduli, yield strengths, compressive strengths) amplified by a factor of 10,000, effectively making those elements

rigid. When combined with a change in the pile head condition ($\text{PHRS} = 0$, for example), the feature shifts the effective pin from the pile head to the bottommost node within the rigidized zone, conceptually representing the embedded portion of the pile as a rigid extension of the cap, with the connection occurring at the base of the embedded zone.

To evaluate whether this feature could reproduce the observed partial fixity, three model runs were performed on specimen W-10-10 using $\text{PHRS} = 1$ and rigidization lengths of 0, 6, and 12 inches, with all other model parameters held constant. Figure 216 shows the resulting moment versus rotation curves alongside the experimental result.

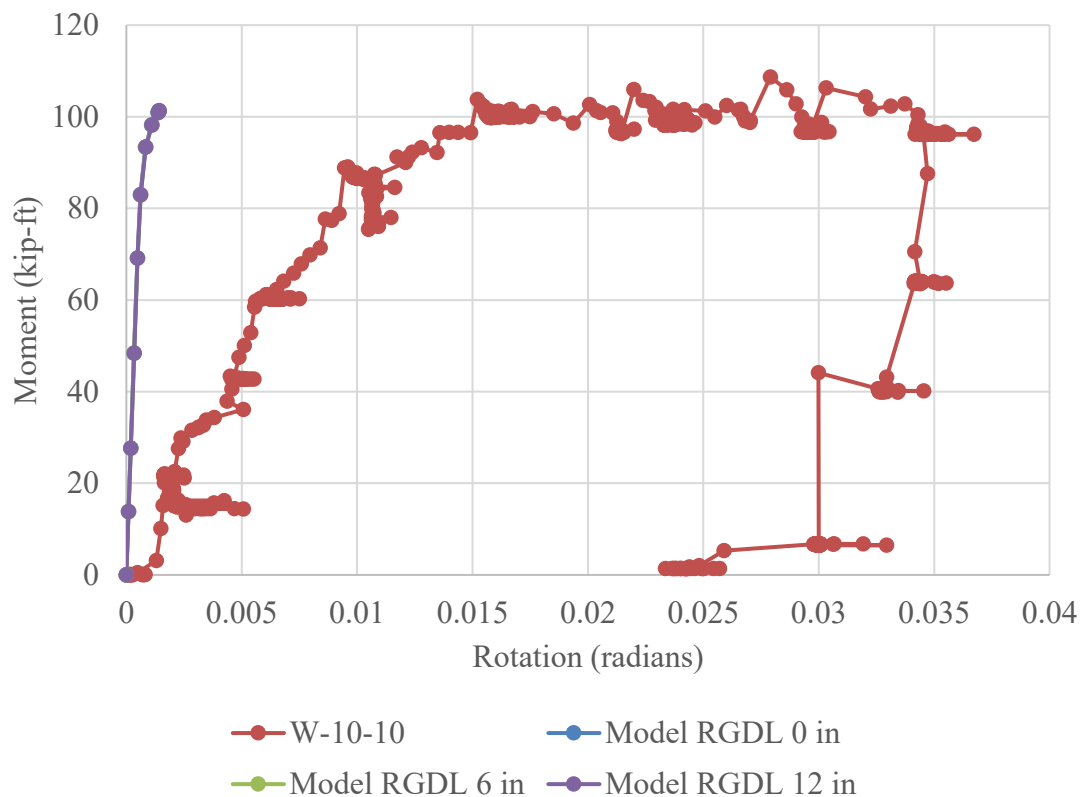


Figure 216. Moment vs rotation curves for W-10-10 load test and FB-Multiplier model with rigidized lengths 0 in, 6 in and 12 in.

As observed, the three model curves are indistinguishable. The numerical output confirms that the pile displacements was identical across all three runs at every load case, regardless of rigidization length. Changing the rigidization length from 0 to 6 to 12 inches

produced no change in connection rotational stiffness, moment capacity, or load-displacement response.

The Pile Top Rigidization feature was therefore not pursued further. As with PHRS, the feature was not designed to model partial pile-to-cap fixity from plain embedment, and it lacks the capability to represent the connection stiffness and degradation behavior documented experimentally.

2.3 Adopted Approach: Rotational Spring at the Connection

The fundamental limitation of PHRS is the coupling of rotational stiffness and moment capacity through a single scalar. The adopted approach in this work resolves this by removing PHRS from the equation entirely. PHRS is set to zero, producing a pinned condition at the pile-to-cap interface node. A discrete rotational spring is then manually inserted between the cap node and the first pile node, located 1 inch below the cap node, to represent the connection behavior independently of the pile section properties.

The initial linear spring stiffness K_θ is derived from the empirical regression developed in Chapter 7:

$$\frac{K_\theta}{EI} = 0.78 (\text{ft} * \text{rad})^{-1} \left(\frac{L_e}{D} \right)^2 \quad (50)$$

A linear spring captures only the initial stiffness phase of the connection. The experimental moment-rotation curves in Chapter 7 show that connection stiffness degrades after a threshold moment is reached, either when the pile steel begins to yield or when the connection concrete bearing zone begins to soften, whichever governs first. A linear spring cannot represent this degradation, and applying it through the full rotation range would overestimate the connection's rotational resistance at large deformations.

To address this, the spring is defined as a nonlinear moment-rotation curve rather than a single stiffness value. The construction of this curve uses only four points derived from pile section properties, embedment length, and connection capacity, and is the subject of Section 4. In combination with the Chapter 7 regression for initial stiffness and the Marcakis and Mitchell (1980) formulation for connection capacity, this approach

produces a complete load-deformation characterization of the connection from a small set of standard inputs.

The results of applying this spring definition to the individual laboratory specimens are presented in Section 3. The generalized construction guideline for the nonlinear spring is formalized in Section 4, followed by its application to full-scale pile bents in Section 5.

3. Linear Modeling of Individual Specimens

This section goes into detail about how the individual specimens built and tested in chapters 4 and 5 were numerically modeled in FB-MultiPier.

3.1 Model Configuration

Each specimen was modeled in FB-MultiPier v6.0.0 (Bridge Software Institute, University of Florida) as an isolated single pile connected to a bent cap. The model does not include adjacent piles, a superstructure, or soil resistance. Isolating the single-pile configuration allows the load-displacement response to be controlled exclusively by the pile section properties and the connection spring, consistent with the intent of the laboratory tests.

All analyses were performed as static, load-controlled, nonlinear analyses. The pile section was modeled with user-defined nonlinear material behavior (described in Section 3.3). No soil stiffness was applied. The numerical solver was used with a force convergence tolerance of 0.10 kips and a maximum of 100 iterations per load increment. The PHRS parameter was set to zero for all models, producing a pinned condition at the cap-to-pile interface node. Moment transfer between the pile and cap is provided entirely through the discrete rotational spring described in Section 3.6.

3.2 Geometry

The pile was modeled as a single beam-column element assembly oriented along the vertical axis in the FB-MultiPier coordinate system, with the cap connection at $Z = 0$ and the pile tip at $Z = 120$ in (10 ft). The total modeled pile length of 10 ft matches the physical pile length used in all specimens.

The pile mesh consists of 119 beam elements at a uniform spacing of 1 in, providing sufficient resolution to capture the nonlinear moment distribution along the pile. The element connecting the cap node (Node 1) to the first pile node (Node 326) spans exactly 1 in. This element provides the node at which the discrete rotational spring is attached (Section 3.6).

The lateral load was applied at Node 409, located 84 in (7 ft) from the cap-to-pile interface, matching the hydraulic jack attachment point in the laboratory test setup. This defines the effective moment arm for all load cases. The pile tip (Node 444 at $Z = 120$ in) was left free, consistent with the test setup in which the pile stub beyond the hydraulic jack attachment point was not restrained.

For all W-series specimens, the load was applied along the X-axis, bending the pile about its weak axis. For the S-series specimen, the load was applied along the Y-axis, bending the pile about its strong axis.

The pile cap was modeled as a 72 in $\times$ 36 in shell element mesh (13 nodes $\times$ 7 nodes on a 6-in grid) with a uniform thickness of 36 in and material properties corresponding to the concrete compressive strength of each specimen (Section 3.3). The pile was positioned at the center of the cap plan ($X = 36$ in, $Y = 18$ in).

Figure 217 shows the schematic of the model geometry, both its “physical” form as well as the nodes and elements.

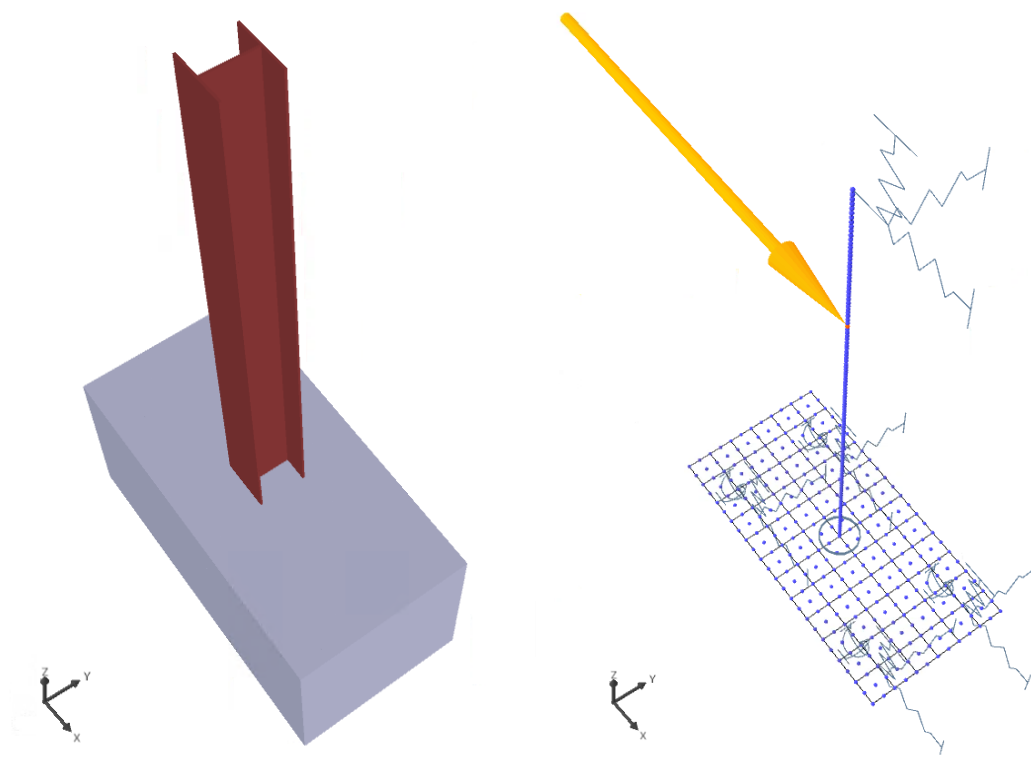


Figure 217. Geometry of the FB-Multiplier model created.

3.3 Material Properties

The pile section was assigned a user-defined nonlinear stress-strain curve, derived from coupon test results obtained for the HP10×42 and HP12×53 sections used in the experimental program. The curve includes a linear elastic branch ($E = 29,000$ ksi), a yield plateau beginning at approximately 59 ksi for the HP10×42 and 54.5 ksi for the HP12×53, a strain hardening branch, and an ultimate stress of 80.5 ksi (HP10) and 71.75 ksi (HP12) followed by rupture. Table 10 (in chapter 4) summarizes the measured material properties for each section and Figure 218 shows the stress-strain curves used, based on the methodology proposed by Karthik and Mander (2011).

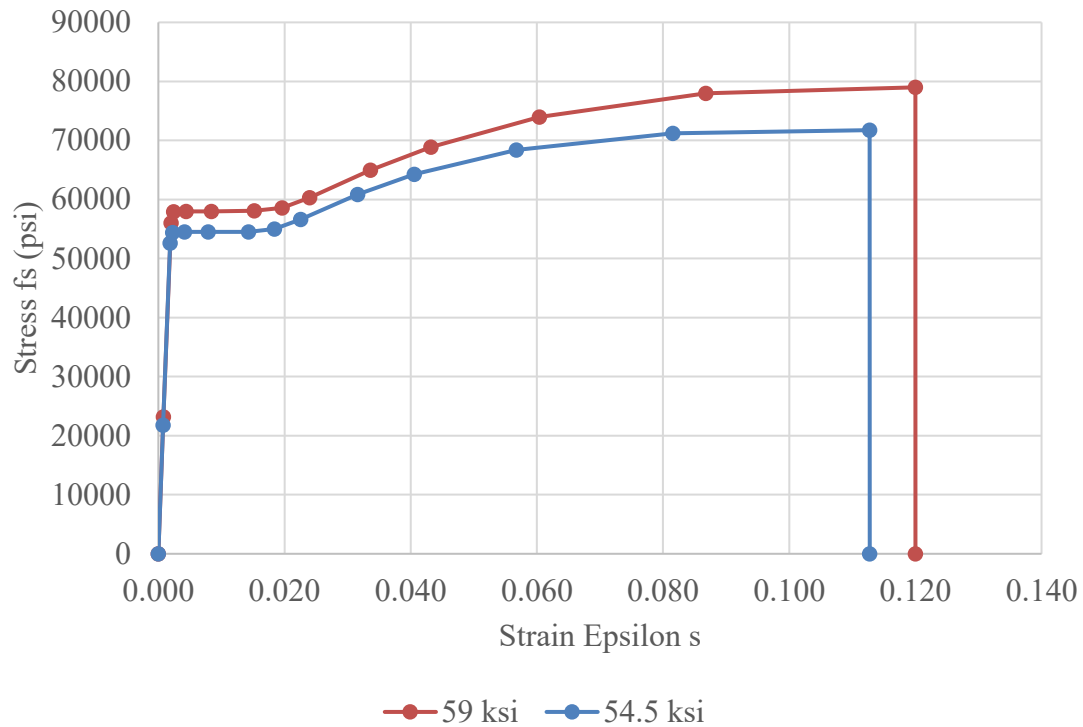


Figure 218. Stress-strain relationship adopted for HP10 (59 ksi) and HP12 (54.5 ksi) piles.

The cap was modeled as a nonlinear concrete shell, as is the software's default. The stress-strain relationship used for the concrete was also determined according to Karthik and Mander (2011). An example of the relationship is seen in Figure 219 for a concrete specimen with compressive strength $f'_c = 7619$ psi.

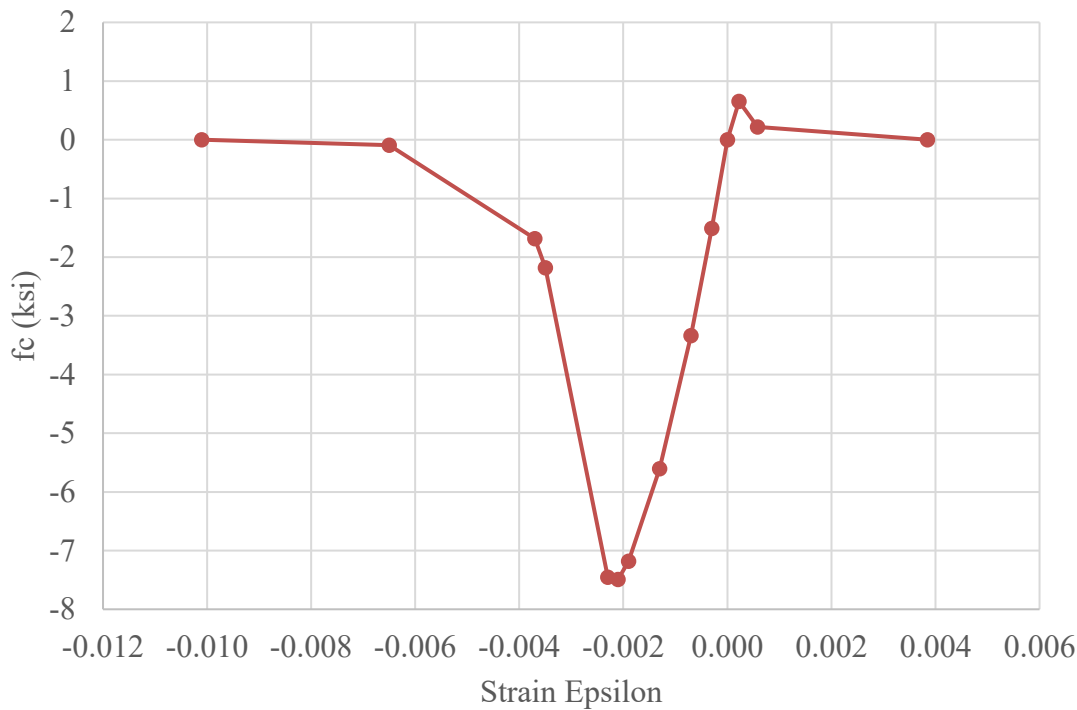


Figure 219. Stress-strain relationship adopted for concrete specimen with $f'_c = 7619$ psi (negative sign indicates compression).

3.4 Model Boundaries

The boundary conditions replicate the laboratory test setup, in which the cap block was anchored to the laboratory's strong floor at four anchor locations while the pile projected freely from the cap face.

As seen in Figure 217, four cap nodes in the cap plane were restrained in all six degrees of freedom using high-stiffness springs (1×10^9 kips/in and kip-ft/rad). These four nodes correspond to the anchor pattern of the actual cap block, positioned symmetrically about the pile at ± 24 in in the X-axis and ± 12 in in the Y-axis.

The pile-to-cap interface is modeled as a pin (PHRS = 0), which eliminates direct moment transfer between Node 1 (the cap-connection node at $Z = 0$) and the pile. All connection moment transfer passes through the discrete rotational spring defined in Section 3.6.

The pile tip (Node 444, $Z = 120$ in) was left essentially unrestrained. Due to the software requirements, nominal spring stiffnesses of 1×10^{-7} kips/in were assigned at the

pile tip in all six degrees of freedom, providing numerical stability to the software solver, but without contributing any meaningful resistance.

3.5 Soil Properties

The individual specimens were tested in a controlled laboratory setting without soil confinement. No soil resistance was applied in the models. The FB-MultiPier soil stiffness setting was set to "No soil," and pile tip springs were set to 1×10^{-7} kips/in, as described in the previous section. All lateral resistance in the model is provided through the discrete rotational spring at the cap connection.

3.6 Connection Spring Definition

The connection between the pile and the cap block is represented by a discrete spring element inserted between Node 1 (the cap interface node at $Z = 0$) and Node 326 (the first pile node, located 1 in below the cap face). Only the rotational degree of freedom about the Y-axis is assigned a nonzero stiffness for the W-series specimens. For the S-series specimen, the rotational degree of freedom about the X-axis only is assigned a nonzero stiffness. All translational and out-of-plane rotational stiffnesses are set to zero.

The spring is initially defined as linear. Its stiffness is determined according to Table 22 (chapter 7), mimicking the obtained rotational stiffness for each specimen. Figure 220 shows in detail the inserted spring between each node in the model.

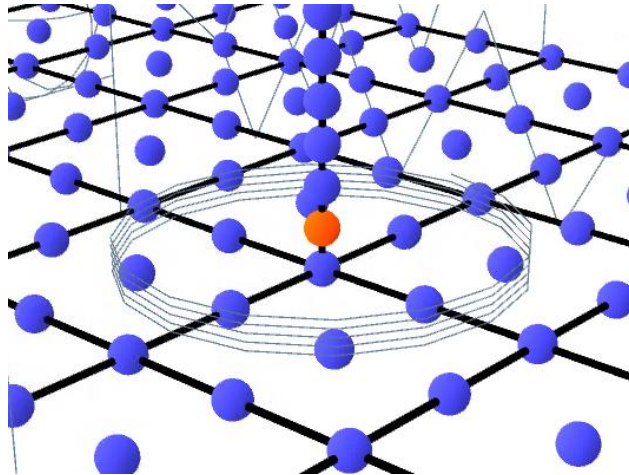


Figure 220. Rotational Spring between the model cap and pile nodes.

3.7 Results

The rotational stiffness values measured experimentally in chapter 7 were assigned as linear spring inputs for each specimen, and the resulting model M- θ response was compared against the experimental curves. This confirms that the spring modeling framework correctly implements a discrete rotational spring in FB-MultiPier with the specified stiffness, and establishes the boundary of applicability of a linear spring representation for these connections. Figures 221 through 228 show the results.

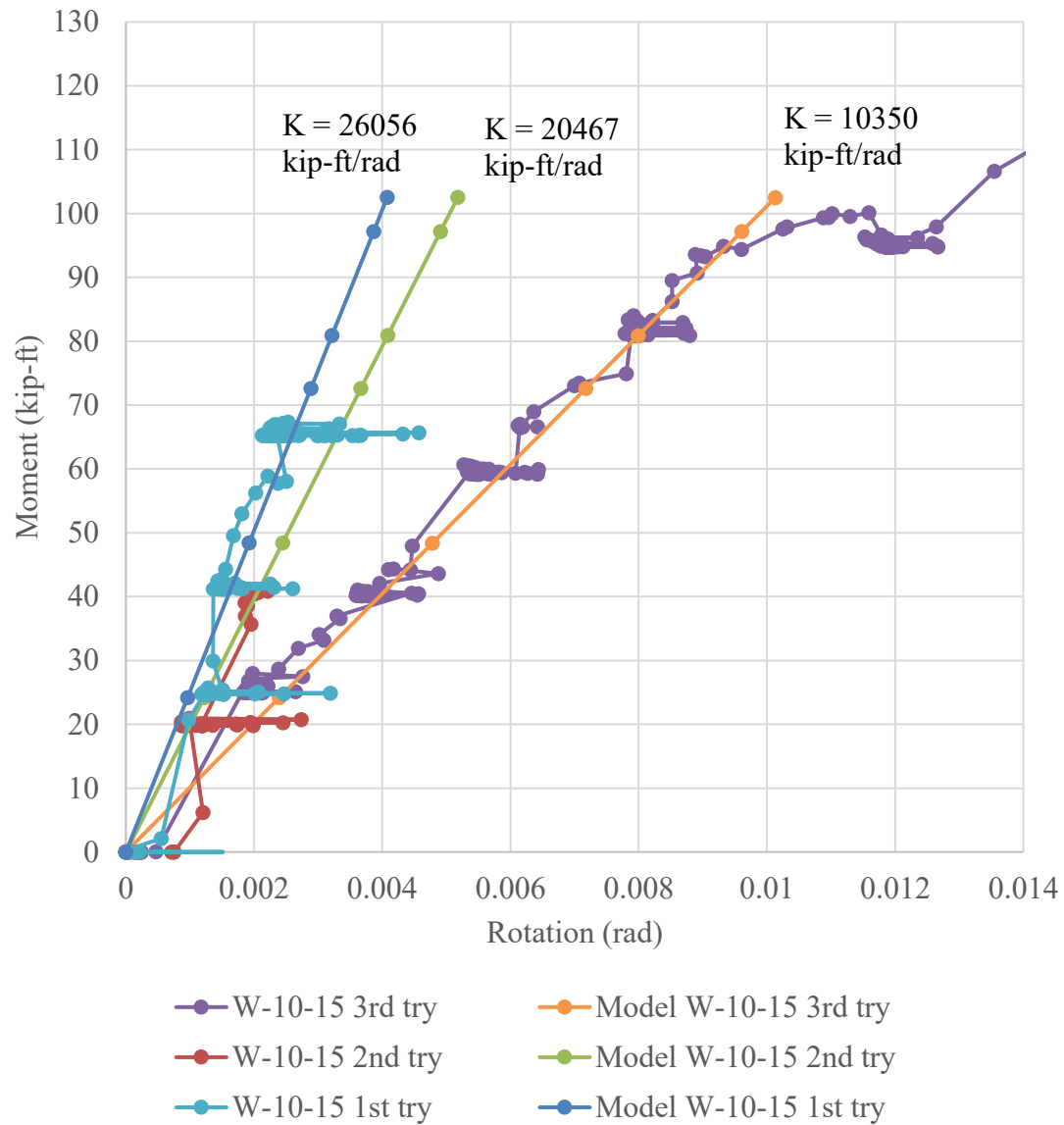


Figure 221. Linear Rotational Spring results compared to the actual M- θ curve for specimen W-10-15 (1st, 2nd and 3rd tries).

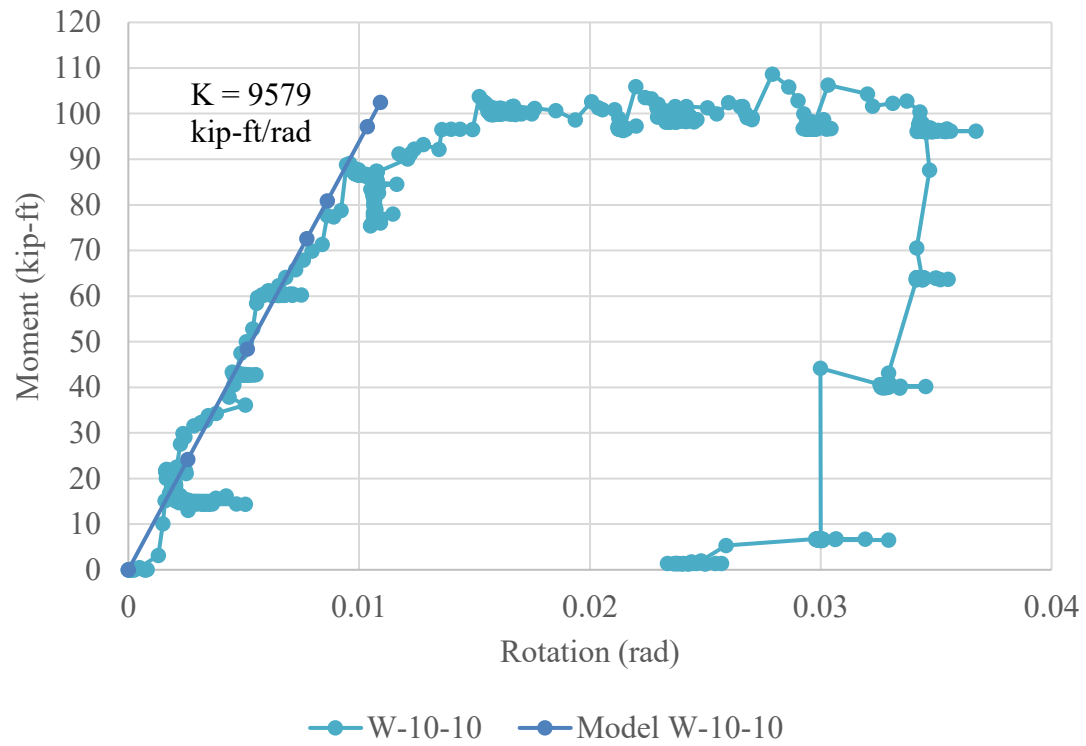


Figure 222. Linear Rotational Spring results compared to the actual $M-\theta$ curve for specimen W-10-10.

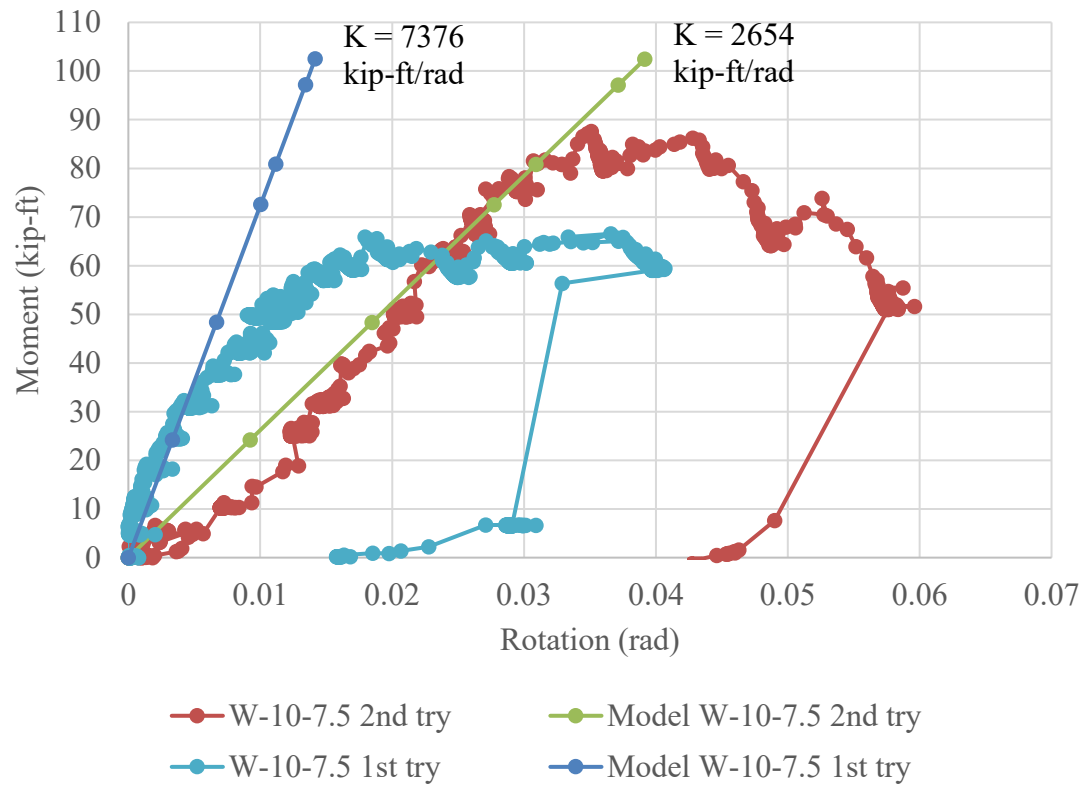


Figure 223. Linear Rotational Spring results compared to the actual M- θ curve for specimen W-10-7.5 1st and 2nd tries.

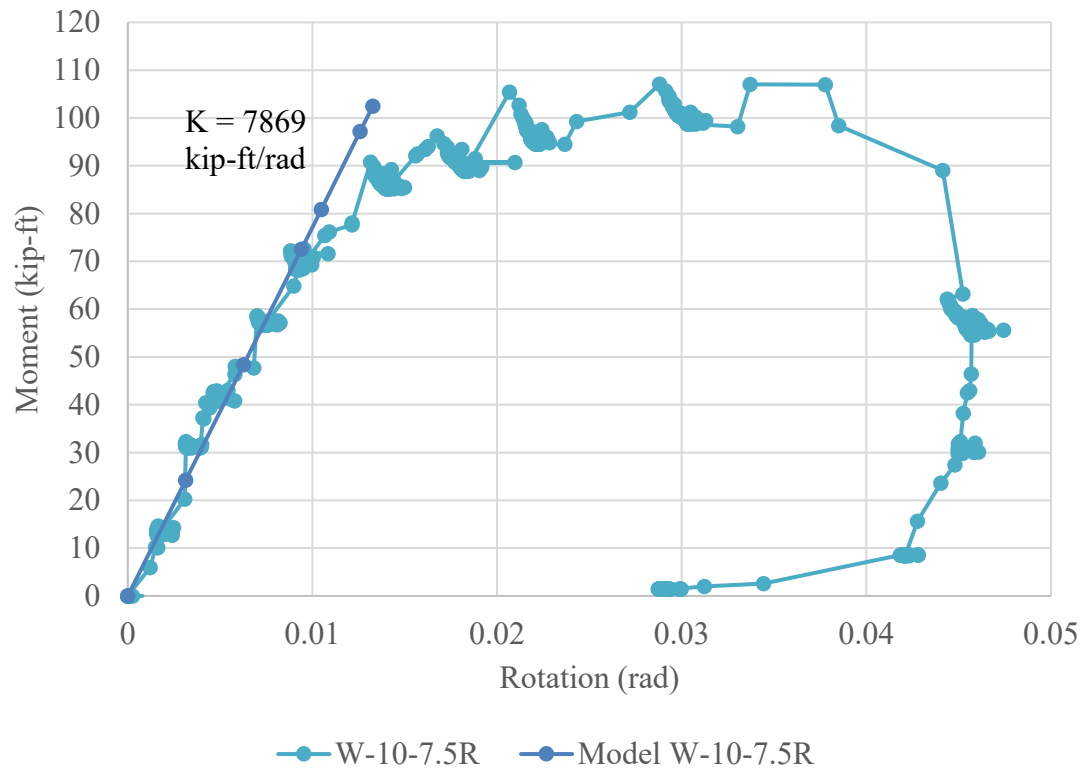


Figure 224. Linear Rotational Spring results compared to the actual M- θ curve for specimen W-10-7.5R.

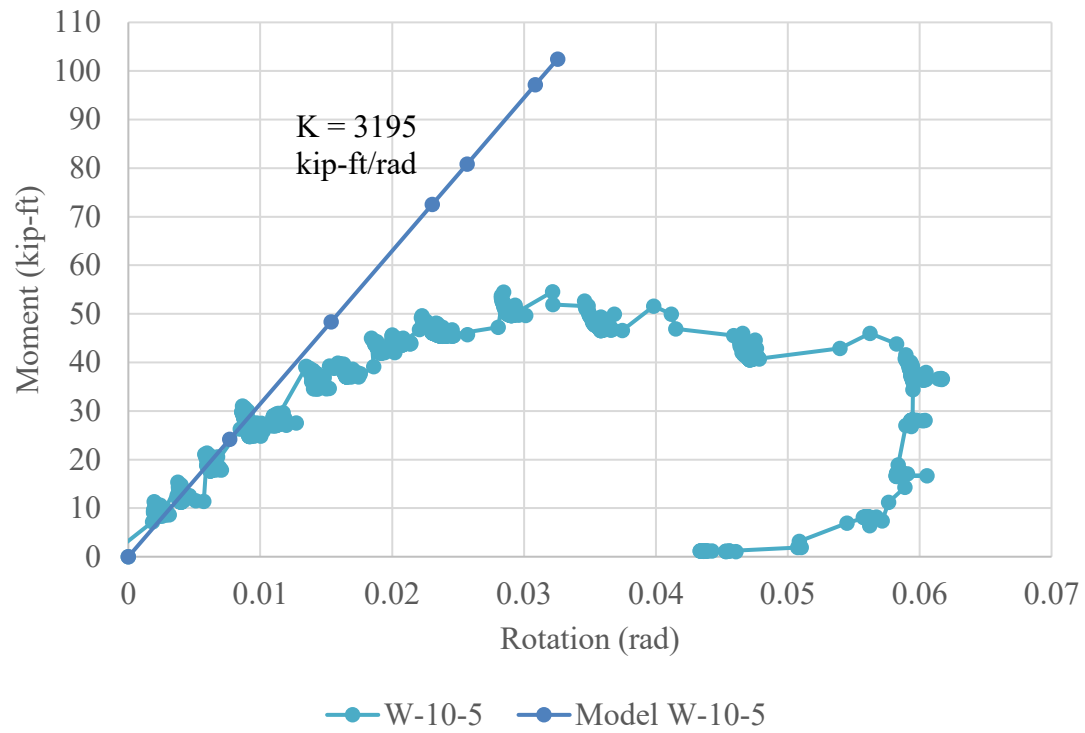


Figure 225. Linear Rotational Spring results compared to the actual M- θ curve for specimen W-10-5.

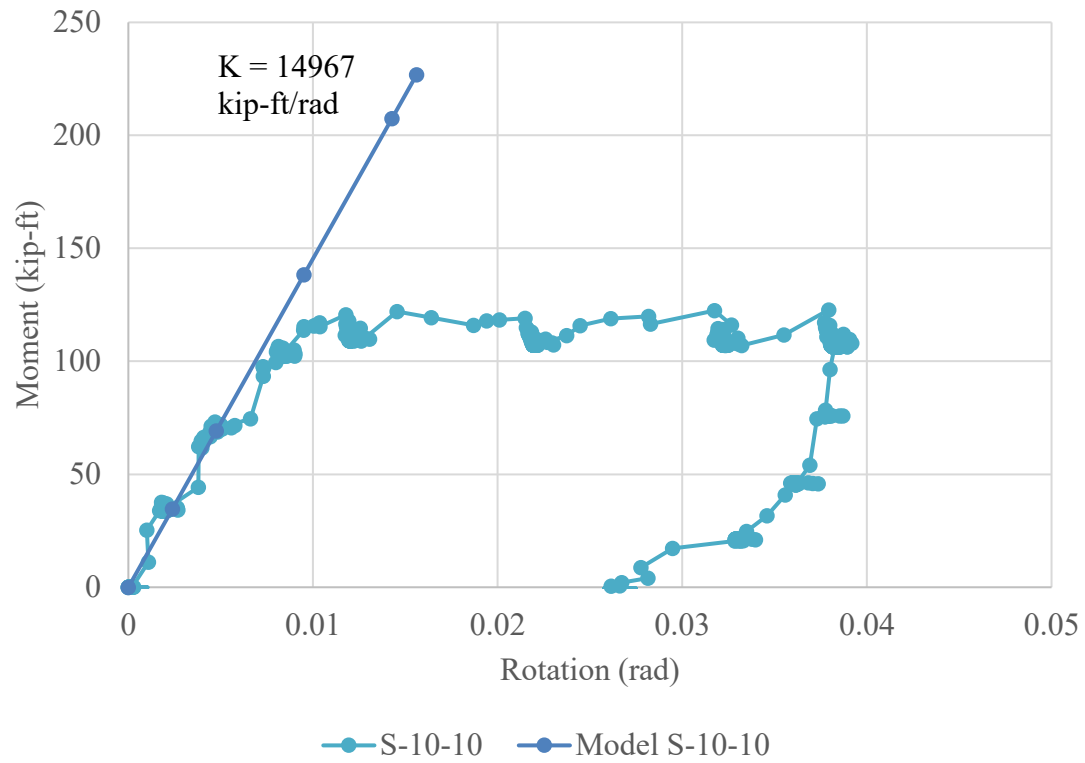


Figure 226. Linear Rotational Spring results compared to the actual $M-\theta$ curve for specimen S-10-10.

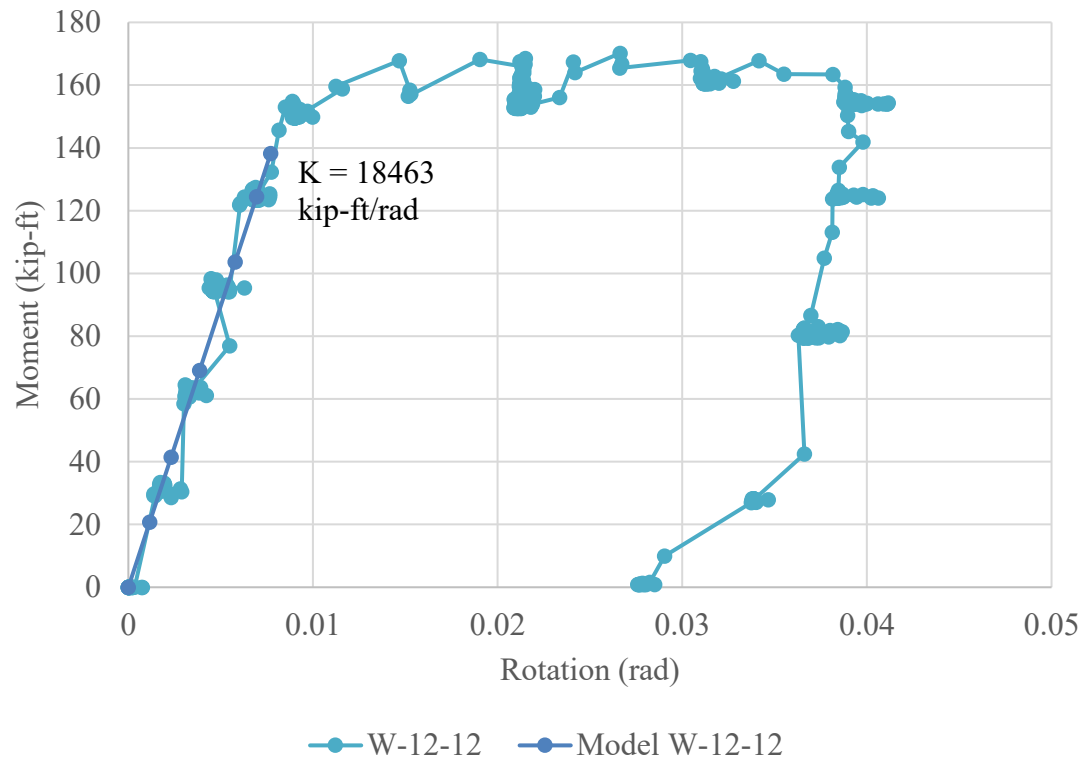


Figure 227. Linear Rotational Spring results compared to the actual $M-\theta$ curve for specimen W-12-12.

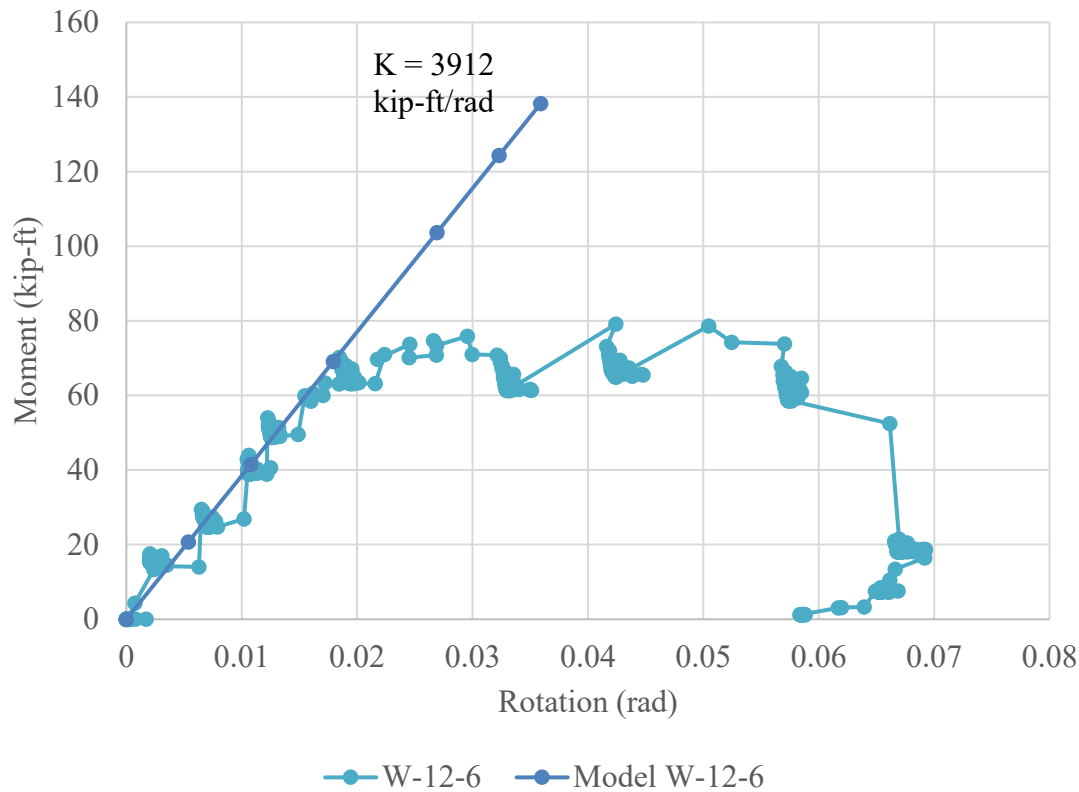


Figure 228. Linear Rotational Spring results compared to the actual M-θ curve for specimen W-12-6.

In all cases, the model initial stiffness line matches the slope of the experimental M-θ curve in the initial stiffness phase, consistent with the use of experimentally measured K_{θ} values as direct input. Beyond this phase, the linear spring continues to increase without bound while the experimental curves degrade, either as the pile steel begins to yield or as the concrete bearing zone softens. The linear spring has no mechanism to represent this degradation, and the model diverges progressively from the measured response as rotation increases.

For steel-yielding specimens (W-10-10, W-12-12, and W-10-15 3rd try), as pile stresses approach the yield plateau, the model approaches convergence limits, and the analysis terminates near the pile's plastic moment capacity. For concrete-bearing specimens (W-10-7.5R, W-10-5, W-12-6, S-10-10), the model has no termination criterion from connection capacity. The linear spring produces unlimited moment growth, and the divergence from the experimental plateau is unbounded.

These results confirm that a discrete rotational spring correctly represents connection behavior within the initial stiffness phase, and that the initial stiffness alone is insufficient to model the full response. The nonlinear spring guideline proposed in Section 4 addresses this limitation by adding a degraded stiffness segment and a moment capacity plateau, both derived from standard section and connection properties.

4. Connection Spring Characterization Guideline

The linear spring applied in Section 3 captures only the initial stiffness phase of the moment-rotation response. The experimental results presented in Chapter 7 demonstrate a consistent three-phase behavior: an initial stiffness phase, a transition phase with degraded rotational stiffness, and a plateau at the governing connection capacity, either steel yielding or concrete failure. A linear spring applied beyond the initial phase overestimates the connection's rotational resistance at large deformations and misrepresents the moment redistribution in pile bents subjected to significant lateral loads.

To represent this full behavior with a minimum number of input parameters, a four-point nonlinear spring definition is proposed. The curve is constructed entirely from standard pile section properties, embedment geometry, and connection capacity.

4.1 Empirical Calibration

Two empirical parameters control the shape of the nonlinear spring curve: a degraded stiffness factor and nonlinearity onset moment. Both were calibrated against the specimens tested in this program. Table 33 summarizes the calibration data.

Table 33. Calibration of empirical Nonlinear spring parameters

Specimen	Le/d	Failure Mode	M _{max} achieved (kip-ft)	K _θ (kip-ft/rad)	K _{degraded} (kip-ft/rad)	K _{degraded} / K _θ	Degraded Curve Onset Moment (kip-ft)	Elastic Moment M _y (kip-ft)	Onset/ M _{max}
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W-10-10 Undamaged	1	Steel yielding	101	9579	3913	0.408	70	69	-
W-12-12 Undamaged	1	Steel yielding	160	18463	9336	0.506	115	95	-
W-10-15 Pre- damaged	1.5	Steel yielding	105	10350	4671	0.451	70	69	-
W-10-7.5R Undamaged	0.75	Concrete bearing	98	7869	2673	0.340	71	-	0.72
W-10-5 Undamaged	0.5	Concrete bearing	51	3195	1319	0.413	28	-	0.55
W-12-6 Undamaged	0.5	Concrete bearing	70	3912	1936	0.495	50	-	0.71
S-10-10 Undamaged	1	Concrete bearing	110	14967	8000	0.535	75	-	0.68

The ratio of the degraded-phase slope to the initial slope ranged from 0.34 to 0.54 across the seven specimens, with a mean of 0.450 and a coefficient of variation of 15%. No systematic dependence of this ratio on L_e/d , failure mode, or pile section was observed across this dataset. The factor of 0.450 is therefore adopted as a single value applicable to both failure modes within the tested range of $L_e/d = 0.5$ to 1.5.

For the four specimens governed by concrete bearing failure, the moment at which the $M-\theta$ curve first departed from linear behavior averaged 67% of the observed connection capacity, with a range of 55% to 72%. Three of the four specimens clustered between 68% and 72%, the exception was W-10-5 ($L_e/d = 0.5$), where onset occurred at only 55%. A round threshold of 70% is adopted, since it is conservatively consistent with the established nonlinear behavior of concrete in compression: the concrete stress-strain relationship exhibits onset of nonlinearity at approximately $0.75\text{--}0.80 \times f'_c$, usually attributed to accelerated micro-cracking (Wight and MacGregor, 2016).

For steel-yielding specimens, the onset trigger is the elastic yield moment of the pile section, $M_y = F_y \times S$. Specimens W-10-10 and W-10-15 showed $M-\theta$ departure from

linearity at essentially M_y . Specimen W-12-12 showed a later departure, at 125 kip-ft compared to $M_y = 95$ kip-ft. The discrepancy for W-12-12 is attributed to the gradual nature of steel yielding in bending: plasticity progresses from the extreme fiber inward, and the associated slope change in the $M-\theta$ curve is initially subtle, making graphical identification of the onset moment sensitive to the local curvature of the experimental curve. Setting the trigger at $F_y \times S$ is therefore conservative for sections with higher shape factors, as it terminates the initial phase earlier than experimentally observed.

4.2 Required Inputs and Computed Intermediate Quantities

The following parameters are required to construct the spring curve:

Pile section properties:

- Modulus of elasticity $E = 29,000$ ksi
- Moment of inertia I (in⁴)
- Elastic section modulus S (in³)
- Plastic section modulus Z_p (in³)
- Yield strength F_y (ksi)

Connection geometry and concrete:

- Embedment length L_e (in)
- Pile section depth d (in)
- Concrete compressive strength, f'_c (ksi)
- Effective bearing width b (in) as defined in Marcakis and Mitchell (1980).

Four quantities are derived from the inputs before the curve is assembled:

- Initial rotational stiffness from the Chapter 7 regression:

$$K_\theta/EI = 0.78 \text{ (ft-rad)}^{-1} \times (L_e/d)^2$$

For strong-axis bending, it is recommended to use a factor of 0.36, instead of 0.78.

For previously loaded connections, a factor ranging from 0.32 to 0.45 should be used instead of 0.78, depending on the severity of prior damage.

- Elastic yield moment of the pile: $M_y = F_y * S$
- Pile Plastic Moment: $M_p = F_y * Z_p$
- M_{conn} : Connection concrete bearing capacity from Marcakis and Mitchell (1980):

$$V_c = \frac{0.85 f'_c b l_e}{1 + 3.6 \frac{e}{l_e}} \quad (51)$$

Moment can be calculated by multiplying the force by the distance to the cap bottom surface.

For previously loaded connections, it is advisable to consider a reduction in the moment connection capacity of approximately 15%-20%.

4.3 Four-Point Curve Construction

The nonlinear spring is defined by four (θ, M) points:

- Point 1, Origin, (θ_1, M_1) :

$$M_1 = 0$$

$$\theta_1 = 0$$

- Point 2, End of Initial Stiffness phase, (θ_2, M_2) :

The end of the initial stiffness phase is controlled by either the onset of steel pile yielding or the onset of loss of concrete bearing capacity, whichever comes first.

$$M_2 = \min (M_y, 0.70 * M_{conn})$$

$$\theta_2 = M_2 / K_\theta$$

- Point 3, Controlling Capacity, (θ_3, M_3) :

Point 3 defines the “capacity plateau”. The rotation θ_3 is reached after a degradation in the stiffness, with a slope of $0.45 \cdot K_\theta$, calibrated as described in the previous sections. The maximum moment will be controlled by either the plastic moment of the pile or the connection moment capacity, whichever is lower.

$$M_3 = \min (M_p, M_{\text{conn}})$$

$$\theta_3 = \theta_2 + [(M_3 - M_2)/(0.45 \cdot K_\theta)]$$

- Point 4, Plateau Extent, (θ_4 , M_4):

Point 4 extends the constant-moment plateau to 1.5 times the rotation at capacity. This provides numerical stability in the post-peak regime without implying that the connection can sustain its peak moment indefinitely at large rotations. In reality, most connections soften beyond the plateau, especially for concrete-bearing failures.

$$M_4 = M_3$$

$$\theta_4 = 1.5 \cdot \theta_3$$

Figure 229 shows an example of the the resulting M- θ curve schematically, illustrating how the four points define the three linear segments. Figures 230 through 240 illustrate how the “fitted curve” would compare against the experimental M- θ curves.

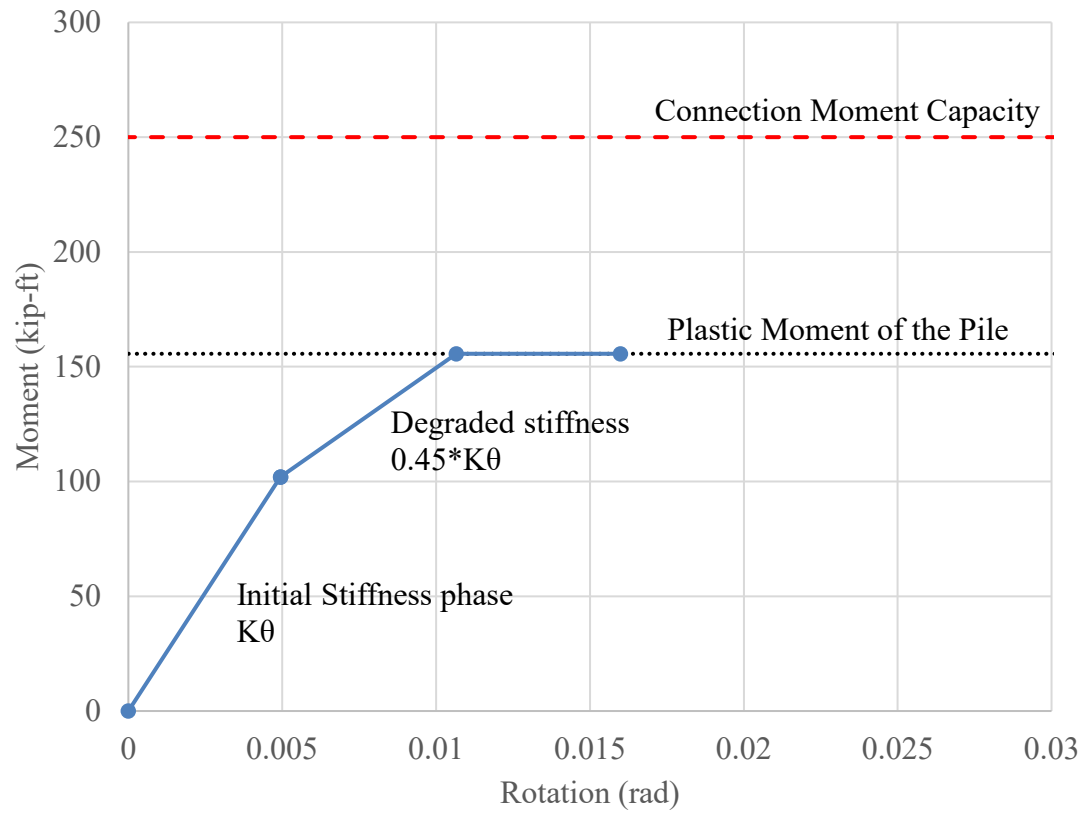


Figure 229. Fitted Moment vs Rotation curve for nonlinear rotational stiffness spring.

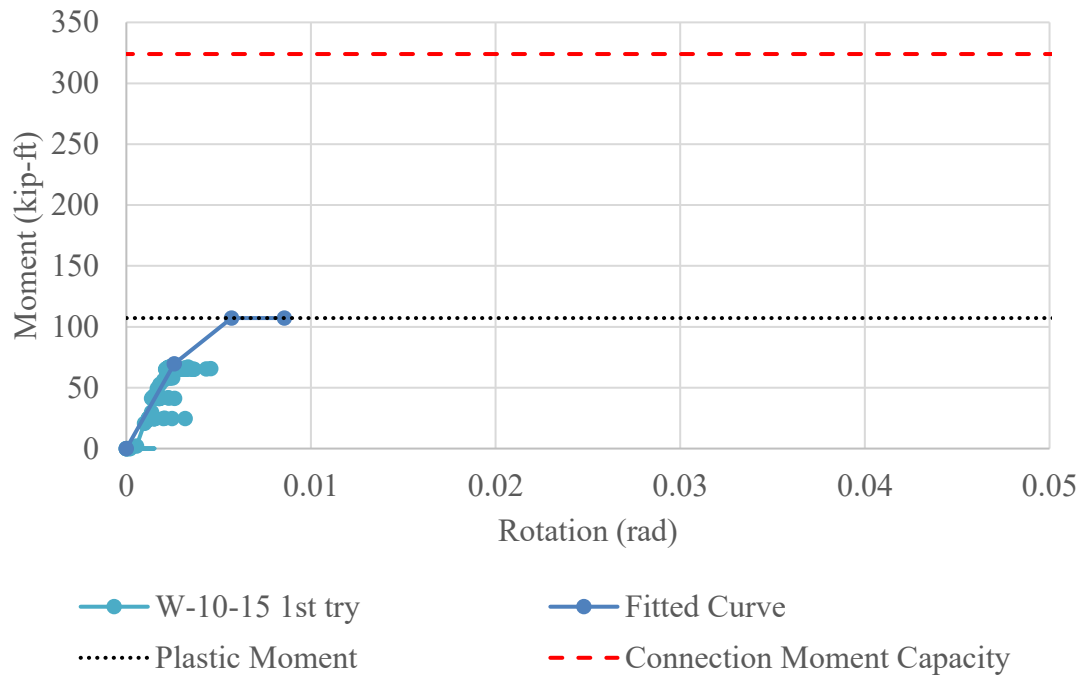


Figure 230. Application of fitted Moment vs Rotation curve to specimen W-10-15 1st try.

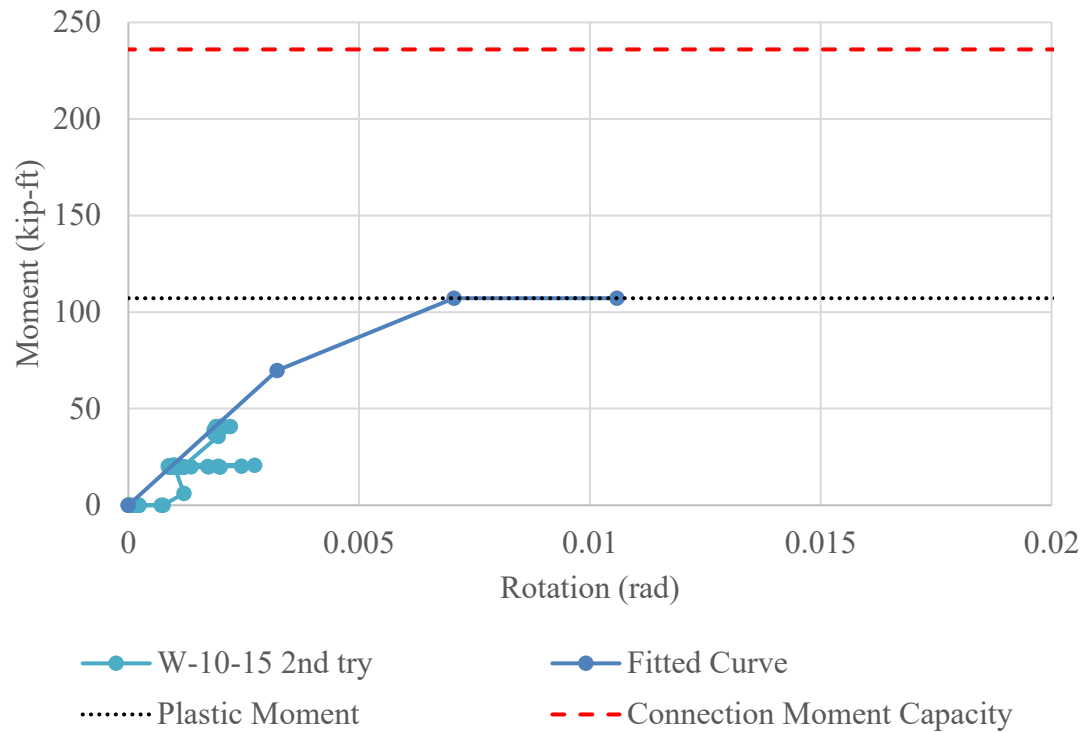


Figure 231. Application of fitted Moment vs Rotation curve to specimen W-10-15 2nd try.

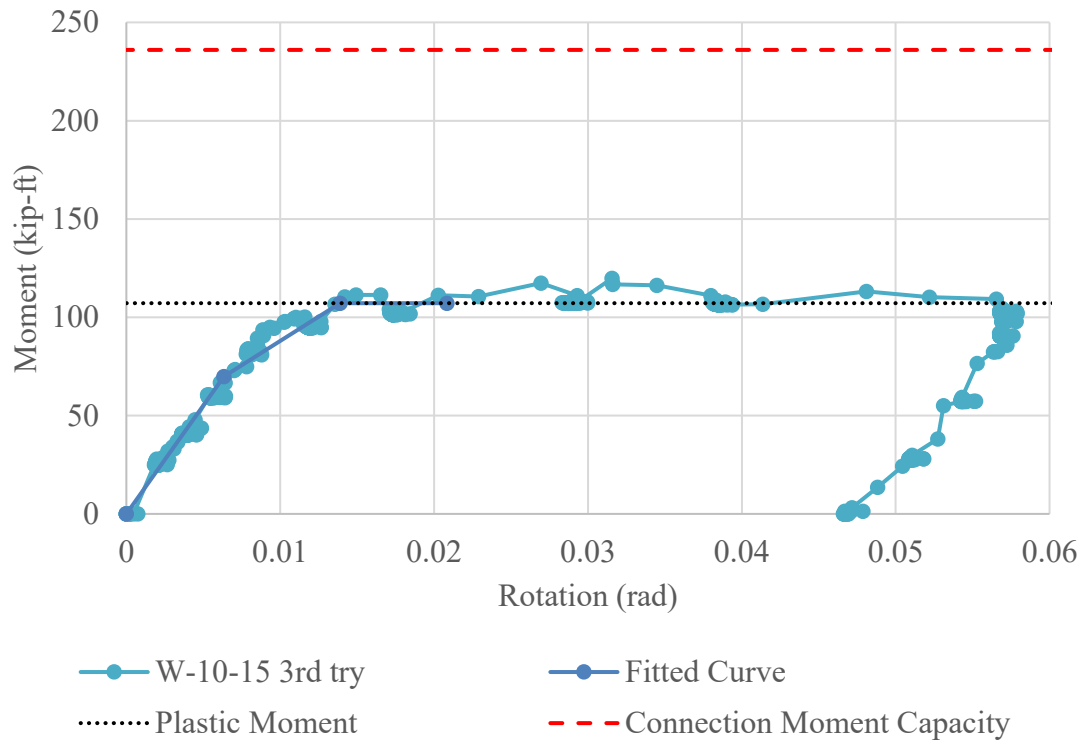


Figure 232. Application of fitted Moment vs Rotation curve to specimen W-10-15 3rd try.

As observed, both the first and second load tests on the W-10-15 connection were halted before the applied moment left the initial stiffness phase of the $M-\theta$ response. The guideline curves shown in Figures 230 and 231 project behavior beyond what the tests reached. The comparison is therefore limited to the initial slope of the measured curve, and the nonlinear portion of the guideline cannot be validated against these two tests.

The third load test was conducted on a connection that had accumulated damage from two prior loading cycles. To account for the stiffness reduction caused by this prior loading history, the rotational stiffness regression coefficient was reduced from 0.78 to 0.32, consistent with the upper range of stiffness reductions documented in Chapter 7 for pre-damaged connections. The connection moment capacity was also reduced by 20% relative to the undamaged prediction to reflect the degraded concrete bearing zone. This reduction did not alter the controlling failure mode: the reduced M_{conn} remained above the pile's plastic moment, and $M_3 = M_p$ continued to control the plateau level. With these adjustments, the guideline yields a good fit to the 3rd-try experimental $M-\theta$ curve, as shown in Figure 232.

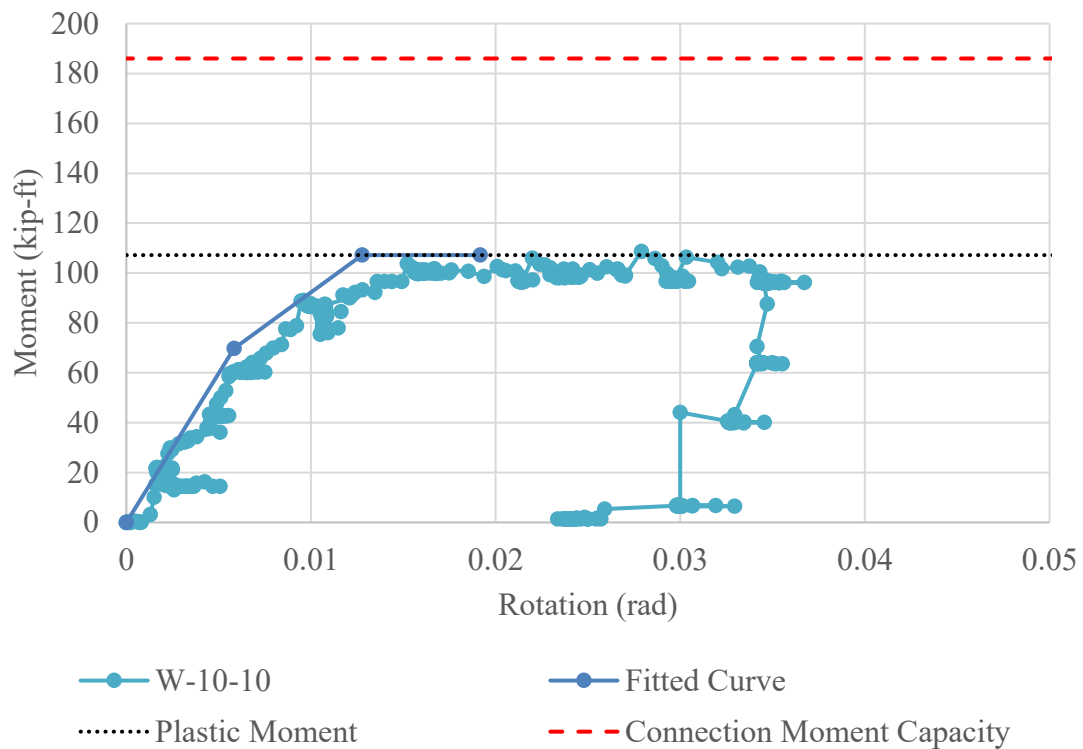


Figure 233. Application of fitted Moment vs Rotation curve to specimen W-10-10.

For the undamaged W-10-10 specimen, the guideline was applied using the standard regression coefficient of 0.78 and the undamaged connection capacity. The predicted curve is slightly stiffer than the measured response in the initial stiffness phase, as shown in Figure 233. The guideline moderately overestimates the initial rotational stiffness for this specimen, producing a conservative moment estimate at a given rotation. The overall fit is good.

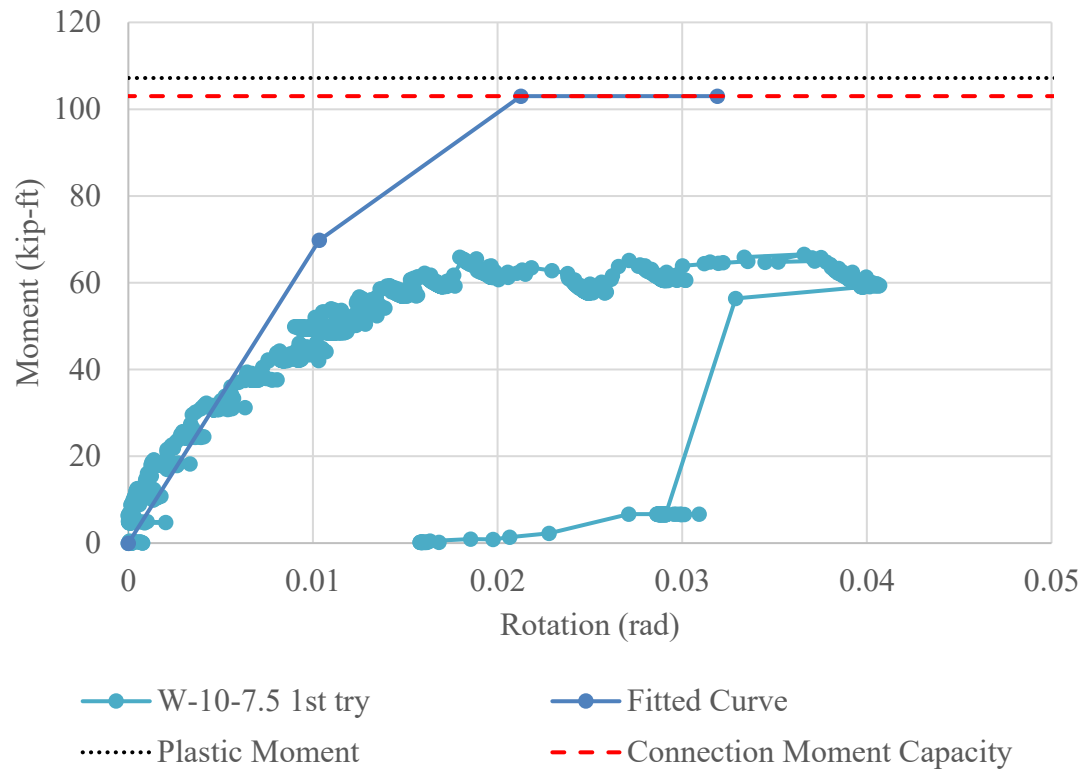


Figure 234. Application of fitted Moment vs Rotation curve to specimen W-10-7.5 1st try.

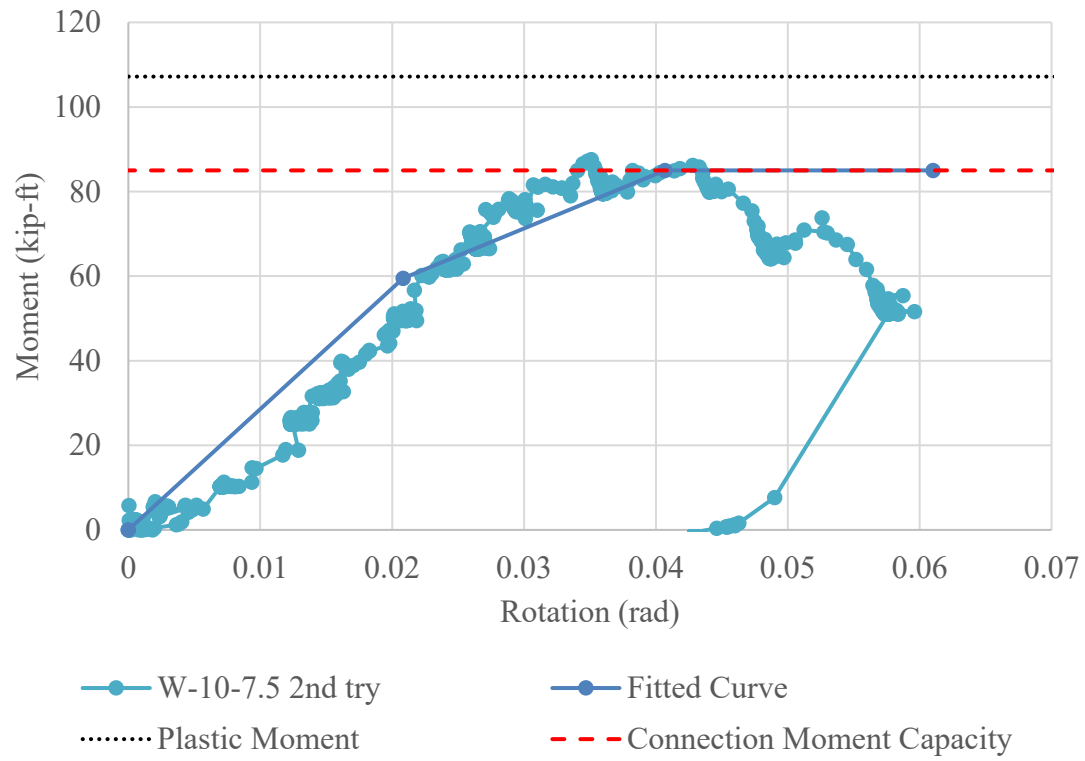


Figure 235. Application of fitted Moment vs Rotation curve to specimen W-10-7.5 2nd try.

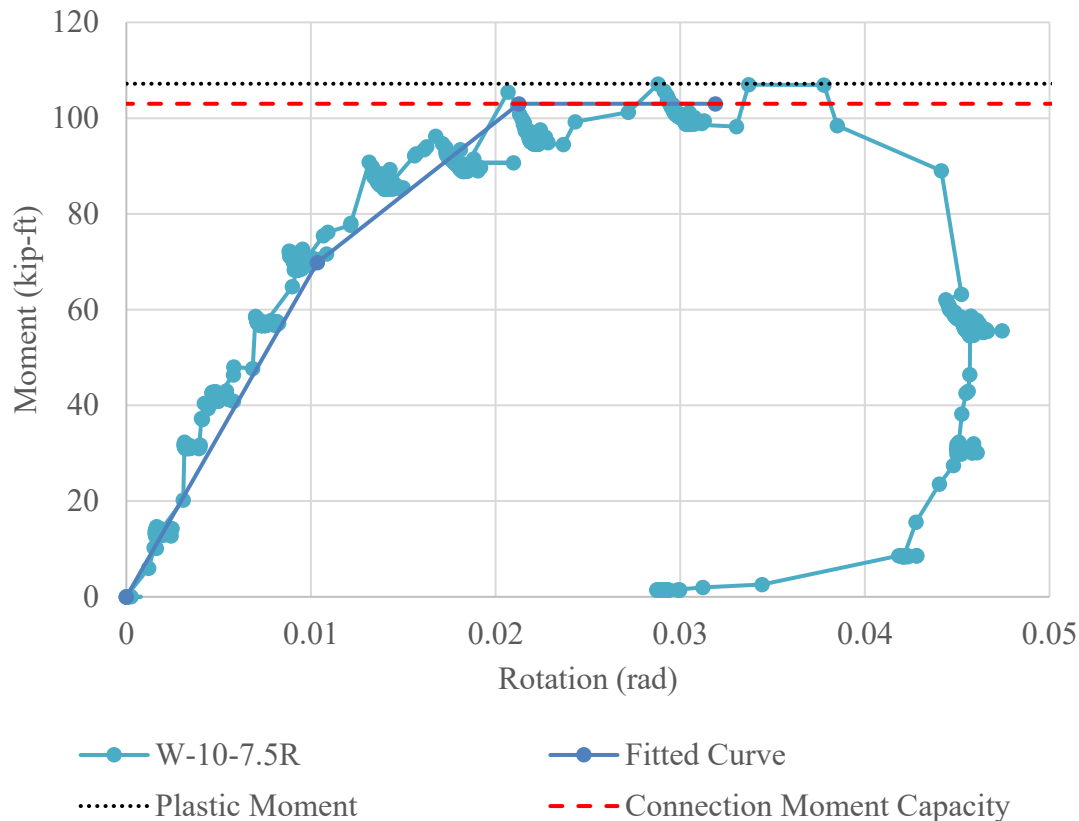


Figure 236. Application of fitted Moment vs Rotation curve to specimen W-10-7.5R.

The first load test on the W-10-7.5 connection produced unreliable results due to instability of the reaction frame during the test. The measured $M-\theta$ response does not reflect the true behavior of the connection, and the guideline is not recommended to be applied to this test (Figure 234).

The second load test was performed on the same connection after it had sustained significant damage from the first test. The regression coefficient was reduced from 0.78 to 0.33 to account for the accumulated stiffness loss, and the connection moment capacity was reduced by 20%. Given the extent of the prior damage, the guideline yields reasonable agreement with the measured response, as shown in Figure 235.

W-10-7.5R was an undamaged specimen cast at $L_e/d = 0.75$ and tested to failure by concrete bearing. The guideline was applied using the standard 0.78 regression coefficient and the undamaged connection capacity. The predicted curve is slightly conservative: the initial stiffness phase is modestly less stiff than the data. For this

specimen, the predicted connection moment capacity is marginally below the pile's plastic moment, making M_{conn} the governing plateau level. The overall agreement is good, as shown in Figure 236.

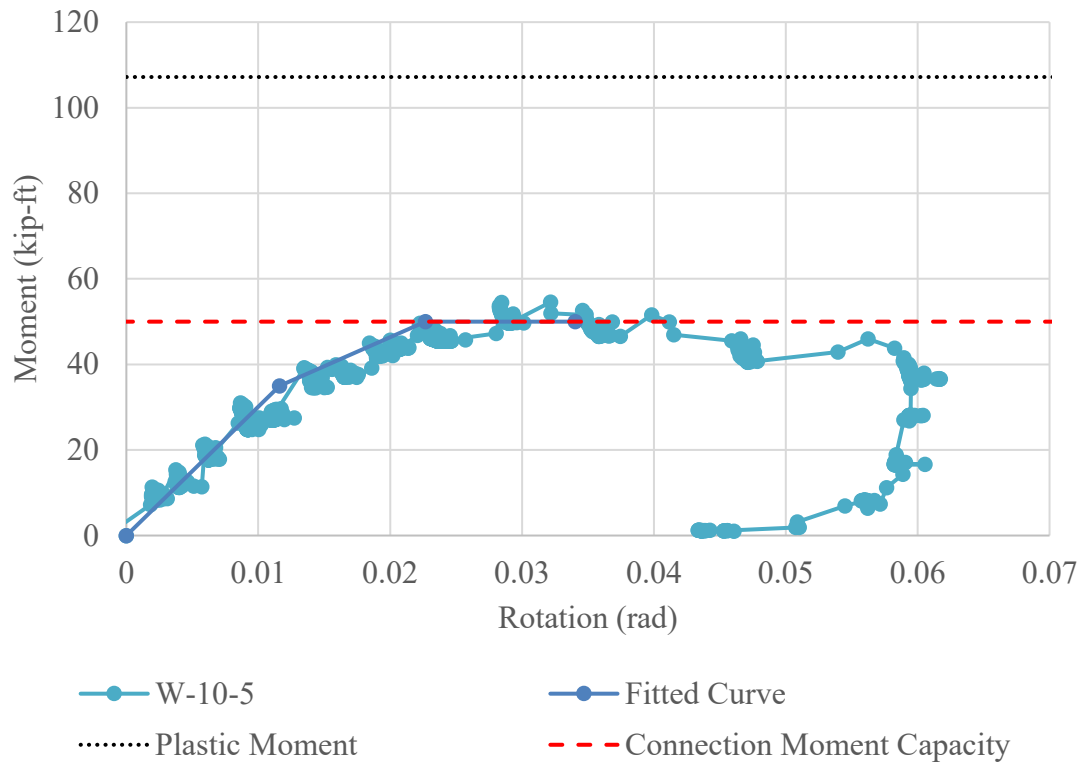


Figure 237. Application of fitted Moment vs Rotation curve to specimen W-10-5.

The guideline yields a good fit to the W-10-5 experimental M- θ curve without modification to either the regression coefficient or the connection capacity, as shown in Figure 237.

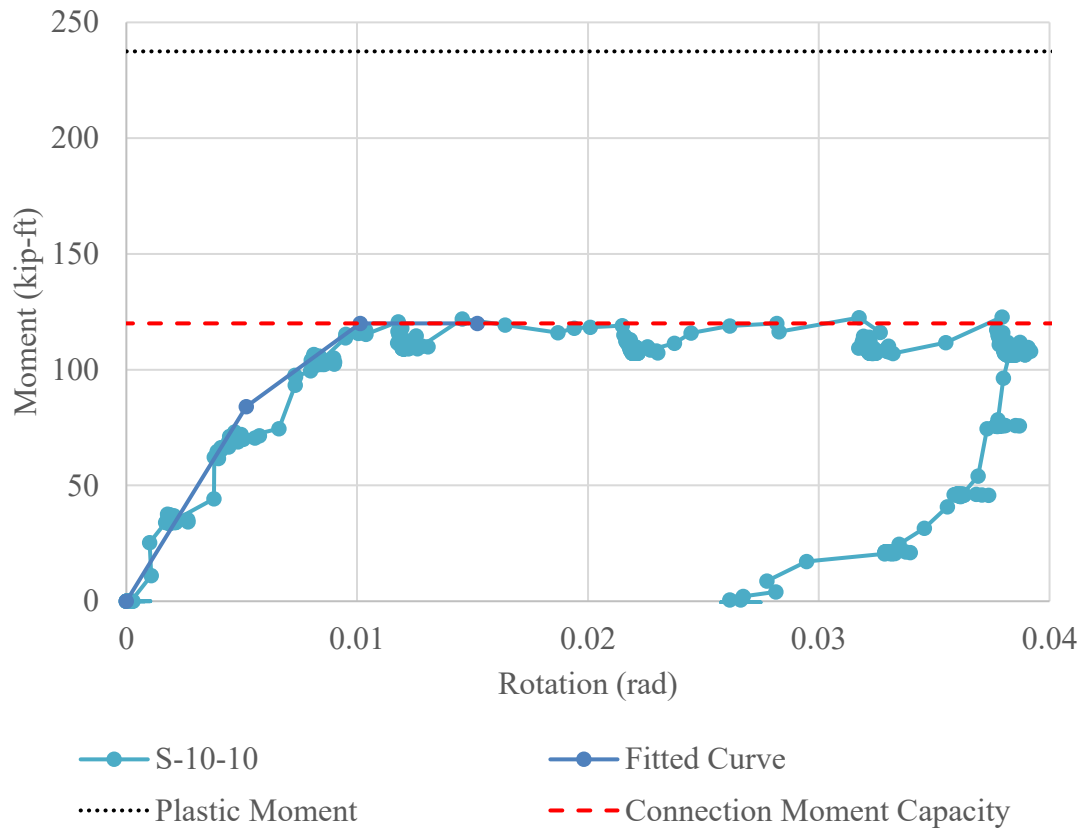


Figure 238. Application of fitted Moment vs Rotation curve to specimen S-10-10.

S-10-10 introduces two sources of complexity absent in the other specimens: it was loaded in strong-axis bending, and it failed by concrete bearing at a L_e/d ratio where the guideline is sensitive to the assumed connection capacity.

For strong-axis bending, the rotational stiffness regression coefficient was reduced from 0.78 to 0.36 in place of the weak-axis calibration. For the connection capacity, the Marcakis and Mitchell (1980) formulation overpredicts the strong-axis bearing resistance. A 25% reduction to the Marcakis and Mitchell (1980) prediction was found necessary to produce agreement with the measured response. This reduction is not supported by existing analytical models and it is an empirical correction derived from the single strong-axis specimen in this program and should be treated as a preliminary finding. With these two adjustments, the guideline yields a good fit to the S-10-10 M- θ curve, as shown in Figure 238.

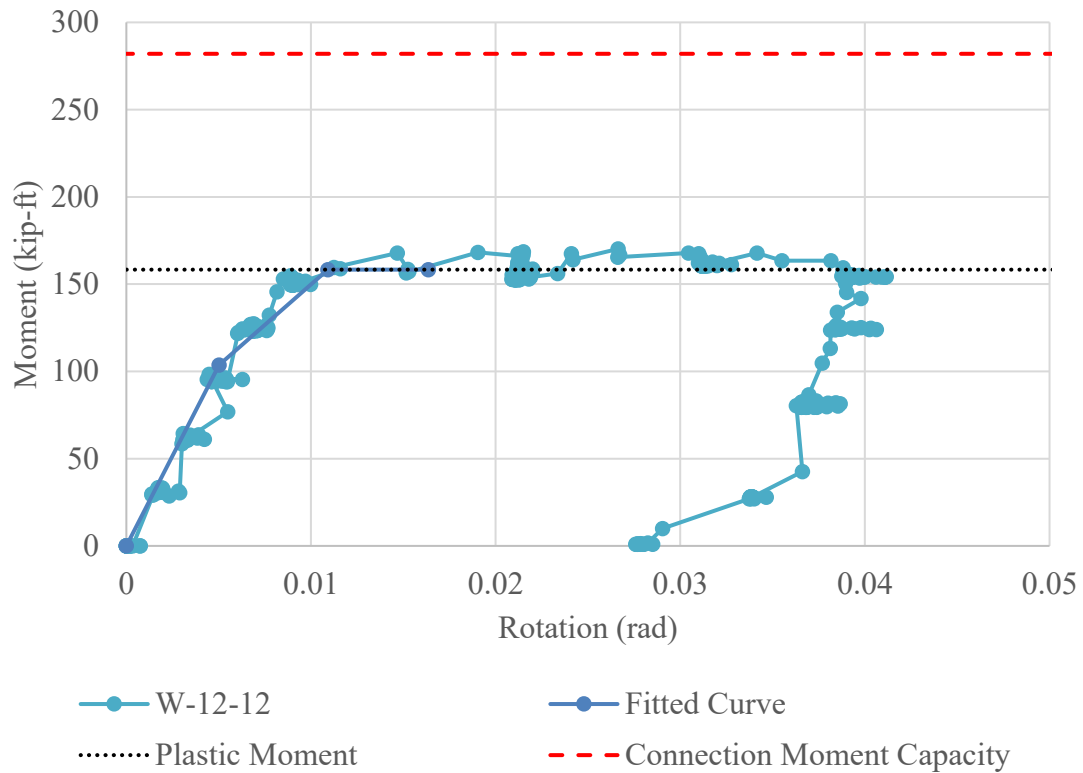


Figure 239. Application of fitted Moment vs Rotation curve to specimen W-12-12.

The guideline was applied to W-12-12 using the standard 0.78 coefficient and the undamaged connection capacity. The predicted curve agrees well with the experimental response across the initial stiffness phase, the degraded segment, and the capacity plateau, as shown in Figure 239.

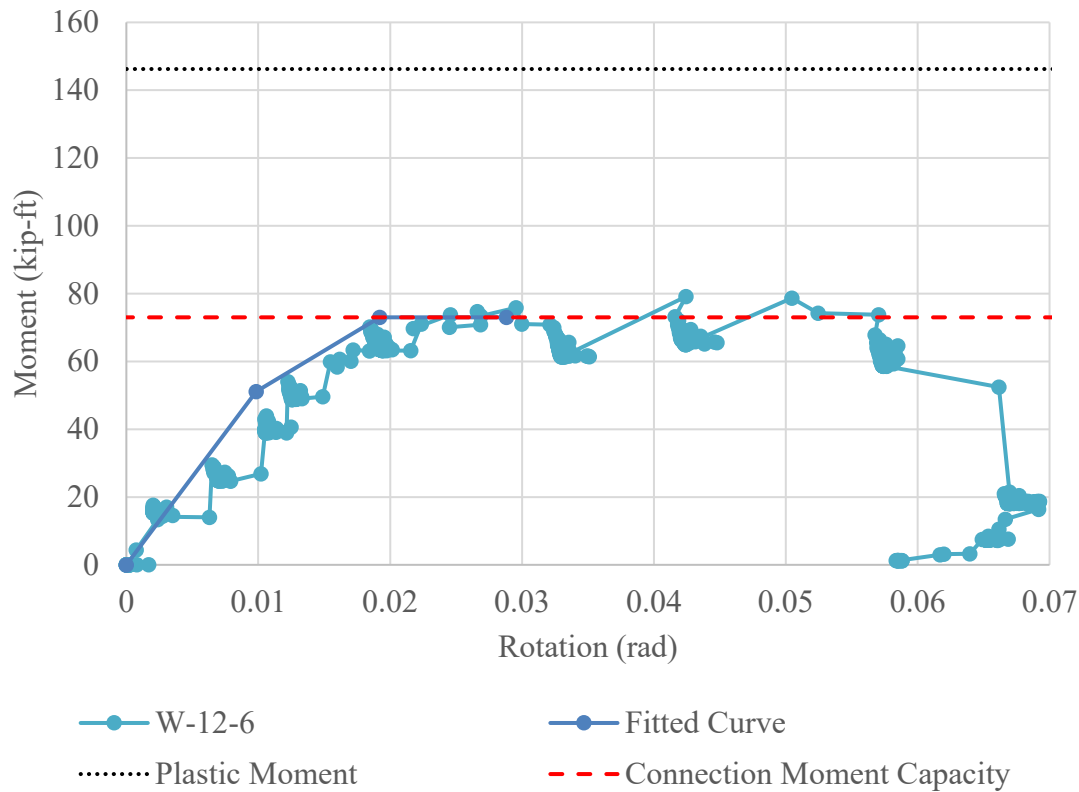


Figure 240. Application of fitted Moment vs Rotation curve to specimen W-12-6.

Finally, the guideline applied to W-12-6 is slightly unconservative: the predicted curve is modestly stiffer than the measured response in the initial stiffness phase, as shown in Figure 240. The governing capacity is predicted correctly, and the fit is otherwise acceptable, though the practitioner should be aware that the guideline may overestimate the initial rotational stiffness for this pile section at $L_e/d = 0.5$.

4.4 Application to Experimental Specimens

The nonlinear spring guideline was applied to each model specimen in FB-Multiplier. The numerical model uses identical geometry, material, and boundary conditions as described in Section 3, with the linear spring replaced by the four-point $M-\theta$ curve defined in Section 4.3. The resulting four-point $M-\theta$ curves are compared against the experimentally measured responses in Figures 241 through 248. Load tests W-10-15 1st and 2nd tries, and W-10-7.5 1st try are not depicted in this section as no relevant information can be extracted from them.

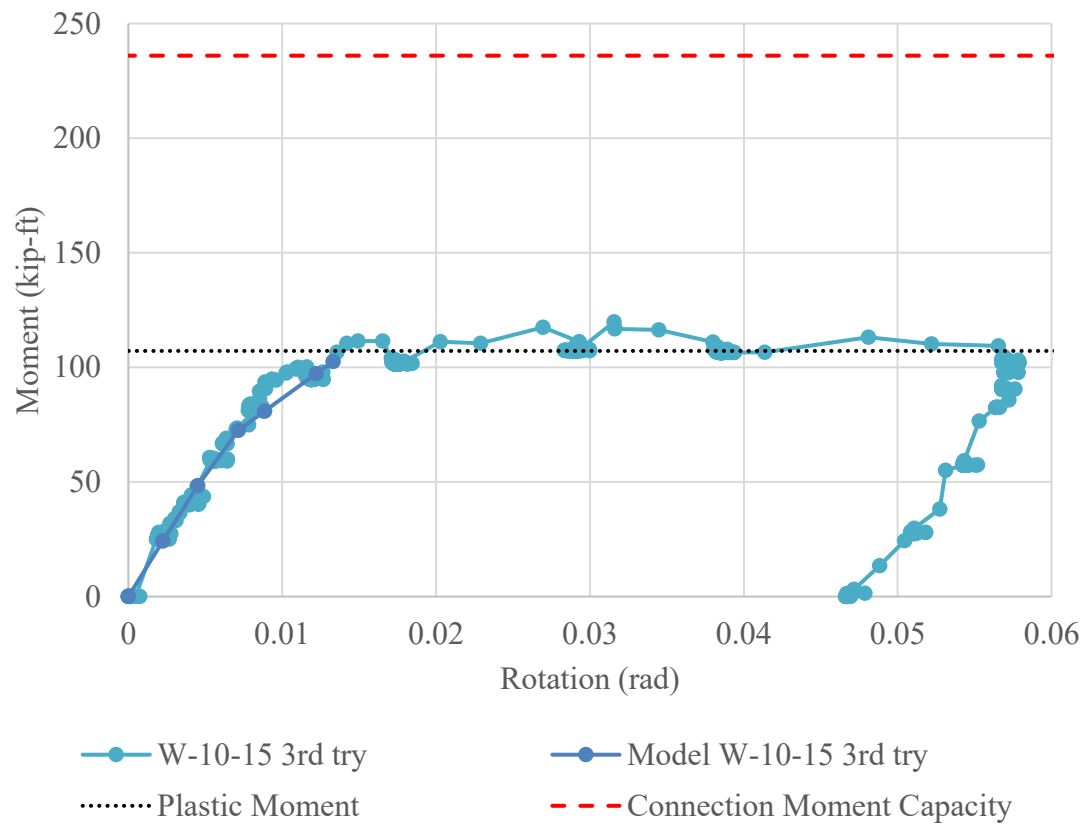


Figure 241. FB-Multiplier Model $M-\theta$ curve compared to experimental W-10-15 3rd try.

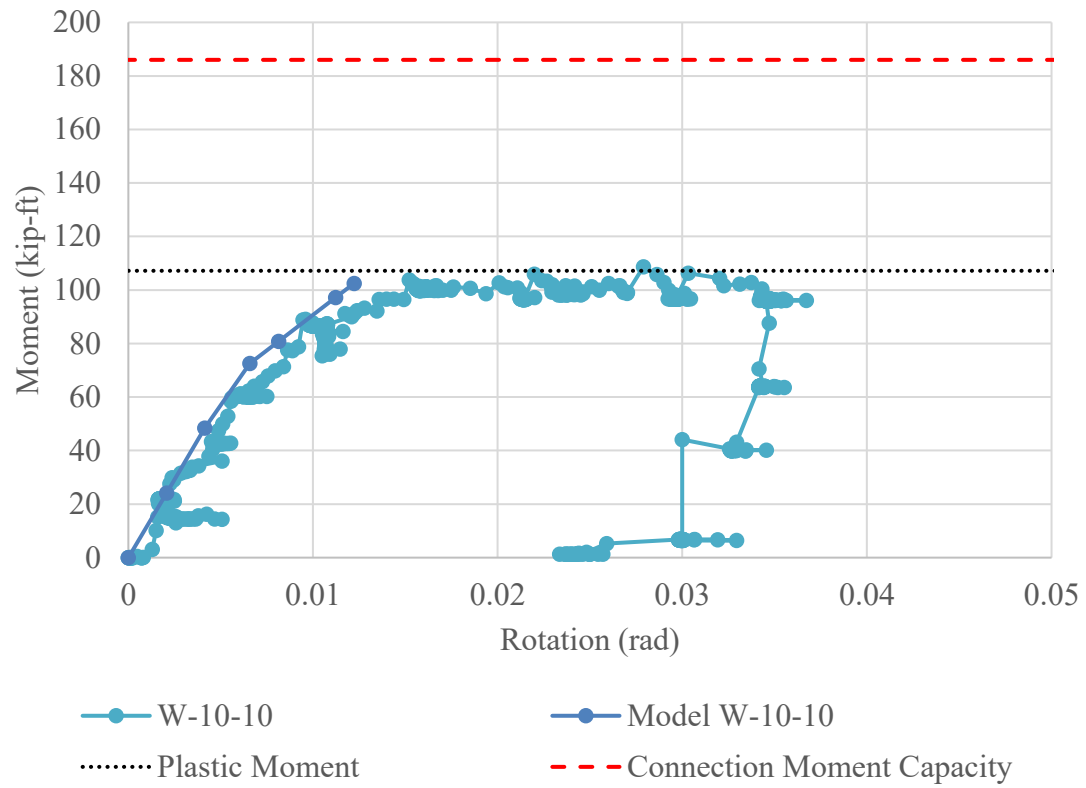


Figure 242. FB-Multiplier Model M- θ curve compared to experimental W-10-10.

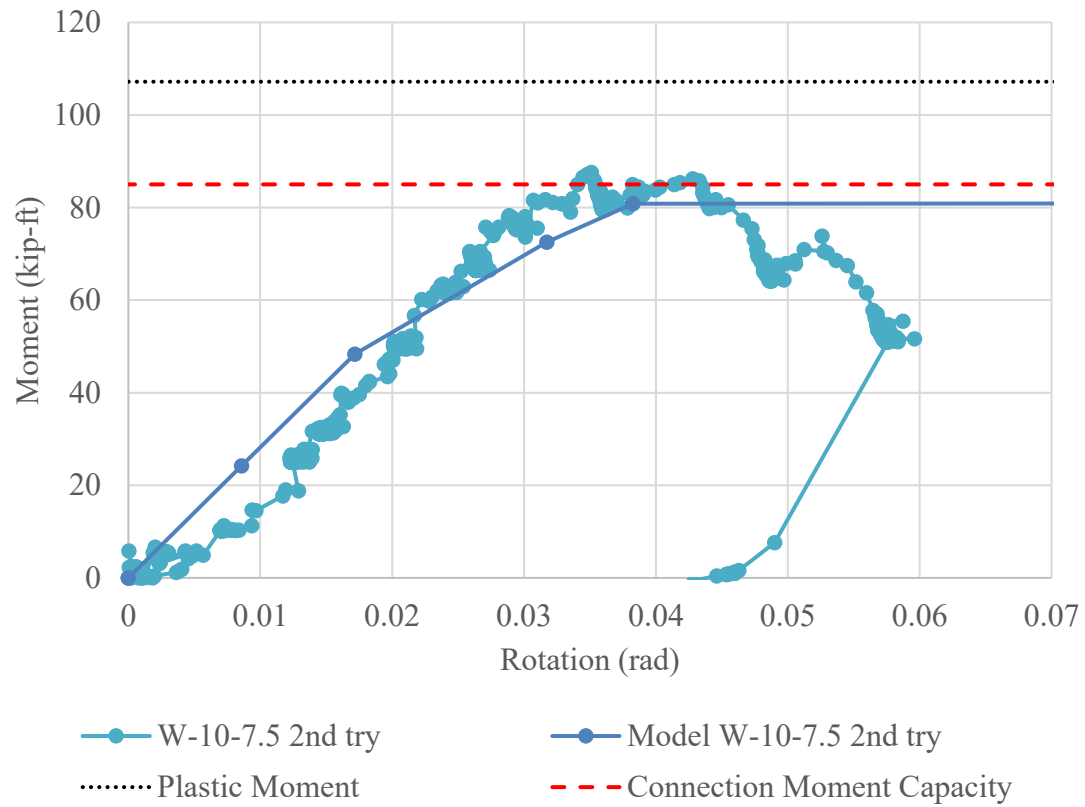


Figure 243. FB-Multiplier Model M- θ curve compared to experimental W-10-7.5 2nd try.

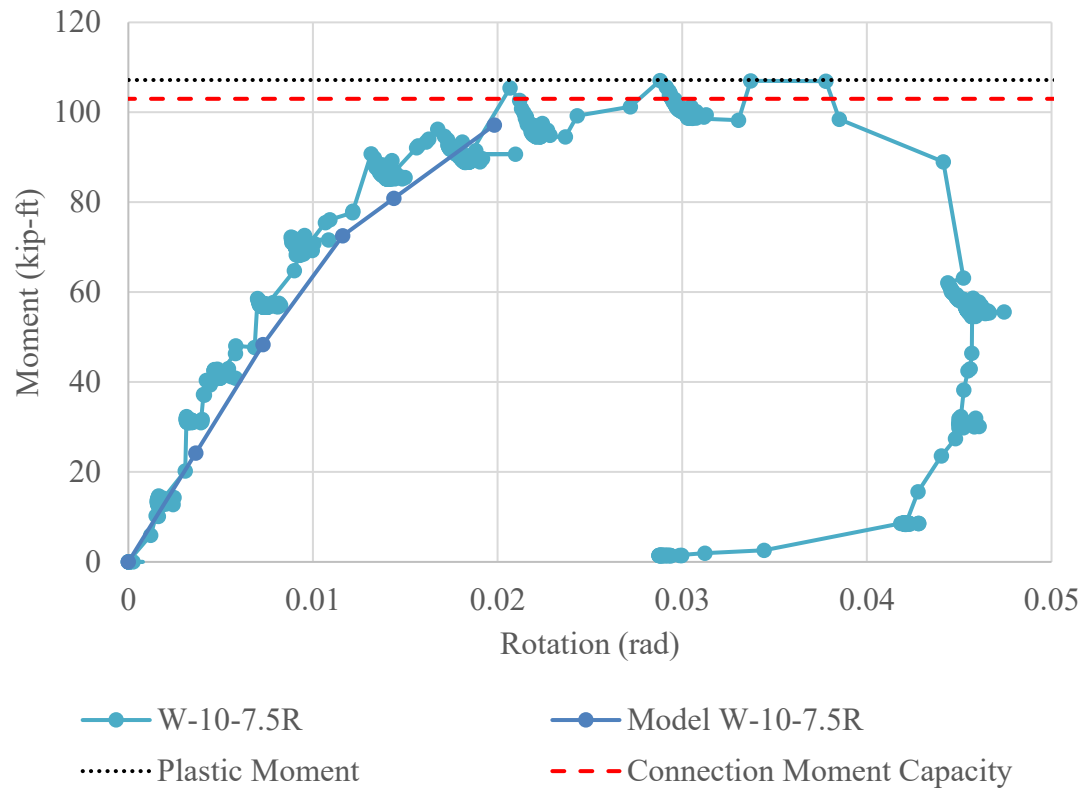


Figure 244. FB-Multiplier Model M-θ curve compared to experimental W-10-7.5R.

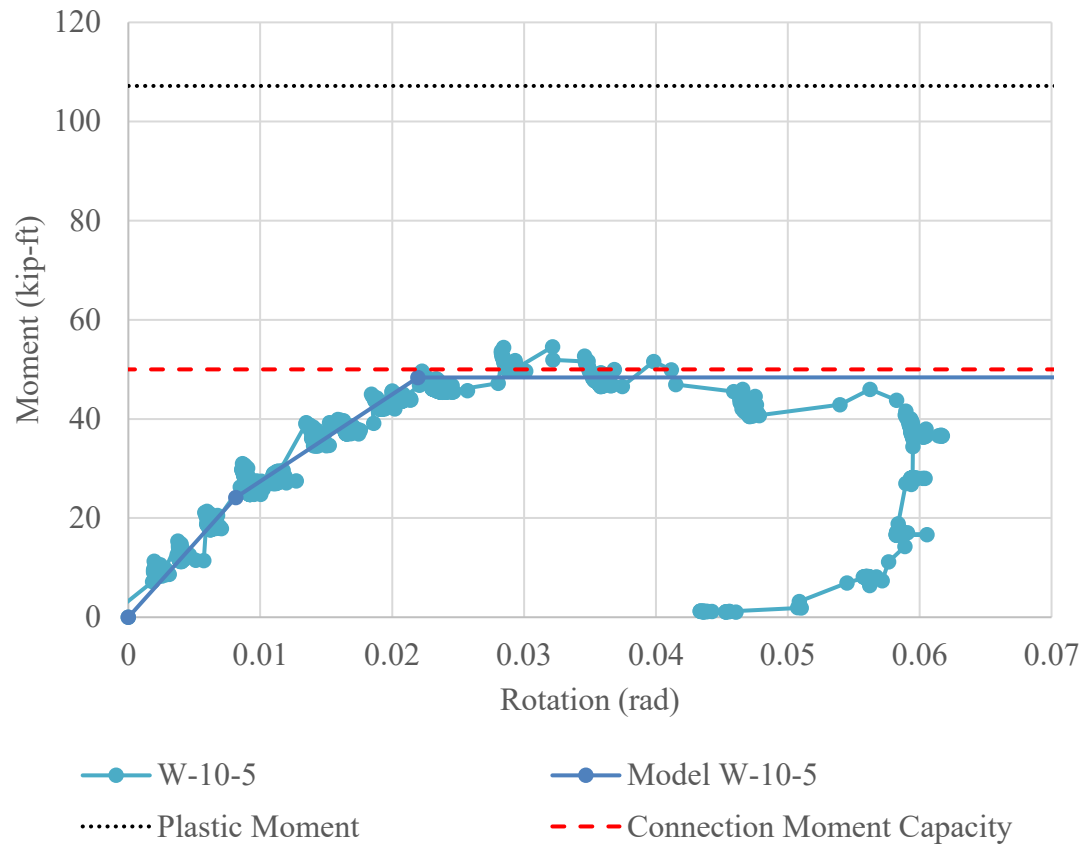


Figure 245. FB-Multiplier Model M-θ curve compared to experimental W-10-5.

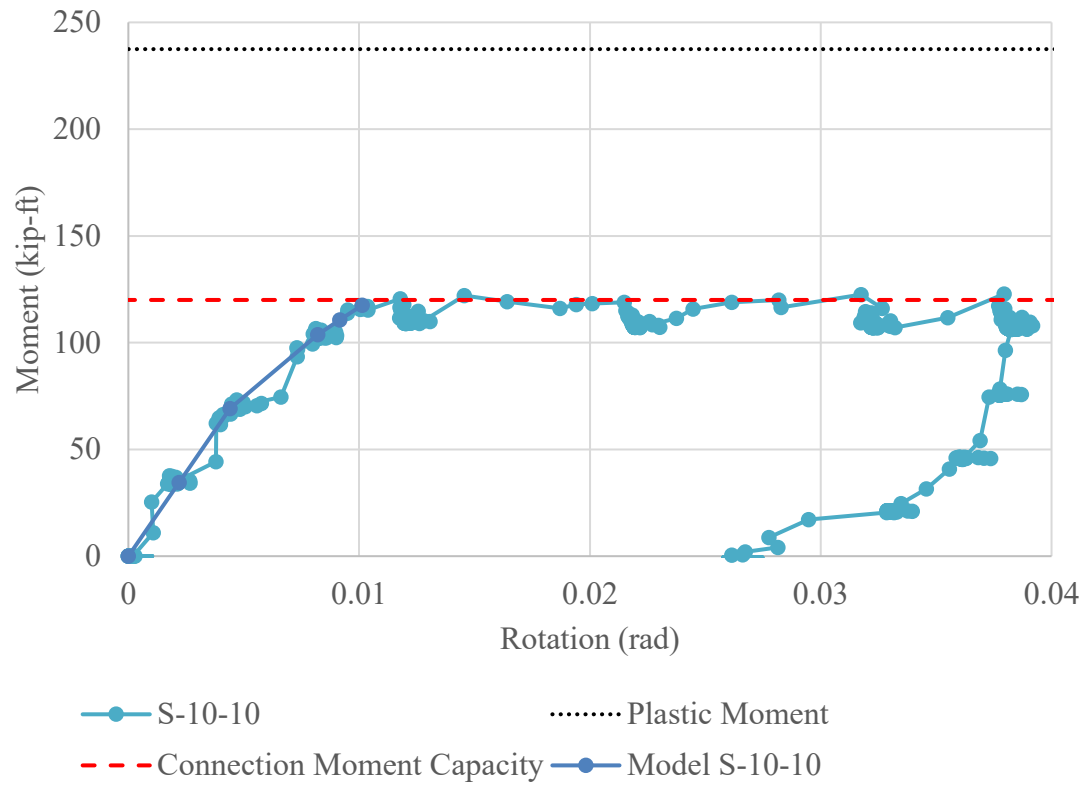


Figure 246. FB-Multiplier Model M- θ curve compared to experimental S-10-10.

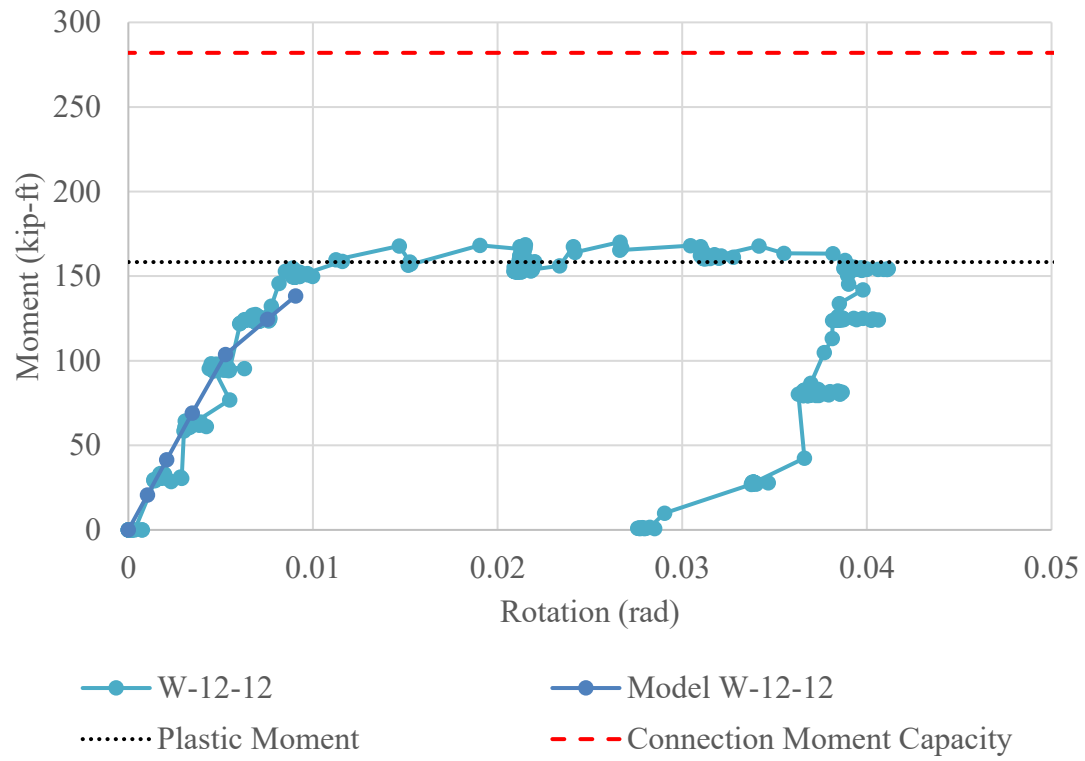


Figure 247. FB-Multiplier Model M- θ curve compared to experimental W-12-12.

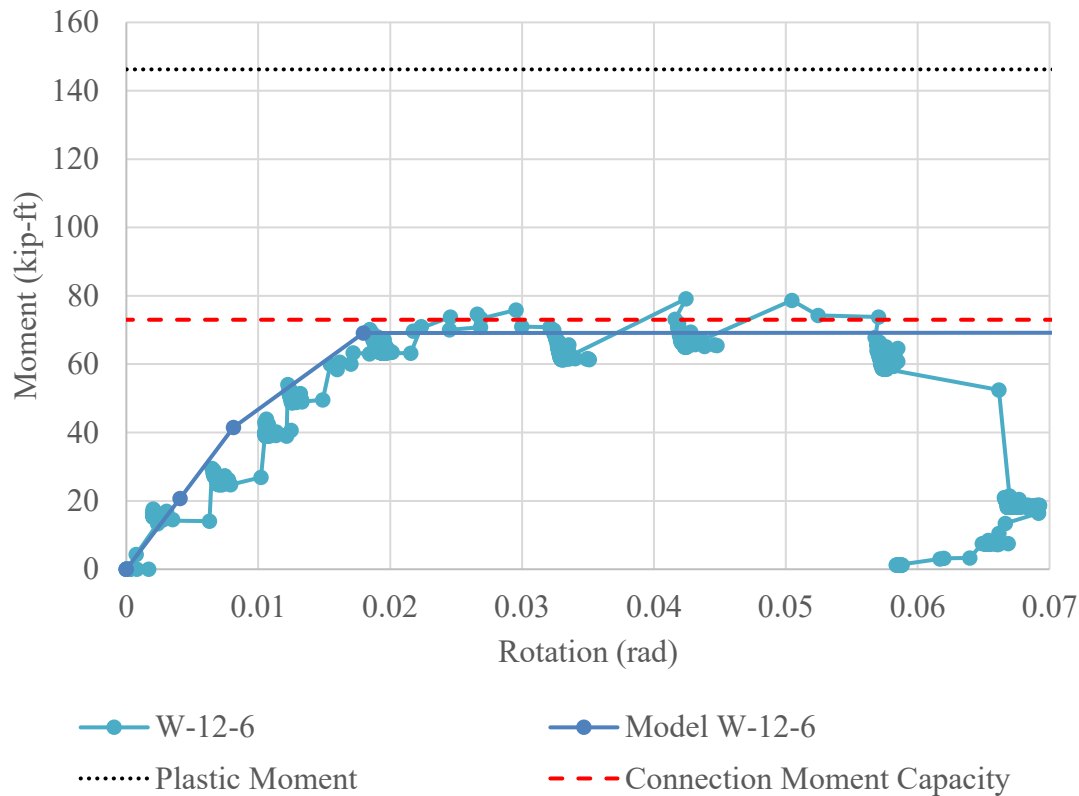


Figure 248. FB-Multiplier Model M- θ curve compared to experimental W-12-6.

As observed, the model M- θ curve fits well the experimental results. For steel-yielding specimens, the model stops converging as the software automatically calculates the pile's yield limit. For concrete-bearing specimens, the model was also a good fit and captured the entire nonlinear curve, since FB-Multiplier still does not have the capability to consider the connection moment capacity. One caveat, though, is that as the model approaches the “plateau level” imposed by the custom M- θ non-linear spring, rotation tends to increase to very large values.

4.5 Limitations

This guideline was calibrated against the HP10×42 and HP12×53 pile sections, over the range $L_e/d = 0.5$ to 1.5. The degradation factor of 0.450 and the 70% transition threshold are empirical values derived from this dataset and should not be extrapolated to different pile sections, connection geometries, or bending orientations without additional experimental validation.

5. Application to Experimental Pile Bents (Marshall et al, 2017, and Anderson et al, 2018)

The pile bents built and tested by Marshall et al. (2017) and Anderson et al. (2018) are reexamined here through the lens of connection behavior and the framework developed in this chapter. The individual specimen models in Section 3 and Section 4 were designed to isolate and calibrate the connection spring, using controlled boundary conditions and no soil.

The pile bent models presented here introduce all the features of a real bridge substructure: soil-pile interaction, multiple piles acting together, a reinforced concrete bent cap, and pile batter. This context allows the connection spring guideline to be evaluated under conditions that more closely reflect field practice.

Three configurations are analyzed: the vertical pile bent ($L_e = 18$ in), the battered pile bent ($L_e = 12$ in), and a theoretical variant of the battered bent with a reduced embedment depth ($L_e = 6$ in). All three use the nonlinear four-point spring defined in Section 4.

5.1 Model Configuration

The pile bent models were built using the software's PILE BENT structural type, which models the pile group together with the bent cap beam as a coupled system. Unlike the individual specimen models in Section 3, which isolated a single pile with no soil or adjacent piles, these models include all four piles, the reinforced concrete bent cap, and nonlinear soil resistance.

All analyses were performed as static, load-controlled, nonlinear analyses with both soil stiffness and pile material stiffness set to nonlinear. The solver was used with a convergence tolerance of 1.0 kip and a maximum of 350 iterations per load step. The PHRS parameter was set to zero for all four piles in each model, and connection moment transfer is provided entirely through the nonlinear four-point rotational spring defined in Section 4.3.

5.2 Geometry

All three models share almost the same bent cap geometry: four piles at a center-to-center spacing of 96 in (8 ft), with 24-in (2-ft) cantilever overhangs on each end. The total cap length between cantilever tips is 336 in (28 ft). The applied lateral load is concentrated at the tip of the right cantilever (Node 37, located 24 in beyond the outer pile), which represents the load transfer point from the superstructure.

For the vertical pile bent, all four piles are completely vertical. Pile lengths reflect the as-built geometry of the Marshall et al. (2017) experiment, ranging from approximately 344 to 396 in (28.7 to 33.0 ft) between piles due to the differing embedment depths and site conditions.

For the battered pile bent and the $L_e = 6$ in theoretical case, the two outer piles (Piles 1 and 4) are battered at a 1.5/12 slope. The two interior piles (Piles 2 and 3) are vertical. Pile lengths are approximately 408 in (34.0 ft) for all four piles.

The discrete rotational spring is placed between the first pile node (located 1 in below the cap) and the cap-connection node (at $Z = 0$) for each pile, consistent with the individual specimen models in Section 3.

All dimensions were extracted from the works of Marshall et al. (2017) and Anderson et al. (2018). All modeled piles were HP12x53, with dimensions extracted from AISC Steel Construction Manual (2023).

The constructed models are shown in figures 249 and 250.

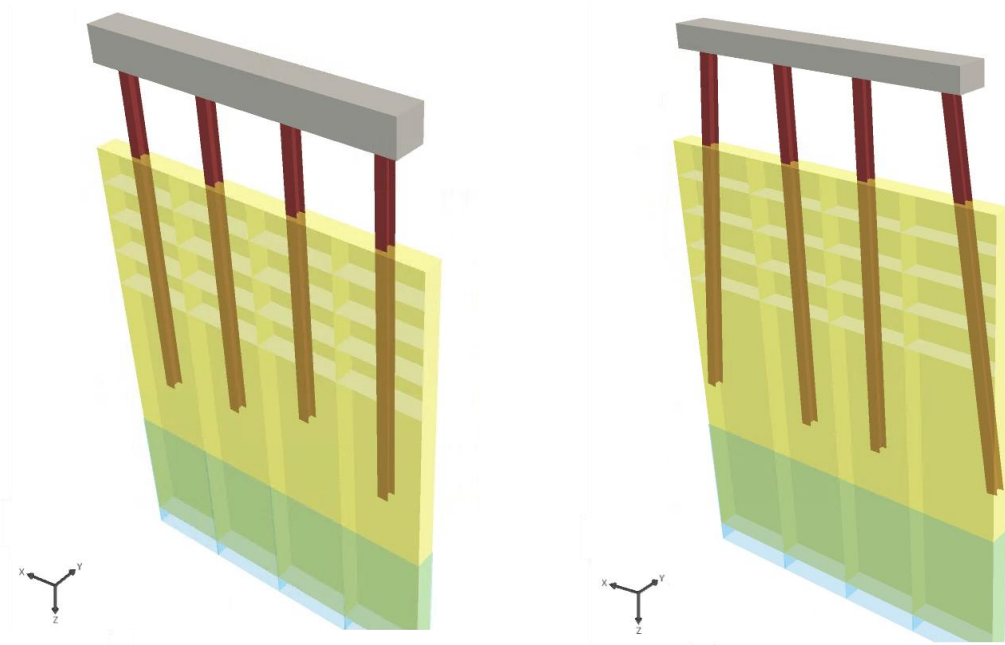


Figure 249. Model Vertical (left) and Battered (right) Pile Bents adapted from Marshall et al. (2017) and Anderson et al. (2018) - Load applied from left to right.

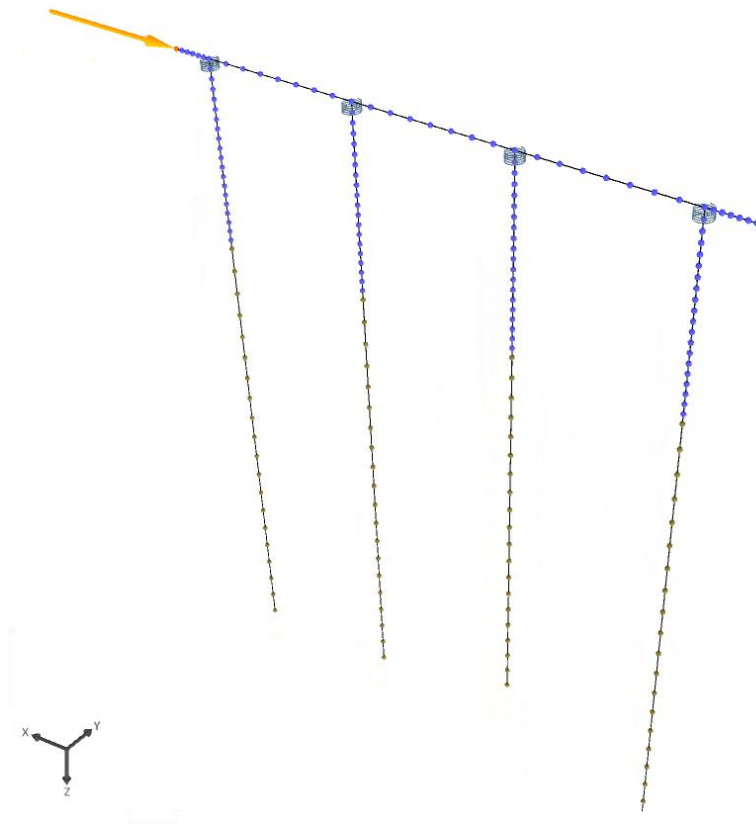


Figure 250. Model pile bent converted into nodes and elements and load application point.

5.3 Material Properties

All piles in the three models are HP12×53 sections and use the properties as seen in AISC Steel Construction Manual (2023). The steel material uses a nonlinear stress-strain curve with yield stress $F_y = 58$ ksi and elastic modulus $E = 29,000$ ksi, consistent with the coupon tests reported by Marshall et al. (2017). The entire stress-strain curve was built using the Karthik and Mander (2011) stress-strain relationship.

The reinforced concrete bent cap is a 36×36 in section modeled with nonlinear material behavior using the Karthik and Mander (2011) stress-strain relationship. The concrete compressive strength is $f'_c = 5.2$ ksi ($E_c = 5,658$ ksi). The longitudinal reinforcement consists of 10 bars with a total area of 13.68 in^2 , with mild steel yield stress $f_y = 60$ ksi. The reinforcement layout replicates the as-built cap cross-section reported in the experimental programs.

5.4 Model Boundaries

In contrast to the individual specimen models, no explicit fixed boundary conditions were applied to the bent cap or piles. Lateral resistance is provided by the combination of soil springs acting along the embedded pile length, pile flexural stiffness, and the discrete rotational springs at the cap connections.

The pile heads (cap-connection nodes) are pinned ($PHRS = 0$), and all moment transfer at those nodes flows through the rotational springs at the connections.

5.5 Soil Properties

Soil resistance is modeled using nonlinear p-y curves applied along the embedded length of each pile. The soil surface was specified at $Z = 9.300$ ft (111.6 in) below the cap, consistent with the reported site conditions in both experimental programs. As seen in Marshall et al. (2017), the load tests were conducted at the Auburn University National Geotechnical Experimentation Site (AUNGES), and its stratigraphy is comprised mostly of silty sand or sandy silt, at the location where the pile bents were built. The authors provided SPT and Dilatometer tests performed in the site. From these, the p-y curves seen in Figure 251 were derived, using API (1993) method.

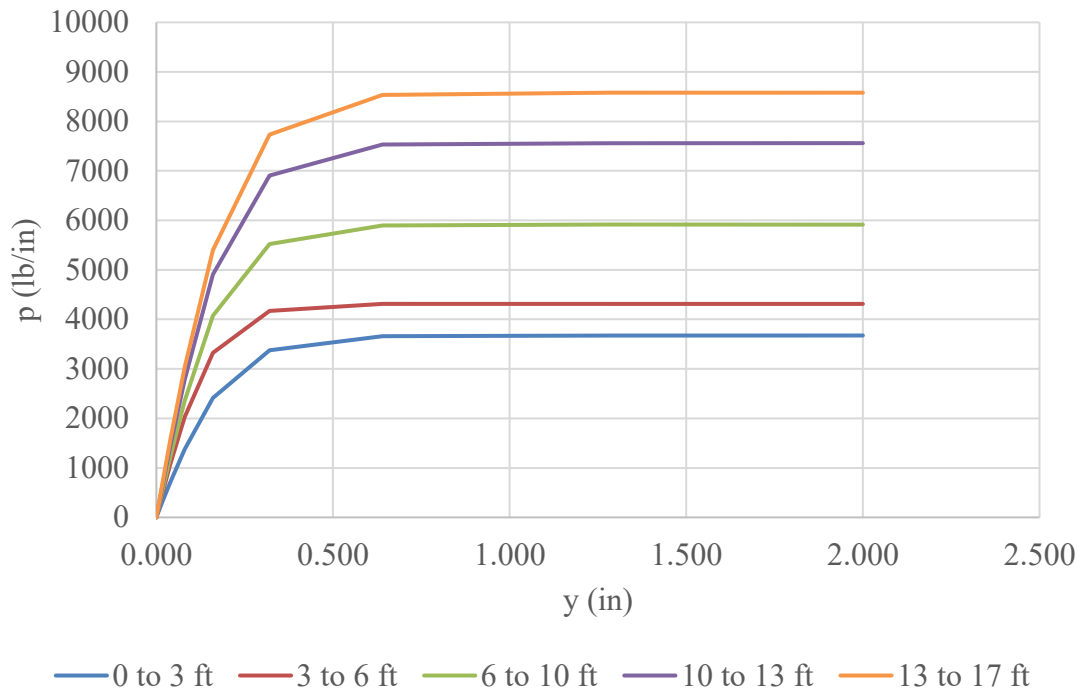


Figure 251. Derived p-y curves for the soil where the Pile Bents were installed in the work of Marshall et al. (2017).

5.6 Vertical Pile Bent, $L_e = 18$ in

The vertical pile bent model with $L_e = 18$ in of embedment corresponds to the as-built configuration of the full-scale bent tested by Marshall et al. (2017). Fourteen lateral load cases were applied at the end of the bent cap (Node 37), with load magnitudes matching the experimental test sequence: 4.7, 13.7, 22.6, 32.1, 41.9, 52.3, 60.0, 70.8, 79.4, 89.8, 98.2, and 107.5 kips for load cases 1 through 12.

The rotational springs at the connection were built according to section 4 guideline for a pile embedment of 18 in. Important to note that 18-in embedment falls outside of the guideline scope.

Figure 252 shows the model load vs displacement curves measured at the load application point and a comparison to the experimental curve.

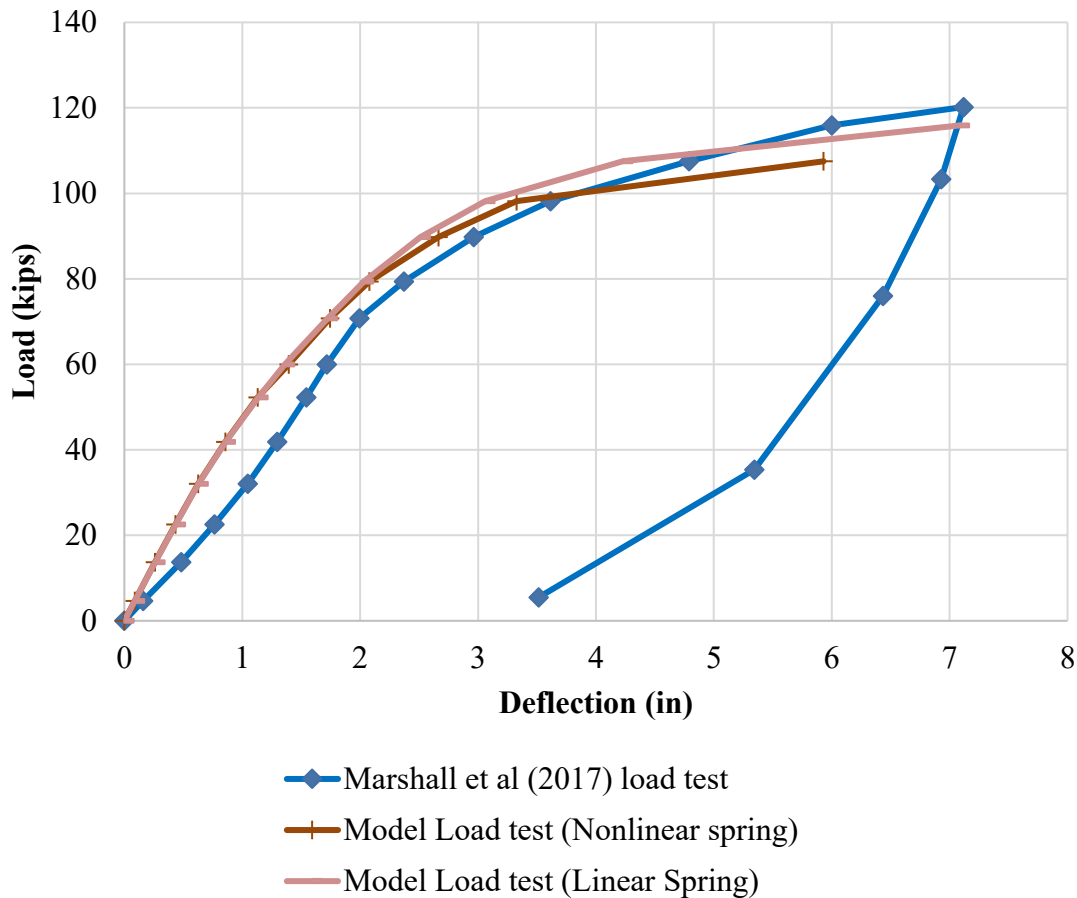


Figure 252. Comparison between Experimental and Model load vs displacement curves for the Vertical Pile Bent ($L_e = 18$ in).

For the vertical pile bent case, it is clear that using either a nonlinear or a linear rotational spring at the connection yielded similar results. The linear spring, however, yielded a slightly closer result to the real data. Both curves were slightly stiffer than the actual load test.

However, for this depth of pile embedment into the cap, the steel pile yields before the connection fails. This means that regardless of the chosen rotational spring type, the FB-Multiplier model is limited by the pile's structural plastic moment.

Figure 253 shows the moment distribution of the four piles as the lateral load increases. Pile 4 is the one closest to the load application point. Moment is drawn on the tension side.

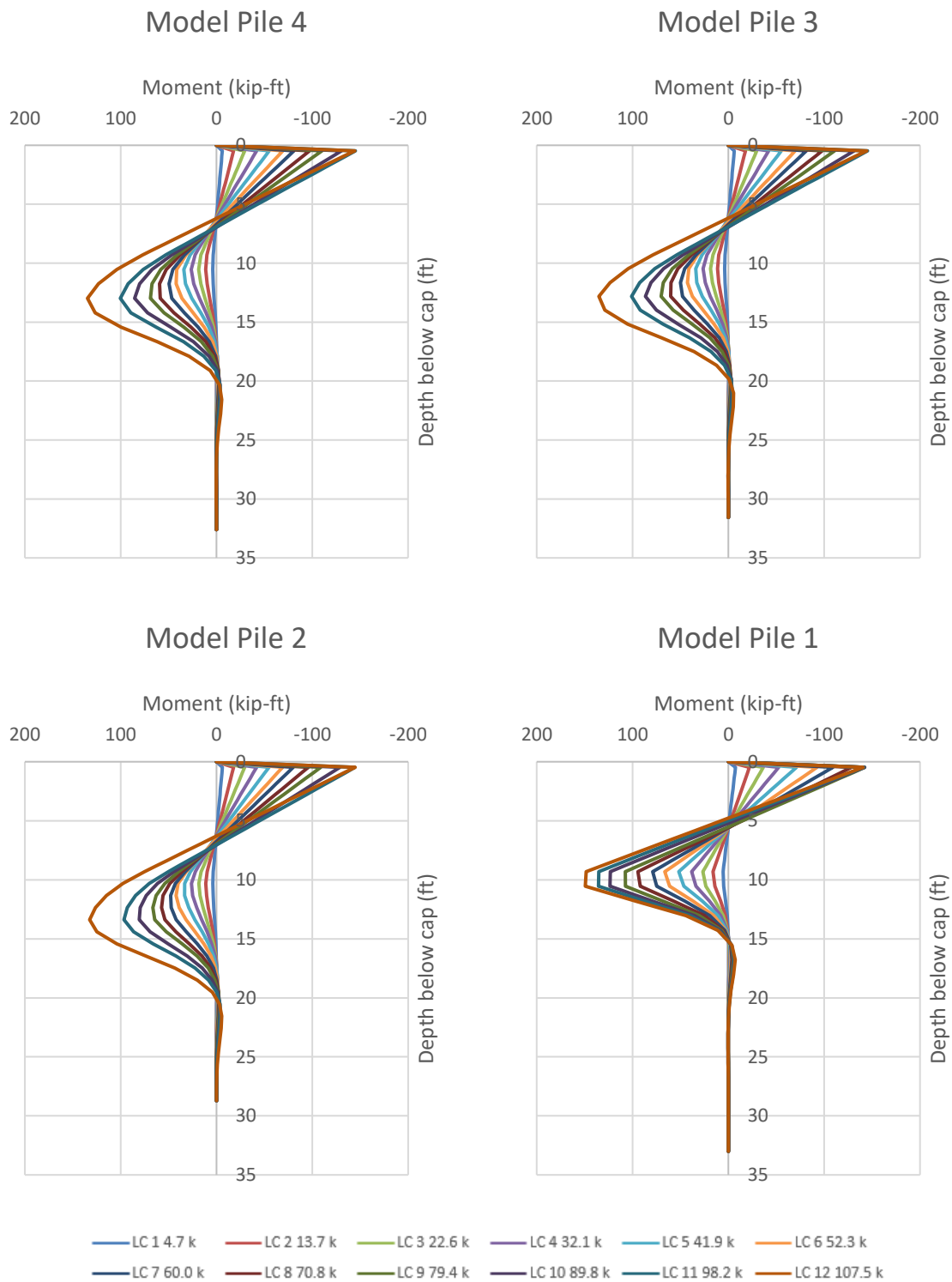


Figure 253. Moment profile for piles in the Vertical Pile Bent ($L_e = 18$ in).

The moment profiles show that the connection was able to carry the full plastic moment of the pile. At approximately 9.3 ft (ground level), the piles also had a large moment load, reaching their plastic limit as well.

5.7 Battered Pile Bent, $L_e = 12$ in

The battered pile bent model with $L_e = 12$ in of embedment corresponds to the as-built configuration of the full-scale bent tested by Marshall et al. (2017). Fourteen lateral load cases were applied at the end of the bent cap (Node 37), with load magnitudes matching the experimental test sequence

The rotational springs at the connection were built according to section 4 guideline for a pile embedment of 12 in.

Figure 254 shows the model load vs displacement curves measured at the load application point and a comparison to the experimental curve.

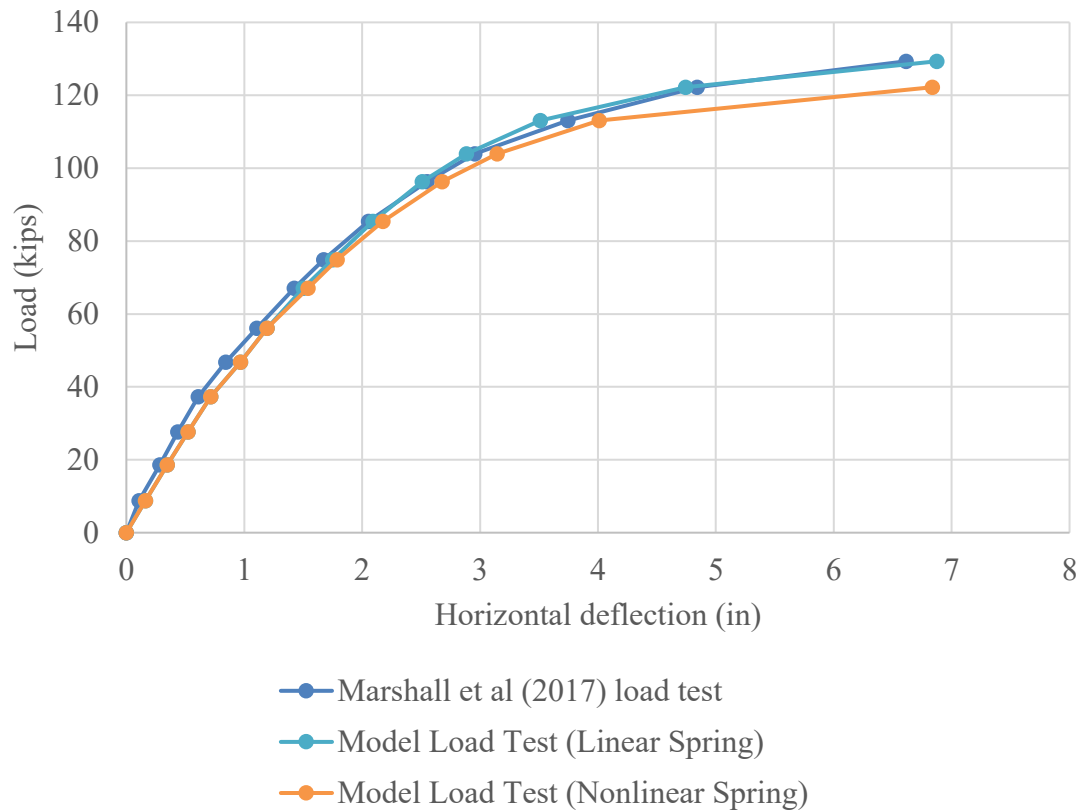


Figure 254. Comparison between Experimental and Model load vs displacement curves for the Vertical Pile Bent ($L_e = 12$ in).

Similar to the vertical pile bent, both modeled curves are similar to the experimental curve, the nonlinear one being slightly more conservative. Again, since the pile yields before the connection fails, the choice of rotational spring does not change the results much.

Figure 255 shows the moment distribution of the four piles as the lateral load increases. Pile 4 is the one closest to the load application point. Moment is drawn on the tension side.

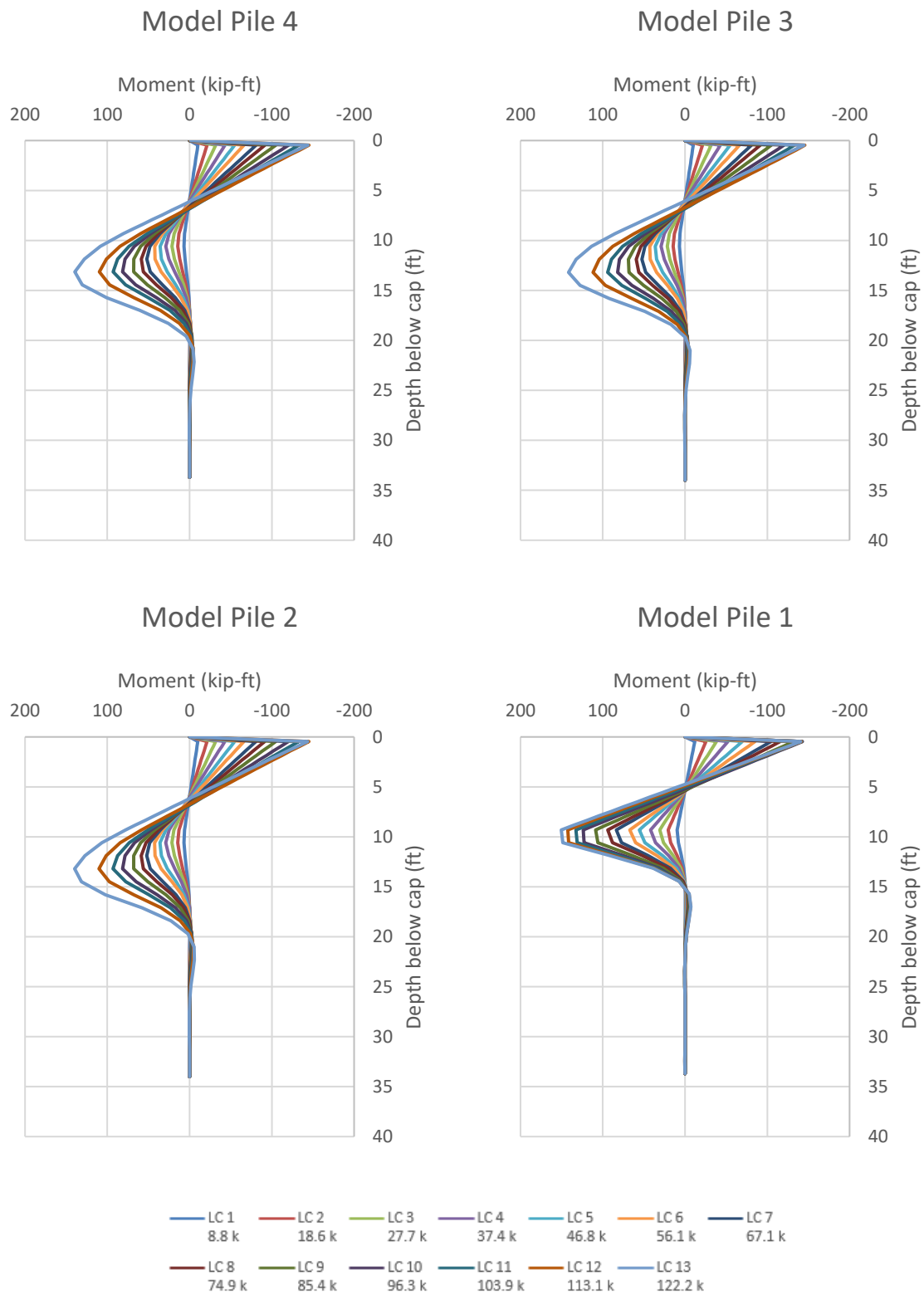


Figure 255. Moment profile for piles in the Battered Pile Bent ($L_e = 12$ in).

As observed, the moment profiles for the battered pile bent are very similar to the vertical pile bent moment profiles. The piles also carried the full plastic moment to the connection.

5.8 Theoretical Case: Battered Pile Bent with $L_e = 6$ in

In this section, a theoretical pile bent with the same characteristics of the previous section is modeled, this time with the intent of inputting a rotational spring simulating a 6-in pile embedment. The purpose of this exercise is to observe how a pile bent with shallow pile embedment into the cap (6 in) behaves under lateral load, when compared to the first two cases.

The rotational springs at the connection were built according to Section 4 guidelines for a pile embedment of 6 in.

Figure 256 shows the model load vs displacement curves measured at the load application point, with the use of a linear rotational spring and a nonlinear rotational spring at the pile-to-cap connections. Naturally, no experimental curve is shown.

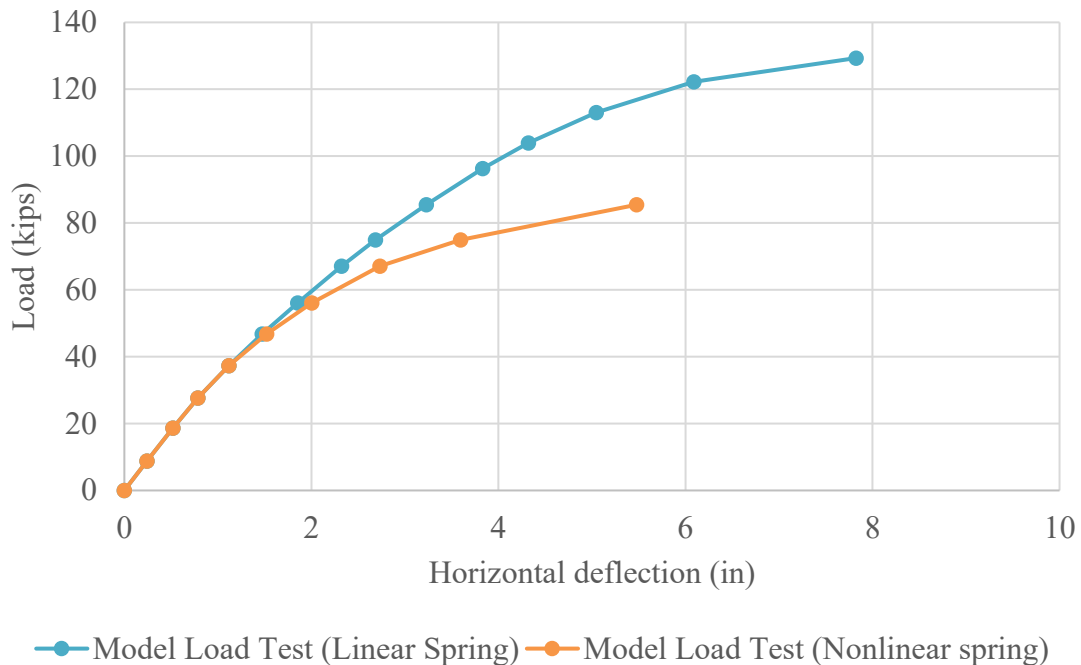


Figure 256. Model load tests for a theoretical battered pile bent with 6-in pile embedment.

As expected, using as 6-in equivalent linear or nonlinear spring largely changes the outcome of the load test. With a linear spring, the FB-Multiplier carries the model until plastic moment capacity of the pile. With a nonlinear spring, on the other hand, the model stops converging closer to 90 kip. This case shows the importance of using a nonlinear spring as a pile-to-cap connection. In a hypothetical case, with a large plastic moment but embedded only 1 ft into the cap, as is the design choice for many DOTs, this represents a real risk of underestimating the connection capacity when using a software such as FB-Multiplier, as well as underestimating its rotational behavior.

Figure 257 shows the moment distribution of the four piles as the lateral load increases. Pile 4 is the one closest to the load application point. Moment is drawn on the tension side.

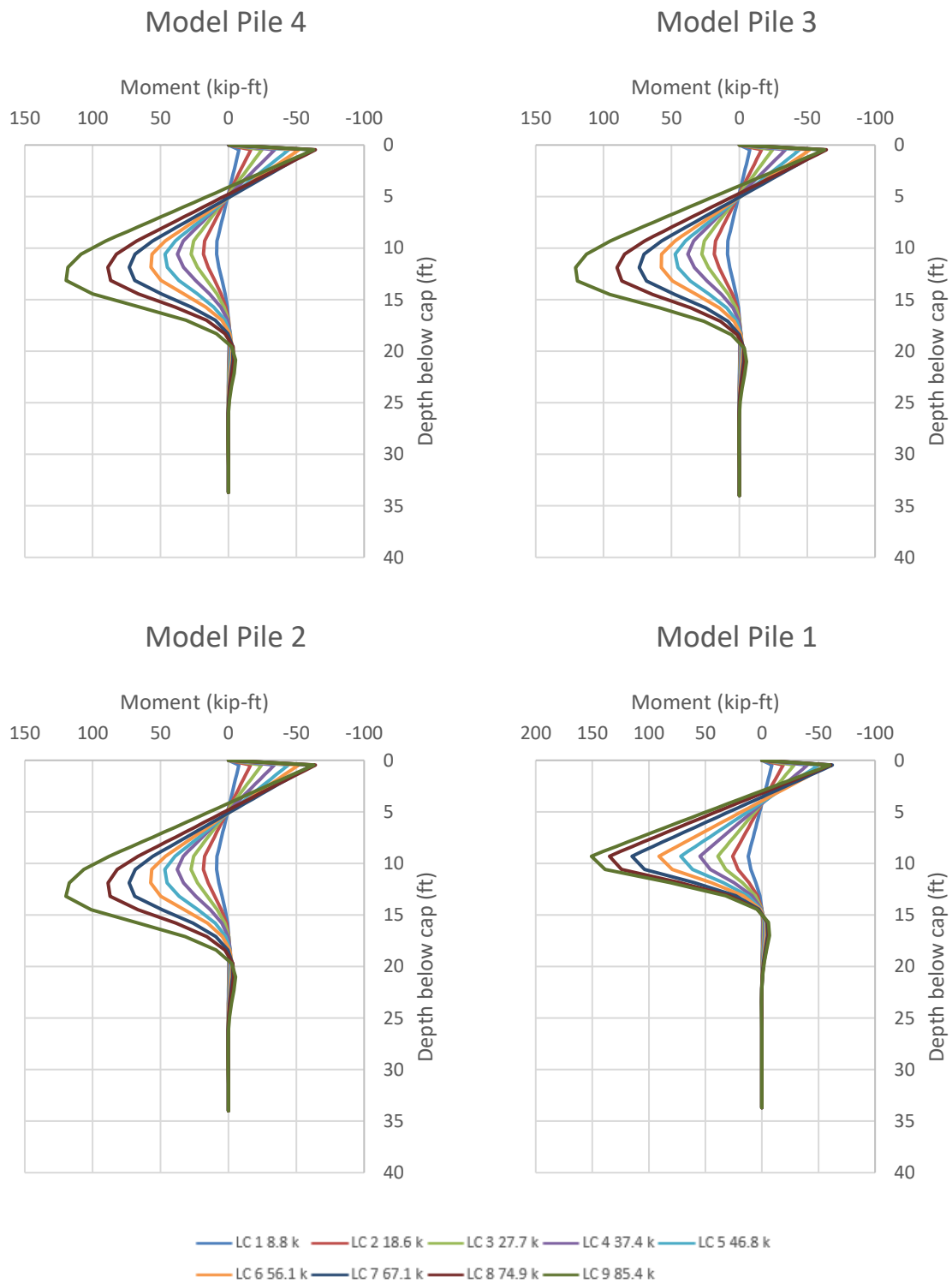


Figure 257. Moment profile for piles in the Theoretical Battered Pile Bent ($L_e = 6$ in).

As observed, for a theoretical 6-in embedment, the piles were able to carry less than half the moment as seen for 18-in or 12-in embedment cases. The maximum demand, therefore, shifts to ground level (9.3 ft) causing pile yield at this point. Furthermore, in pile 1, at the connection the moment demand reaches its peak at around 60 kip-ft, indicating that it is approaching connection failure long before the pile midsection achieves its plastic limit.

5.9 Comparison across Embedment Depths

Three embedment depths were analyzed to span the range of connection behavior observed in this research program: $L_e = 18$ in ($L_e/d = 1.5$) for the vertical pile bent, $L_e = 12$ in ($L_e/d = 1.0$) for the battered pile bent, and $L_e = 6$ in ($L_e/d = 0.5$) as a theoretical battered configuration. The first two correspond to as-built experimental configurations and the third was introduced to examine bent-level response when concrete bearing governs the connection. Figure 258 overlays the load-displacement curves for the three configurations under both spring formulations.

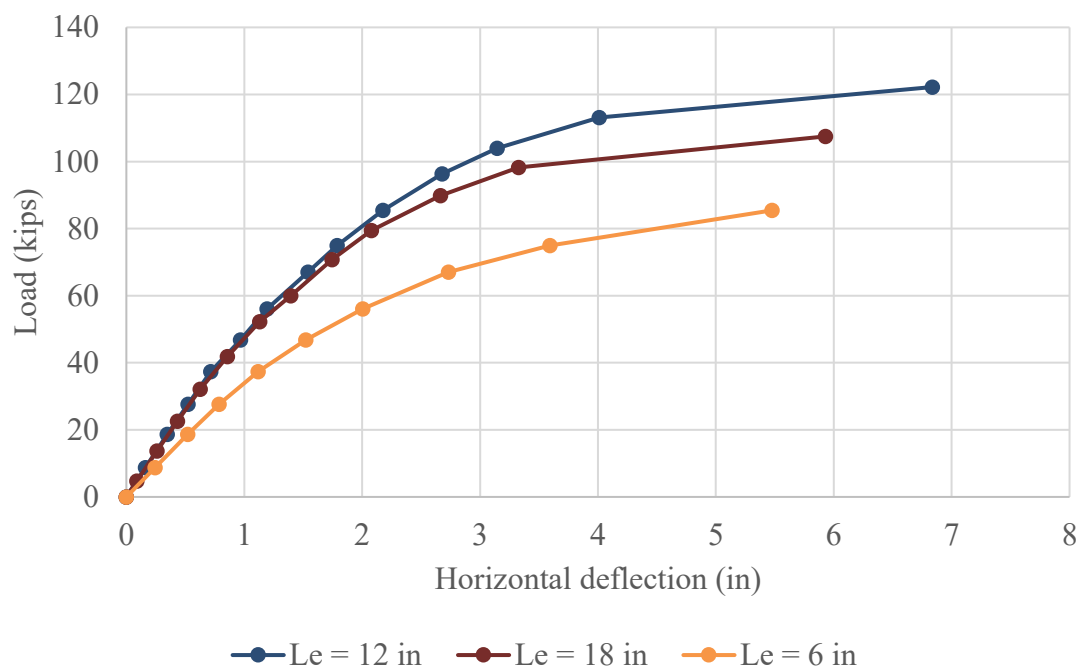


Figure 258. Model load vs displacement curves for pile bents with varied embedment depths, with nonlinear connection springs.

The critical difference between the three cases is the controlling failure mode at the connection. For $L_e = 18$ in and $L_e = 12$ in, the spring capacity moment equals the pile's plastic moment (approx. 155 kip-ft), because the pile steel yields before the concrete bearing zone reaches its limit. In these cases, the pile is the structural “weak link”, not the connection. As a direct consequence, the linear and nonlinear spring formulations produce nearly identical load-displacement responses, and the choice of spring type has minimal effect on the predicted system capacity.

The $L_e = 6$ in case is the opposite. The spring maximum moment is approx. 70 kip-ft, controlled by a theoretical concrete bearing capacity rather than the pile plastic moment. The nonlinear spring prevents the connection from transmitting moment beyond what the concrete bearing zone can sustain. Without this constraint, the linear spring would allow unbounded moment growth at the pile-to-cap interface, allowing the model to continue loading until the pile's own plastic limit is reached, overestimating the bent's lateral capacity. This has a direct practical consequence: FB-MultiPier has no built-in mechanism to enforce a concrete bearing limit at the pile-to-cap connection. Any model

using a linear spring for a shallow-embedment connection will be non-conservative in its prediction of lateral capacity by a margin that grows as L_e/d decreases.

A secondary effect of reduced embedment is the redistribution of maximum moment demand along the pile. For the 18-in and 12-in cases, the maximum pile moment occurs at the pile head, where the spring saturates at M_p . Simultaneously, the pile body also approaches M_p at the soil surface (9.3 ft depth), reflecting double-curvature demand consistent with partial fixity. For the 6-in case, the connection moment is capped at 70 kip-ft, and the maximum moment in the pile body (119 kip-ft at ground level) approaches the plasticity limit. The critical section shifts from the pile-to-cap interface to the pile at ground level, and the structural failure mechanism changes from connection-controlled to pile-controlled at the soil surface, but at a much lower applied load.

Taken together, the three cases establish a clear pattern for pile bents supported by HP12 piles: at $L_e/d \approx 1.0$ or above, the pile section governs and spring type is inconsequential. At $L_e/d \approx 0.5$, concrete bearing governs the connection and the nonlinear spring is essential for a correct prediction of system behavior. The boundary between these regimes lies within the tested L_e/d range and is consistent with the transition in governing failure mode documented in the individual specimen program (Chapter 7).

Chapter 9: Conclusion

1. Summary

Steel H-pile to concrete bent cap connections with plain pile embedment are standard in bridge construction across the United States, especially for short-span bridges. However, the relationship between embedment depth and the degree of rotational fixity at the connection has not been systematically quantified for HP pile sections prior to this work.

State transportation departments specify embedment depths ranging from 6 to 24 inches with limited experimental basis specific to HP piles, and no unified analytical framework existed to predict how embedment depth governs either the connection moment capacity or the initial rotational stiffness. This dissertation addressed that gap through a full-scale experimental program designed to characterize the rotational behavior of HP pile-to-concrete bent cap connections across a range of embedment depths, for two pile sections and two bending orientations.

Eight specimens were fabricated and subjected to eleven lateral load tests following a modified ASTM D3966 load protocol. The experimental matrix varied embedment depth from 5 to 15 inches for the HP10x42 section, corresponding to embedment-to-pile-depth ratios (L_e/d) of 0.5 to 1.5, and from 6 to 12 inches for the HP12x53 section ($L_e/d = 0.5$ and 1.0). One specimen investigated strong-axis bending of the HP10x42 section at $L_e/d = 1.0$, representing a connection subjected to loading in the bridge longitudinal direction. A comprehensive instrumentation program including MEMS tiltmeters, MEMS inclinometers, string potentiometers, slip meters, vibrating wire load cells, and strain gauges on the pile, reinforcement, and concrete cap surface was deployed to characterize failure modes, rotation profiles, moment distributions, and steel and concrete bearing behavior across all test configurations.

The experimental results were analyzed along the four research objectives defined in Chapter 1: (1) characterization of connection failure modes and moment capacities and

their dependence on embedment depth; (2) evaluation of existing analytical methods for predicting connection moment capacity and rotational stiffness against the experimental results; (3) development of an empirical relationship between embedment depth and rotational stiffness through a normalized regression analysis, together with an assessment of where the measured behavior falls within the pinned-to-fixed fixity spectrum; and (4) implementation of the experimentally derived connection behavior in FB-MultiPier through a four-point nonlinear connection spring guideline, validated against the individual laboratory specimens and the full-scale pile bents tested by Marshall et al. (2017) and Anderson et al. (2018).

2. Conclusions

The following conclusions are drawn from the experimental and analytical work presented in this dissertation, organized by the four research objectives defined in Chapter 1:

Regarding Objective 1 (characterization of governing failure modes and moment capacities):

- (1) A transition in governing failure mode occurs between $L_e/d = 0.75$ and $L_e/d = 1.0$ for the HP10x42 pile in weak-axis bending. Specimens at $L_e/d \geq 1.0$ failed by steel flange local buckling with ductile post-peak behavior and post-test concrete crack widths of 0.03 mm or less. Specimens at $L_e/d \leq 0.75$ failed by concrete cracking and crushing at the connection, with post-test crack widths exceeding 1 mm and rapid loss of load-carrying capacity after the peak. The HP12x53 series exhibited the same directional pattern: concrete failure at $L_e/d = 0.5$, steel failure at $L_e/d = 1.0$, consistent with the transition occurring at a similar embedment-to-pile-depth ratio across both pile sections.
- (2) Concrete-bearing failure becomes progressively more brittle with decreasing embedment depth. Specimen W-10-5 ($L_e/d = 0.5$), for example, exhibited diagonal cracking and interface gap formation at very low load levels, while deeper-embedment specimens showed no cracking at similar load levels. After reaching

peak load, shallow-embedment specimens did not sustain a load plateau, in contrast to the steel-failure specimens.

- (3) Strong-axis bending produces a rotational stiffness approximately 56% higher than weak-axis bending at the same pile section and embedment depth ($L_e/d = 1.0$), attributed to the larger and more uniformly distributed bearing area engaged when both flange faces bear against the concrete simultaneously. Despite this higher stiffness advantage, the strong-axis connection fails by concrete bearing at $L_e/d = 1.0$ (unlike the weak-axis bending) because the pile's strong-axis structural capacity is approximately twice its weak-axis capacity, making concrete bearing the controlling limit before steel yields. This illustrates that the controlling failure mode depends not only on embedment depth but on bending orientation. This creates a possible design conflict: connections adequately designed for bridge transverse loading may behave differently under longitudinal loading.

Regarding Objective 2 (evaluation of existing analytical methods):

- (4) Among the five analytical methods evaluated for predicting connection moment capacity, Kappes et al. (2021) provided the most balanced predictions for weak-axis concrete-bearing failures, with a mean predicted-to-actual ratio of 0.927 across the three undamaged concrete-bearing specimens. Marcakis and Mitchell (1980) is a reasonable alternative due to its inherent conservatism, but is unconservative for the strong-axis configuration.

Regarding Objective 3 (relationship between rotational stiffness and embedment depth, and position within the fixity spectrum):

- (5) No tested connection behaved as a pinned condition. Every embedment depth tested, including the shallowest ($L_e/d = 0.5$), produced measurable moment transfer at the pile head and a decreasing moment profile with embedment depth. The degree of fixity computed using the framework of Shishlov et al. (2022) ranged from 50% at $L_e/d = 0.5$ to 93% at $L_e/d = 1.5$ across the undamaged specimen series. No tested connection behaved as a fixed condition either. In design,

assuming either pinned or fixed can render the connection conservative or unconservative, depending on the perspective. For a fixed assumption, deflection is underestimated. For a pinned assumption, deflection will be overestimated, and the moment demand, underestimated.

- (6) The normalized rotational stiffness of plain HP-pile embedment connections follows a simple power-law relationship with the embedment-to-pile-depth ratio. The regression is as follows:

$$\frac{K_{\theta}}{EI} = 0.78 (\text{ft} * \text{rad})^{-1} \left(\frac{L_e}{D} \right)^2 \quad (52)$$

Where:

K_{θ} = Rotational stiffness (kip-ft/rad)

E = Pile modulus of elasticity (kips/ft²)

I = Pile moment of inertia (ft⁴)

L_e = pile embedment into the concrete cap (in).

D = Pile depth or width (in)

It was fit to seven undamaged weak-axis data points from the HP10x42 and HP12x53 series. The relationship is validated for $L_e/d = 0.5$ to 1.5 and for HP10 and HP12 pile sections, and should not be extrapolated beyond these bounds without additional experimental validation.

In normalized stiffness regression curve, the exponent of 2.0 is physically consistent with the assumed bearing mechanism and may not be an empirical coincidence. The moment transmitted to the connection at the cap surface is resisted inside the embedment by a moment formed by a couple due to the two concrete bearing areas. This moment equals the bearing force magnitude times the distance between them. This distance is proportional to L_e , and so are the bearing forces. Since rotational stiffness $K_{\theta} = M/\theta$ and both the moment arm and

the bearing force are each proportional to L_e at a fixed θ , the connection stiffness therefore scales as L_e^2 .

- (7) Moment capacity and rotational stiffness are independent properties governed by independent mechanisms, and this research showed they cannot be optimized simultaneously by the same design decisions. Deep embedment maximizes both stiffness and ductility, being controlled by steel yielding. Shallow embedment reduces stiffness and produces brittle concrete failure. Strong-axis orientation maximizes stiffness but triggers concrete bearing failure. Pre-damage degrades stiffness far faster than capacity. In every dimension examined, the two properties diverge. Current practice treats the connection as either pinned or fixed, collapsing this complexity into a single parameter. The research provides the experimental basis for treating connection stiffness and connection capacity as two separate design checks, each with its own limit state, just as flexure and shear are treated separately in beam design, for example.

Regarding Objective 4 (implementation in a practice-oriented numerical modeling framework):

- (8) Regarding numerical modeling with FB-MultiPier, the Pile Head Rotational Stiffness (PHRS) parameter and the Pile Top Rigidization feature are unsuitable for representing the partial fixity of plain pile embedment connections. PHRS couples initial rotational stiffness and moment capacity through a single scalar, preventing independent calibration of either property. Reducing PHRS to match the experimentally measured initial stiffness eliminates the connection's moment-carrying capacity and produces numerical instability at applied loads far below the observed failure. Since PHRS is the primary built-in parameter available to FB-MultiPier users for modeling non-fixed pile-to-cap connections, practitioners who apply PHRS to represent partial fixity face a systematic modeling inconsistency regardless of the value chosen, as no single PHRS value simultaneously produces correct initial stiffness and correct moment capacity. This is a structural limitation

of the parameter's formulation, implying that existing bridge models relying on PHRS to represent HP-pile partial fixity may carry systematic errors.

- (9) A four-point nonlinear connection spring guideline, calibrated against the experimental program, captures the full moment-rotation behavior of plain HP-pile embedment connections using a minimum set of standard inputs. The guideline uses the normalized rotational stiffness regression for initial stiffness, the Marcakis and Mitchell (1980) formulation for connection capacity, and two empirically calibrated parameters: a degraded stiffness factor of 0.450 and a transition onset at the lesser of the pile elastic yield moment (M_y) or 70% of the connection moment capacity. The guideline was validated against all eight tested specimens and the full-scale pile bents of Marshall et al. (2017) and Anderson et al. (2018), providing good agreement across both steel-governed and concrete-controlled failure modes. This guideline is proposed to be applied to FB-Multiplier software as a simple parameter where the user, upon selecting steel H-piles, simply determines the desired embedment length into the cap.

Without a moment limit at the connection, FB-MultiPier continues loading the connection beyond its physical concrete bearing capacity, overestimating the bent's lateral capacity by a margin that grows with decreasing embedment. FB-MultiPier has no built-in mechanism to enforce a concrete bearing capacity limit at the pile-to-cap interface. A non-linear spring applied to the pile-to-cap interface solves this issue, leading to a better behavior prediction.

3. Additional Findings

In addition to the conclusions above, the experimental program produced several significant observations:

- (1) Prior lateral loading reduces connection rotational stiffness by 19 to 65% per loading event, with the magnitude of degradation correlating more closely with the peak rotation imposed during prior loading. By contrast, connection moment capacity is reduced by only approximately 13% under comparable prior loading

conditions. This difference has a direct implication for bridge asset management: connections at in-service bridges that have experienced lateral loading from construction, traffic, or hydraulic events may exhibit substantially lower initial rotational stiffness than laboratory measurements on undamaged specimens would suggest without a proportional reduction in moment capacity. It is noted that these degradation data originated from specimens that were re-tested after prior interrupted or completed load tests, rather than from a purpose-designed damage protocol.

- (2) The TILT-HEAD instrument - a tiltmeter mounted to a rod welded to the pile head inside the cap and extending through the side cap - produced a physically coherent progression across the embedment range that independently corroborates the failure mode transition. At $L_e/d = 1.5$, TILT-HEAD registered negligible rotation throughout the entire test, indicating that the deep embedment suppressed pile head movement entirely. At $L_e/d = 1.0$ and 0.75 , counterclockwise rotation (opposite to the applied load direction) developed at intermediate-to-high load levels, reflecting the geometric behavior of the measurement rod as the pile head began to rotate: as the pile rotates in the direction of load, the rod tip rotates in the opposite direction in a lever-arm effect. The counterclockwise magnitude was larger at $L_e/d = 0.75$ than at $L_e/d = 1.0$, consistent with shorter embedment providing less rotational restraint and allowing greater rigid-body rotation of the embedded pile segment. At $L_e/d = 0.5$, the initial counterclockwise signal reversed to clockwise rotation at approximately 6 kips applied load, indicating that the concrete bearing zone had broken down sufficiently to release the constraint on the measurement rod.
- (3) The leading-face concrete surface compressive stress measured by the CONC BC strain gauge increased systematically with decreasing embedment depth across the W-10 series: from approximately 0.9 ksi in W-10-15 ($L_e/d = 1.5$), to 2.5 ksi in W-10-10 ($L_e/d = 1.0$), 3.5 ksi in W-10-7.5R ($L_e/d = 0.75$), and 4.0 to 4.5 ksi in W-10-5 ($L_e/d = 0.5$). The same escalation trend was observed in the W-12 series.

This progressive escalation in surface bearing stress provides direct experimental evidence of the stress condition driving the failure mode transition: at $L_e/d \geq 1.0$, the concrete bearing demand is modest and does not govern. Below $L_e/d = 0.75$, it approaches the concrete compressive capacity and becomes the controlling failure mechanism.

- (4) Reinforcing bar gauge measurements (R1 and R2) revealed that cap reinforcement demand is fundamentally different in the two failure mode regimes. In steel-failure specimens, rebar stresses at failure ranged from 0.75 to 2 ksi, indicating that the cap reinforcement cage is essentially unstressed at connection failure. In concrete-failure specimens, rebar stresses ranged from approximately 10 to 40 ksi, approaching yield in the shallowest embedment specimen (W-10-5). This regime-dependence implies that cap reinforcement demands cannot be established without first determining which failure mode controls. No existing design method makes this distinction, representing a gap in current cap reinforcement design guidance for HP-pile to concrete bent cap connections.
- (5) The embedded strain gauges were able to confirm a decreasing moment profile with embedment depth across all specimens and load stages where functioning data were available, providing direct experimental evidence of moment transfer from the connection interface progressively into the concrete cap. In specimens where the EM-I, EM-M and EM-H gauges remained functional through higher load stages, the measured moments at successive embedded levels decreased consistently with depth, and moment at the pile head (EM-H level) remained non-zero in every case, verifying partial fixity.
- (6) The AASHTO LRFD minimum embedment requirement of 12 inches coincides with the empirically determined failure mode transition boundary between ductile flange-buckling failure and brittle concrete-bearing failure for HP10 and HP12 piles. However, for larger piles such as HP 14, HP 18 or even an 18-in prestressed squared concrete pile, which have larger structural moments, capacity calculations

performed in this work suggest that the connection would be controlled by the concrete capacity rather than steel structural capacity. This observation motivates a re-examination of AASHTO guidelines since a nominal design embedment of 12 in may provide insufficient margin against brittle failure. For the pile sections tested in this program (HP 10 and HP 12), this embedment is sufficiently deep. For larger piles (HP 18, or prestressed concrete piles), this inference is based on capacity calculations rather than direct testing and requires experimental confirmation.

- (7) The degree of fixity metric (Λ), as defined by Shishlov et al. (2022), can be misleading when reported without the accompanying normalized stiffness ratio $K_{\text{partial}}/K_{\text{fixed}}$. Across all tested specimens, the experimental rotational stiffness represents less than 7.5% of the theoretical fixed-head reference stiffness, yet the corresponding Λ ranges from 50 to 93%. This arises from the nonlinear shape of the fixity curve, which rises steeply at low $K_{\text{partial}}/K_{\text{fixed}}$ values. A connection reported as 93% fixed is operating at only 7.5% of its theoretical fixed-head stiffness. A practitioner reading that value would reasonably conclude the connection is nearly rigid, when it is not. Reporting Λ alone, without $K_{\text{partial}}/K_{\text{fixed}}$, does not show the true magnitude of rotational flexibility present in the connection and should be interpreted with caution.
- (8) Previous studies (Castilla et al., 1984; Cheng and Sritharan, 2010) have recommended using embedments of 1.5 to 2 times the pile depth to achieve a fixed-head condition. Results from this research indicate that an embedment of 1.5 times the pile depth does not attain the fixed-head condition, although the failure mode shifts to steel pile yielding for HP10 and HP12 sections. This likely does not occur with larger pile sections. At an embedment of 2 times the pile depth, it is probable that all HP sections fail primarily through steel yielding. However, this does not fulfill the criteria for a fixed-head condition, as the connection's rotational stiffness remains significantly below the theoretical fixed-head value.

- (9) Arvan and Arockiasamy (2023) suggested that for prestressed concrete piles, embedment depths ranging from 1.2 to 1.5 times the pile diameter or width are generally sufficient to achieve an adequate moment capacity. Experimental findings from this research confirm that this relationship also applies to HP10 and HP12 piles. However, for larger cross-sectional areas with increased structural moment capacity, this correlation does not necessarily remain valid.

4. Recommendations for Future Work

Based on the limitations identified during this research and the possible extensions that could apply to its findings, the following recommendations are suggested for future work:

- Additional specimens are needed in the HP12x53 series at intermediate embedment depths, particularly $L_e/d = 0.75$ and $L_e/d = 1.25$. With only two data points ($L_e/d = 0.5$ and 1.0), the stiffness-embedment curve for HP12 relies on few data. Additionally, more specimens with other pile dimensions (HP 14 or HP 18) are necessary.
- Bridge longitudinal loads, such as braking and acceleration forces per AASHTO LRFD, produce moments about the strong axis of the HP pile. Strong-axis bending connections require a dedicated experimental program before design guidance can be formulated with confidence. The single strong-axis specimen (S-10-10) produced findings of practical significance (higher stiffness but brittle concrete failure) but one specimen cannot characterize the full L_e/d range or establish a stiffness regression.
- The effect of sustained axial load on connection rotational stiffness and moment capacity should be investigated. All specimens were tested under purely lateral loading. HP piles in service carry axial dead load continuously, and axial compression affects both the concrete bearing capacity and the pile's structural moment capacity.

- Replicated specimens at matched embedment depths are needed to quantify specimen-to-specimen variability inherent in cast-in-place connections. Scatter in the embedded moment-at-pile-head measurements suggests meaningful variability in the concrete-bearing response that a single specimen per configuration cannot characterize. Similarly, the effect of concrete compressive strength on rotational stiffness can also be investigated.
- A controlled pre-damage experimental program should be conducted to establish a predictive relationship between prior rotation demand and stiffness reduction. The degradation data in this dissertation come from mistakes and re-tests of specimens loaded to failure, not from a systematic damage-then-retest protocol designed to isolate degradation as a function of rotation history.
- The normalized rotational stiffness regression and connection spring guideline should be extended to larger pile sections (HP14x89, HP14x117, HP18x204) which are commonly specified for highway bridges. The EI normalization is expected to hold, but it has only been validated for HP10 and HP12.
- The effective length parameter L_{eff} in the Kappes et al. (2021) methodology should be recalibrated using a broader HP-pile dataset. The values derived in this program ($L_{eff} = 2.0$ times the nominal pile depth) are based on a limited set of weak-axis HP10 and HP 12 results and may not generalize.
- The Hammett (2017) framework should be recalibrated specifically for HP steel piles in plain-embedment bridge bent connections. The link-element traction force assumption in Methods 1-3 should be re-examined in light of the experimental evidence. The currently adopted distributions render results that consistently overestimate rotational stiffness compared to the experimental results.
- The influence of cap geometry on connection stiffness and capacity should be investigated. All specimens used a single 36×36-inch cap cross-section. Cap

width-to-pile-depth ratio, cap height, and concrete strength likely all influence the bearing resistance, but were held constant throughout this program.

- Field measurements of rotational stiffness on in-service bridge bent connections should be pursued to assess whether the undamaged laboratory stiffness values are representative of field conditions, given that real connections may have construction variability, partial corrosion, or prior traffic-induced loading.
- A finite element study should be developed to: (1) model the rod-through-cap geometry of the TILT-HEAD instrument and determine what the tiltmeter actually reads as a function of pile rotation and translation, resolving the geometric ambiguities that prevented a closed-form formula from being validated experimentally, and (2) track the deformation field along the embedded pile length to test the rigid body assumption explicitly.
- This dissertation found that cap reinforcement demand differs by an order of magnitude between failure mode regimes (0.75-2 ksi vs. 10-40 ksi). All specimens used the same cap cross-section and reinforcement layout. A parametric study varying transverse reinforcement ratio, spiral vs. hoop configuration, and cap width-to-pile-depth ratio would establish whether existing AASHTO cap design provisions are adequate for the concrete-bearing failure regime.
- All tests were monotonic. A displacement-controlled cyclic protocol would characterize stiffness and strength degradation under repeated inelastic cycles, directly relevant to seismic assessment of existing bent bridges.
- This experimental program used a pile with no soil restraint. In service, soil provides distributed lateral resistance along the pile below the cap, which reduces the moment demand reaching the connection. Coupled pile-cap-soil models, or physical tests with soil, would quantify how much the soil reduces connection demand and whether the lab stiffness values are conservative or unconservative

relative to field conditions. This is also relevant to scour scenarios, where loss of soil support concentrates moment demand at the connection.

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Appendix A: CR-6 Monitoring Code

Program: Lab_Testing_All_Sensors_V19.CR6

'CR6 Datalogger

'Author: Murilo Tarozzo / mht0028@auburn.edu

'Wiring:

'-----AM16/32-B (4x16) (SN23546) to CR6 Datalogger:

'Control channels

'RES <-> C2

'CLK <-> C1

'G <-> G

'12 V <-> 12 V

'COM channels

'Odd High <-> U1

'Odd Low <-> U2

'Even High <-> U3

'Analog Ground <-> Earth Ground

'-----AM16/32-B (4x16) (SN23505) to CR6 Datalogger:

'RES <-> C4

'CLK <-> C3

'G <-> G

'12 V <-> 12 V

'COM channels

'Odd High <-> U11

'Odd Low <-> U12

'Even High <-> U9

'Analog Ground <-> Earth Ground

'-----AM16/32-B (4x16) (SN23507) to AVW200(SN2592):

'RES <-> RES

'CLK <-> CLK

'G <-> G

'12 V <-> 12 V

'COM channels

'Odd High <-> 1V+

'Odd Low <-> 1V-

'Ground -> Ground

'Even High <-> 1T+

'Even Low <-> 1T-

'-----AVW 200 (SN2592) to CR6 datalogger

'G -> G

'12 V -> SW12_2

'RS232 cable

' Brown -> U9

' White -> U10

' Yellow -> G

'-----3-wired Strain Gauge to 4wfbs120

'Red/white -> Earth Ground

'White -> High

'White -> Low

'-----4wfbs120 to AM16/32-B

'High -> Odd High

'Low -> Odd Low

'Earth Ground -> Analog Ground

'long wire -> Even high

'-----Geokon MEMS Tiltmeter 6160 to AM16/32-B SN 23505

'Red :12 V DC -> SW12_1 of CR6 datalogger

'Red's black :Ground -> port G of CR6 datalogger

'White :Out Diff+ -> Odd High

'White's black :Out Diff- -> Odd Low

'Bare :Shield -> Analog Ground

'Green :Thermistor -> disconnected

'Green's black :Thermistor -> disconnected

'-----Micro_Epsilon String Potentiometer to AM16/32-B SN 23505

'Blue (or White) (Input + Excitation) -> Even High

'Green (Signal) -> Odd High

'Orange and Brown (Ground) -> Even Ground

'-----Sensata Linear Spring Potentiometer (Slipmeter) to AM16/32-B SN 23505

'Note that wire colors and positioning may vary

'Red (Excitation)(next to the actuator) -> Even High

'Black (ground) (the other end) -> Even Ground

'White (Wiper signal) (middle) -> Odd High

'----- Buck converter (regulated to output 5V) to CH200

'Negative input -> ground

'Positive input -> 12 V

'Make sure you use a multimeter in the output, and regulate it to 5V

'-----Rieker inclinometer to AM16/32-B SN 23505 and Buck Converter
'Brown (Excitation +Ub: 5V stable (app. 1mA)) -> Positive output (buck converter)
'Black (ground) -> Negative output (Buck converter)
'Orange (Output signal) -> Odd High (multiplexer)
'Red - disconnected

'-----load cell_Geokon_4900_200-3 (SN 2438883) to AM16/32-B SN 23507
'Red --> 1 Odd High
'Red's Black --> 2 Odd High
'White --> 3 Odd High
'White's Black --> 1 Odd low
'Green --> 1 Even Low
'Green's Black --> 1 even High
'Bare wire --> Ground
'Short 1L and 3L
'Short 1L and 5L
'Short 2H and 4H
'Short 2L and 4L
'Short 2H and 6H
'Short 2L and 6L

'-----load cell_Geokon_4900_200-3 (SN 2438882) to AM16/32-B SN 23507
'Red --> 4 Odd High
'Red's Black --> 5 Odd High
'White --> 6 Odd High
'White's Black --> 4 Odd low
'Green --> 4 Even Low
'Green's Black --> 4 even High
'Bare wire --> Ground
'Short 7L and 9L
'Short 7L and 11L

'Short 8H and 10H

'Short 8L and 10L

'Short 8H and 12H

'Short 8L and 12L

'Declaring general variables

Public BattV

Public PTemp_C

Public MuxCh

Units BattV = volts

Units PTemp_C = celsius

'Declaring variables for Strain Gauges for Steel H Pile, rebars and concrete.

Public SGVolt(16)

Public SG(16)

Units SGVolt() = mvolt/volt

Units SG() = microStrain

'Declaring variables for Geokon Inclinometers

Public MEMS_Output(2)

Public MEMS_Rotation(2)

Units MEMS_Output()=V

Units MEMS_Rotation()=degrees

'Declaring variables for Micro Epsilon String Potentiometers

Public Pot_Output(6)

Public Pot(6)

Units Pot_Output()=V

Units Pot()=in

'Declaring variables for Spring Potentiometer (Slipmeters)

Public Slip_Output(12)

Public Slip(12)

Units Slip_Output()=V

Units Slip()=in

'Declaring variables for Rieker inclinometers

Public Rieker_Output(14)

Public Rieker(14)

Units Rieker_Output()=mV

Units Rieker()=degrees

'Declaring variables for AVW200 and load cell_Geokon_4900_200-3

Public VWResult

Public Gauges_Output(6,6)

'For 1 load cell, we use 3 gauges, 6 values (freq, amp, SNR, noise freq, decay, thermistor)

'we are measuring 3 gauges, with thermistor. If no thermistor, then (3,5)

'Values for sensor 1 would be stored in Array(1,1) through Array(1,6),

'values for sensor 2 stored in Array (2,1) through (2,6),

'values for sensor 3 stored in Array (3,1) through (3,6), etc, etc.

'since we are using 2 load cells, then it must be 6 gauges, 6 values

Public Freq(6) 'Frequency in Hertz

Public Amp(6) 'Amplitude in mV RMS

Public SNRat(6) 'Signal-to-noise ratio

Public NFreq(6) 'Noise frequency

Public DRat(6) 'Decay ratio

Public TR(6) 'Thermistor in ohms

Public LCAvgDigitsLC83

Public LoadCellLC83

Public ThermLC83(3)

Public Temp_avgLC83

Units Freq()=Hz

Units Amp()=mV RMS

Units NFreq()=Hz

Units TR=ohms

Units LoadCellLC83=lbs

Units ThermLC83()=celsius

Units Temp_avgLC83=celsius

Public LCAvgDigitsLC82

Public LoadCellLC82

Public ThermLC82(3)

Public Temp_avgLC82

Public TOTAL_LOAD

Units TOTAL_LOAD=lbs

'Healthy signal: Amplitude (RMS) > ~5 mV

'Weak signal: < 1 mV → sensor may be disconnected or not loaded

'Weak signal: < 1 mV → sensor may be disconnected or not loaded

'Noisy reading: High amplitude but low SNRat → investigate mechanical/electrical noise

'Defining Data Tables

DataTable (Test_Measurements,True,-1)

DataInterval(0,15,Sec,10)

Sample(1,TOTAL_LOAD,IEEE4)

Sample(1,LoadCellLC83,IEEE4)

Sample(1,Temp_avgLC83,IEEE4)

Sample(1,LoadCellLC82,IEEE4)

Sample(1,Temp_avgLC82,IEEE4)

Sample(1,SGVolt(1),IEEE4)

Sample(1,SGVolt(2),IEEE4)

Sample(1,SGVolt(3),IEEE4)

Sample(1,SGVolt(4),IEEE4)

Sample(1,SGVolt(5),IEEE4)

Sample(1,SGVolt(6),IEEE4)

Sample(1,SGVolt(7),IEEE4)

Sample(1,SGVolt(8),IEEE4)

Sample(1,SGVolt(9),IEEE4)

Sample(1,SGVolt(10),IEEE4)

Sample(1,SGVolt(11),IEEE4)

Sample(1,SGVolt(12),IEEE4)

Sample(1,SGVolt(13),IEEE4)

Sample(1,SGVolt(14),IEEE4)

Sample(1,SGVolt(15),IEEE4)

Sample(1,SGVolt(16),IEEE4)

Sample(1,SG(1),IEEE4)

Sample(1,SG(2),IEEE4)

Sample(1,SG(3),IEEE4)

Sample(1,SG(4),IEEE4)

Sample(1,SG(5),IEEE4)

Sample(1,SG(6),IEEE4)

Sample(1,SG(7),IEEE4)

Sample(1,SG(8),IEEE4)
Sample(1,SG(9),IEEE4)
Sample(1,SG(10),IEEE4)
Sample(1,SG(11),IEEE4)
Sample(1,SG(12),IEEE4)
Sample(1,SG(13),IEEE4)
Sample(1,SG(14),IEEE4)
Sample(1,SG(15),IEEE4)
Sample(1,SG(16),IEEE4)
Sample(1,MEMS_Output(1),IEEE8)
Sample(1,MEMS_Output(2),IEEE8)
Sample(1,MEMS_Rotation(1),IEEE8)
Sample(1,MEMS_Rotation(2),IEEE8)
Sample(1,Pot_Output(3),IEEE4)
Sample(1,Pot_Output(4),IEEE4)
Sample(1,Pot_Output(5),IEEE4)
Sample(1,Pot_Output(6),IEEE4)
Sample(1,Pot(3),IEEE4)
Sample(1,Pot(4),IEEE4)
Sample(1,Pot(5),IEEE4)
Sample(1,Pot(6),IEEE4)
Sample(1,Slip_Output(7),IEEE4)
Sample(1,Slip_Output(8),IEEE4)
Sample(1,Slip_Output(9),IEEE4)
Sample(1,Slip_Output(10),IEEE4)
Sample(1,Slip_Output(11),IEEE4)
Sample(1,Slip_Output(12),IEEE4)
Sample(1,Slip(7),IEEE4)
Sample(1,Slip(8),IEEE4)
Sample(1,Slip(9),IEEE4)
Sample(1,Slip(10),IEEE4)

```
Sample(1,Slip(11),IEEE4)
Sample(1,Slip(12),IEEE4)
Sample(1,Rieker_Output(13),IEEE4)
Sample(1,Rieker_Output(14),IEEE4)
Sample(1,Rieker(13),IEEE4)
Sample(1,Rieker(14),IEEE4)
    Sample(1,VWResult,IEEE4)
Sample(1,Freq(1),IEEE4)'LC 83
Sample(1,Amp(1),IEEE4)'LC 83
Sample(1,SNRat(1),IEEE4)'LC 83
Sample(1,NFreq(1),IEEE4)'LC 83
Sample(1,DRat(1),IEEE4)'LC 83
Sample(1,TR(1),IEEE4)'LC 83
Sample(1,Freq(2),IEEE4)'LC 83
Sample(1,Amp(2),IEEE4)'LC 83
Sample(1,SNRat(2),IEEE4)'LC 83
Sample(1,NFreq(2),IEEE4)'LC 83
Sample(1,DRat(2),IEEE4)'LC 83
Sample(1,TR(2),IEEE4)'LC 83
Sample(1,Freq(3),IEEE4)'LC 83
Sample(1,Amp(3),IEEE4)'LC 83
Sample(1,SNRat(3),IEEE4)'LC 83
Sample(1,NFreq(3),IEEE4)'LC 83
Sample(1,DRat(3),IEEE4)'LC 83
Sample(1,TR(3),IEEE4)'LC 83
Sample(1,LCAvgDigitsLC83,IEEE4)'LC 83
Sample(1,ThermLC83(1),IEEE4)'LC 83
Sample(1,ThermLC83(2),IEEE4)'LC 83
Sample(1,ThermLC83(3),IEEE4)'LC 83
Sample(1,Freq(4),IEEE4)'LC 82
Sample(1,Amp(4),IEEE4)'LC 82
```

```
Sample(1,SNRat(4),IEEE4)'LC 82
Sample(1,NFreq(4),IEEE4)'LC 82
Sample(1,DRat(4),IEEE4)'LC 82
Sample(1,TR(4),IEEE4)'LC 82
Sample(1,Freq(5),IEEE4)'LC 82
Sample(1,Amp(5),IEEE4)'LC 82
Sample(1,SNRat(5),IEEE4)'LC 82
Sample(1,NFreq(5),IEEE4)'LC 82
Sample(1,DRat(5),IEEE4)'LC 82
Sample(1,TR(5),IEEE4)'LC 82
Sample(1,Freq(6),IEEE4)'LC 82
Sample(1,Amp(6),IEEE4)'LC 82
Sample(1,SNRat(6),IEEE4)'LC 82
Sample(1,NFreq(6),IEEE4)'LC 82
Sample(1,DRat(6),IEEE4)'LC 82
Sample(1,TR(6),IEEE4)'LC 82
Sample(1,LCAvgDigitsLC82,IEEE4)'LC 82
Sample(1,ThermLC82(1),IEEE4)'LC 82
Sample(1,ThermLC82(2),IEEE4)'LC 82
Sample(1,ThermLC82(3),IEEE4)'LC 82
```

```
EndTable
```

```
BeginProg
```

```
SW12(SW12_1,1)
SW12(SW12_2,1)
SerialOpen(COMU9,38400,4,0,0)
```

```
Scan(15,Sec,1,0)
```

```
Battery(BattV)'Default CR6 Datalogger Battery Voltage measurement 'BattV'
```

PanelTemp(PTemp_C,60)'Default CR6 Datalogger Wiring Panel Temperature
measurement 'PTemp_C'

'Here, we start reading from the Geokon Load Cell SN 2438883, connected to
AM16/32B SN 23507

'The AVW200 instruction is used to set up the datalogger to read measurements from
an AVW200 Series Vibrating Wire Spectrum Analyzer.

'AVW200 (Result, ComPort, NeighborAddr, PakBusAddr, Dest, AVWChan,MuxChan,
Reps, BeginFreq, EndFreq, ExVolt, Therm50_60Hz, Multiplier,
'Offset, AmpThreshold [optional])

AVW200(VWResult,ComU9,0,200,Gauges_Output(1,1),1,1,6,450,6000,1,_60Hz,1,0)

'Outputs for gauge 1 - LC 83

Freq(1) = Gauges_Output(1,1)

Amp(1) = Gauges_Output(1,2)

SNRat(1) = Gauges_Output(1,3)

NFreq(1) = Gauges_Output(1,4)

DRat(1) = Gauges_Output(1,5)

TR(1) = Gauges_Output(1,6)

'Outputs for gauge 2 - LC 83

Freq(2) = Gauges_Output(2,1)

Amp(2) = Gauges_Output(2,2)

SNRat(2) = Gauges_Output(2,3)

NFreq(2) = Gauges_Output(2,4)

DRat(2) = Gauges_Output(2,5)

TR(2) = Gauges_Output(2,6)

'Outputs for gauge 3 - LC 83

Freq(3) = Gauges_Output(3,1)

Amp(3) = Gauges_Output(3,2)
SNRat(3) = Gauges_Output(3,3)
NFreq(3) = Gauges_Output(3,4)
DRat(3) = Gauges_Output(3,5)
TR(3) = Gauges_Output(3,6)

'Outputs for gauge 4 - LC 82

Freq(4) = Gauges_Output(4,1)
Amp(4) = Gauges_Output(4,2)
SNRat(4) = Gauges_Output(4,3)
NFreq(4) = Gauges_Output(4,4)
DRat(4) = Gauges_Output(4,5)
TR(4) = Gauges_Output(4,6)

'Outputs for gauge 5 - LC 82

Freq(5) = Gauges_Output(5,1)
Amp(5) = Gauges_Output(5,2)
SNRat(5) = Gauges_Output(5,3)
NFreq(5) = Gauges_Output(5,4)
DRat(5) = Gauges_Output(5,5)
TR(5) = Gauges_Output(5,6)

'Outputs for gauge 6 - LC 82

Freq(6) = Gauges_Output(6,1)
Amp(6) = Gauges_Output(6,2)
SNRat(6) = Gauges_Output(6,3)
NFreq(6) = Gauges_Output(6,4)
DRat(6) = Gauges_Output(6,5)
TR(6) = Gauges_Output(6,6)

'Converting Output into engineering units - for LC83

ThermLC83(1) = (1/(1.405E-3+2.369E-4*LN(TR(1))+1.019E-7*LN(TR(1))^3))-
273.15'Replace with correct coefficients

ThermLC83(2) = (1/(1.405E-3+2.369E-4*LN(TR(2))+1.019E-7*LN(TR(2))^3))-
273.15'Replace with correct coefficients

ThermLC83(3) = (1/(1.405E-3+2.369E-4*LN(TR(3))+1.019E-7*LN(TR(3))^3))-
273.15'Replace with correct coefficients

Temp_avgLC83 = (ThermLC83(1)+ThermLC83(2)+ThermLC83(3))/3

LCAvgDigitsLC83 = ((Freq(1)^2)/1000 + (Freq(2)^2)/1000 + (Freq(3)^2)/1000)/3

LoadCellLC83=((LCAvgDigitsLC83-(7423.3908))*(-94.31)*1)- (0) '((R1 – R0) × G
× K) - (value)

'R0 is the unloaded LCAvgDigits value. Manually input here: xxxx.xx lb/digit SN
2438883

'Also, I'm subtracting xxx lbs which is the average reading at rest.

'Converting Output into engineering units - for LC82

ThermLC82(1) = (1/(1.405E-3+2.369E-4*LN(TR(4))+1.019E-7*LN(TR(4))^3))-
273.15'Replace with correct coefficients

ThermLC82(2) = (1/(1.405E-3+2.369E-4*LN(TR(5))+1.019E-7*LN(TR(5))^3))-
273.15'Replace with correct coefficients

ThermLC82(3) = (1/(1.405E-3+2.369E-4*LN(TR(6))+1.019E-7*LN(TR(6))^3))-
273.15'Replace with correct coefficients

Temp_avgLC82 = (ThermLC82(1)+ThermLC82(2)+ThermLC82(3))/3

LCAvgDigitsLC82 = ((Freq(4)^2)/1000 + (Freq(5)^2)/1000 + (Freq(6)^2)/1000)/3

LoadCellLC82=((LCAvgDigitsLC82-(7398.623))*(-94.00)*1)- (0) '((R1 – R0) × G ×
K) - (value)

'R0 is the unloaded LCAvgDigits value. Manually input here: xxxx.xx lb/digit SN
2438883

'Also, I'm subtracting xxx lbs which is the average reading at rest.

TOTAL_LOAD = LoadCellLC83 '+' LoadCellLC82 (for S-10-10, I consider only Load cell 83)

PortSet(C2,1)'Turn AM16/32 Multiplexer On (SN 23546)

Delay(0,150,mSec)

For MuxCh = 1 To 16 'Measure Strain Gauges

PulsePort(C1,10000)'Switch to next AM16/32 Multiplexer channel

BrFull(SGVolt(MuxCh),1,mV200,U1,U3,1,2500,True,True,500,60,1,0)'Measuring Quarter Bridge Strain

Next MuxCh

'Converting mVolt/Volt into microStrain

SG(1) = 4000*(SGVolt(1))/(2.11) 'Gauge R1

SG(2) = 4000*(SGVolt(2))/(2.11) 'Gauge R2

SG(3) = 4000*(SGVolt(3))/(2.11) 'Gauge CONC AD (for S-10-10, it's CONC AB)

SG(4) = 4000*(SGVolt(4))/(2.11) 'Gauge CONC BC (for S-10-10, it's CONC CD)

SG(5) = 4000*(SGVolt(5))/(2.11) 'Gauge EM-H-A

SG(6) = 4000*(SGVolt(6))/(2.11) 'Gauge EM-H-B (for S-10-10, it's EM-H-D)

SG(7) = 4000*(SGVolt(7))/(2.11) 'Gauge EM-M-A (for embedments of 10 in, 15 in, 12 in) / EX-C1-D for W-10-7.5R

SG(8) = 4000*(SGVolt(8))/(2.11) 'Gauge EM-M-B (for embedments of 10 in, 15 in, 12 in) / EX-C6-D for W-10-7.5R / (for S-10-10, it's EM-M-D)

SG(9) = 4000*(SGVolt(9))/(2.11) 'Gauge EM-I-A

SG(10) = 4000*(SGVolt(10))/(2.11) 'Gauge EM-I-B (for S-10-10, it's EM-I-D)

SG(11) = 4000*(SGVolt(11))/(2.11) 'Gauge EX-C1-A

SG(12) = 4000*(SGVolt(12))/(2.11) 'Gauge EX-C1-B (for S-10-10, it's EX-C1-D)

SG(13) = 4000*(SGVolt(13))/(2.11) 'Gauge EX-C6-A

SG(14) = 4000*(SGVolt(14))/(2.11) 'Gauge EX-C6-B (for S-10-10, it's EX-C6-D)

SG(15) = 4000*(SGVolt(15))/(2.11) 'Gauge (empty) / EM-H-D for W-10-7.5R

SG(16) = 4000*(SGVolt(16))/(2.11) 'Gauge (empty) / EM-I-D for W-10-7.5R

PortSet(C2,0)'Turn off the AM16/32B SN 23546

Delay(0,150,mSec)

PortSet(C4,1)'Turn on the AM16/32B SN23505

Delay(0,150,mSec)

For MuxCh = 1 To 2'Measure Geokon MEMS Inclinometers

PulsePort(C3,10000)

'VoltDiff (Dest, Reps, Range, DiffChan, RevDiff, SettlingTime, fN1, Mult, Offset)

VoltDiff (MEMS_Output(MuxCh),1,mV5000,U5,False,0,250,0.001,0)'Multiplier

0.001 makes sure we get the Output in Volts, not mVolt

Next MuxCh

'Here do the conversion from MEMS_Output() to MEMS_rotation(), according to manufacturer manual

MEMS_Rotation(1)=(MEMS_Output(1)*3.6375) + 0.2302 'Geokon MEMS Tilt 1 SN 1315141 - Tilt-Cap

MEMS_Rotation(2)=(MEMS_Output(2)*3.6414) - 0.0059 'Geokon MEMS Tilt 2 SN 1315140 - Tilt-C6

'IMPORTANT! remember!

'tilt-cap:

'positive angle - counter-clockwise rotation

'negative angle - clockwise rotation

'tilt-c6:

'positive angle - clockwise rotation (from AD to BC)

'negative angle - counter-clockwise rotation (from BC to AD)

'FOR THE TILTMETERS, THIS MEANS THAT WHEN LOOKING AT THE RESULTS, AND BOTH HAVE POSITIVE ANGLE OF ROTATION, THEY ARE ROTATING IN OPPOSITE DIRECTIONS.

'For S-10-10 only:

'tilt-cap:

'positive angle - counter-clockwise rotation (meaning, in favor of load direction)

'negative angle - clockwise rotation (meaning, against load direction)

,

'tilt-c6:

'positive angle - clockwise rotation (from AB to CD)

'negative angle - counter-clockwise rotation (from CD to AB)

For MuxCh = 3 To 6'Measure Linear String Potentiometers

PulsePort(C3,10000)

'BrFull(Pot_Output(MuxCh),1,mv5000,U11,U9,1,2500,True,True,50000,60,1,0)'Measuring mVolt/volt in potentiometers

BrHalf(Pot_Output(MuxCh),1,mv5000,U5,U7,1,2500,True,50000,60,1,0)

Next MuxCh

'Converting mVolt/Volt into linear distance. Check calibration.

Pot(3) = ((Pot_Output(3))*(4.2545))-0.0015 'Pot sn 62896 - Pot C1

Pot(4) = ((Pot_Output(4))*(4.1490))-0.0033 'Pot sn 62897 - Pot C6

Pot(5) = ((Pot_Output(5))*(4.2754))-0.0146 'Pot sn 38797 - Pot C42

'Pot(6) = ((Pot_Output(6))*(4.4388))+0.0074 'Pot sn 38794 - Pot C84

Pot(6) = 26.305*(Pot_Output(6))^3 - 49.289*(Pot_Output(6))^2 + 67.363*(Pot_Output(6)) + 0.178 'Pot sn 72328 - Pot C84

For MuxCh = 7 To 12'Measure Sensata Potentiometers (Slipmeters)

PulsePort(C3,10000)

'BrHalf (Dest, Reps, Range, SEChan, ExChan, MeasPEX, ExmV, RevEx,
SettlingTime, fN1, Mult, Offset)

BrHalf(Slip_Output(MuxCh),1,mv5000,U5,U7,1,2500,True,50000,60,1,0)

Next MuxCh

'Converting mVolt/Volt into linear distance. Check calibration.

Slip(7) = ((Slip_Output(7))*(1.0555))-0.0002 'Slip-A-D (for S-10-10 SLip C-D)

Slip(8) = ((Slip_Output(8))*(1.0486))-0.0002 'Slip-A

Slip(9) = ((Slip_Output(9))*(1.5343))-0.0003 'Slip-B

Slip(10) = ((Slip_Output(10))*(1.6983))-0.1381 'Slip-C

Slip(11) = ((Slip_Output(11))*(1.5554))-0.0003 'Slip-D

Slip(12) = ((Slip_Output(12))*(1.0563))-0.0002 'Slip-B-C (for S-10-10 Slip A-B)

'REMEMBER:

'The slip pots start with the stroke at maximum by default.

For MuxCh = 13 To 14 'Measure Rieker Inclonometers

PulsePort(C3,10000)

VoltSe(Rieker_Output(MuxCh),1,mV5000,U5,True,50000,60,1,0)

Next MuxCh

'Converting output into degrees. Check calibration.

Rieker(13) = (Rieker_Output(13)*(-0.1111))+278.11 'Rieker #2506099 - TILT-head

Rieker(14) = (Rieker_Output(14)*(-0.1129))+282.98 'Rieker #2506098 - Tilt-C1

'Remember: clockwise rotation is positive.

'For S-10-10 only:

'rieker 14 - clockwise rotation is positive

'rieker 13 - clockwise rotation is positive (meaning, against the load direction).

,

PortSet(C4,0)'Turn off the AM16/32B SN23505

Delay(0,150,mSec)

CallTable Test_Measurements
NextScan
EndProg

Appendix B: Complementary Results of Experimental Specimens

Figures 259 through 316 show complementary results not discussed in chapter 5, for all the tested specimens in this research.

1. Specimen W-10-15

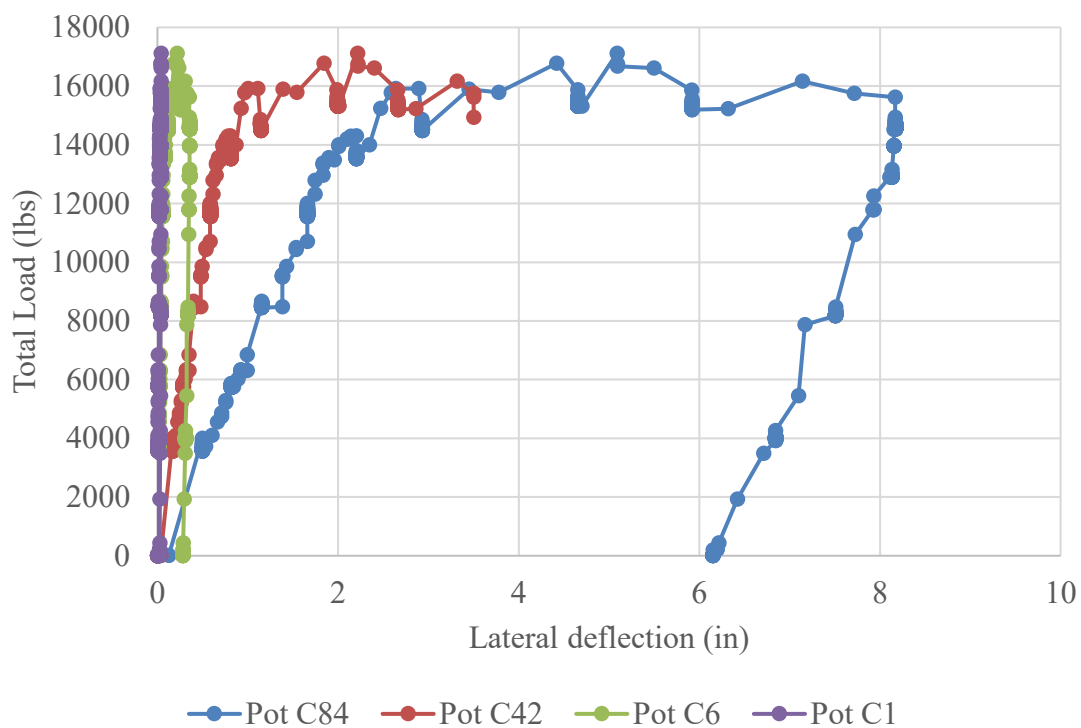


Figure 259. Specimen W-10-15 (3rd try) wire potentiometers measurements at 1 in (C1), 6 in (C6), 42 in (C42) and 84 in (C84, load application point) above the cap surface.

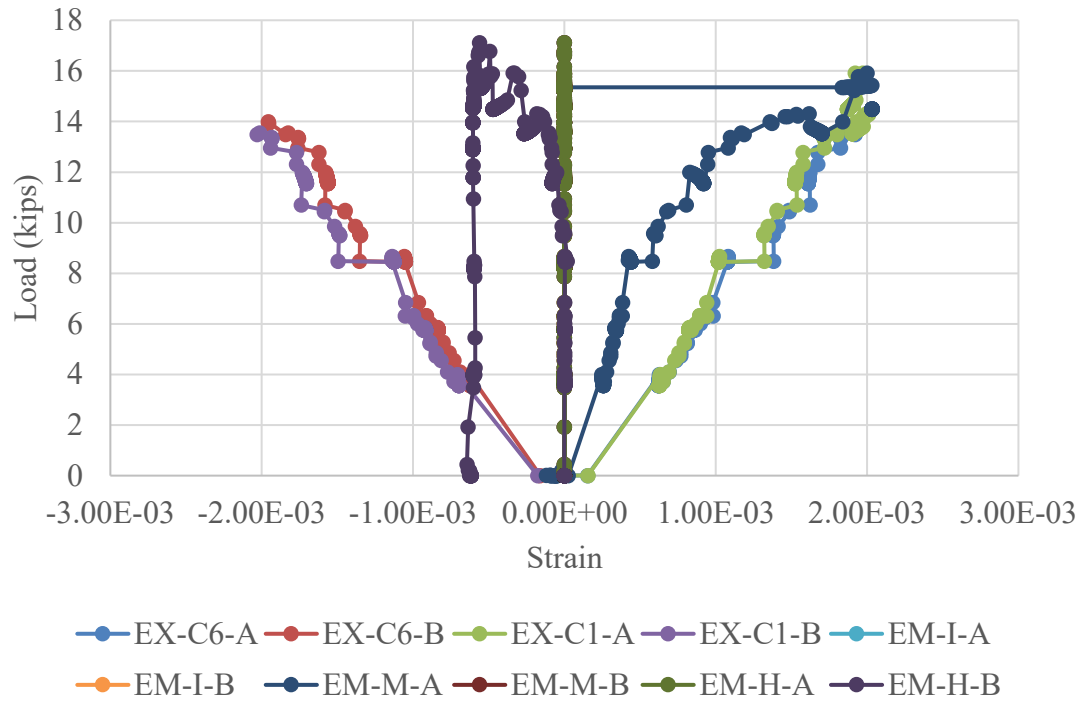


Figure 260. Specimen W-10-15 (3rd try) strain gauges on the steel pile.

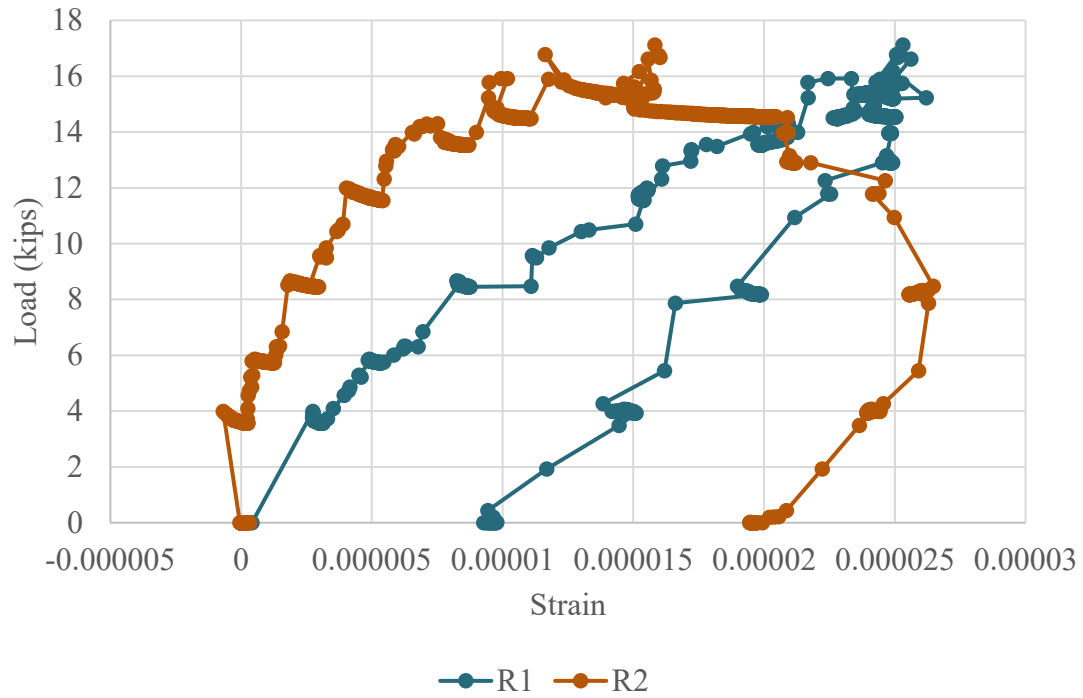


Figure 261. Specimen W-10-15 (3rd try) strain gauges on steel reinforcement bars.

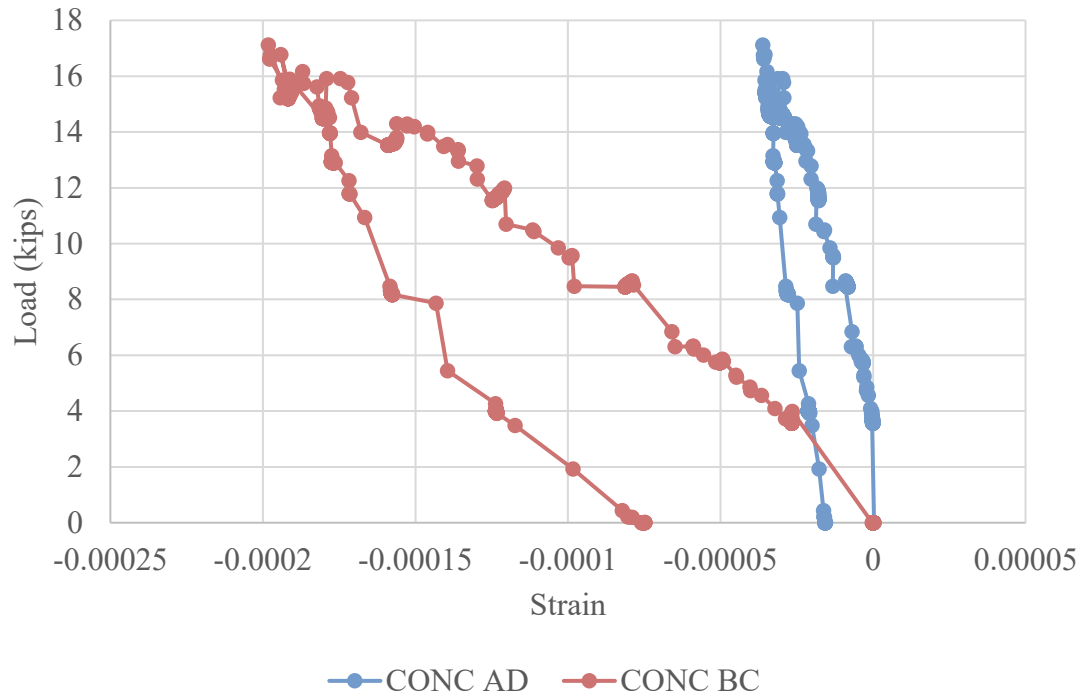


Figure 262. Specimen W-10-15 (3rd try) strain gauges on concrete cap surface.

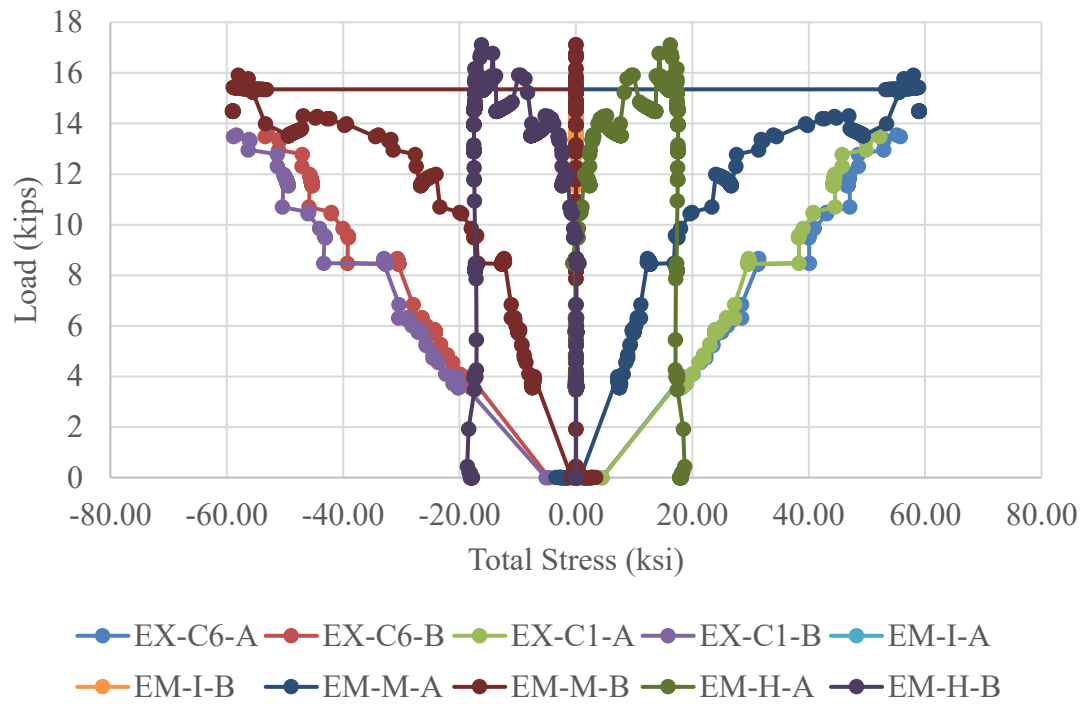


Figure 263. Specimen W-10-15 (3rd try) stress calculated from strain gauges on the steel pile.

2. Specimen W-10-10

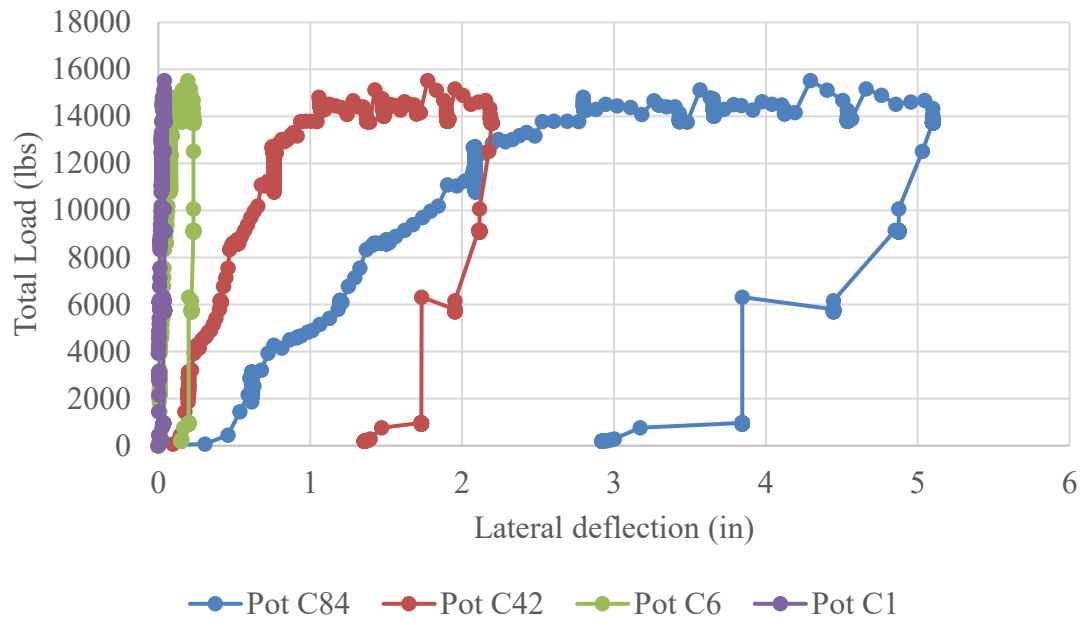


Figure 264. Specimen W-10-10 wire potentiometers measurements at 1 in (C1), 6 in (C6), 42 in (C42) and 84 in (C84, load application point) above the cap surface.

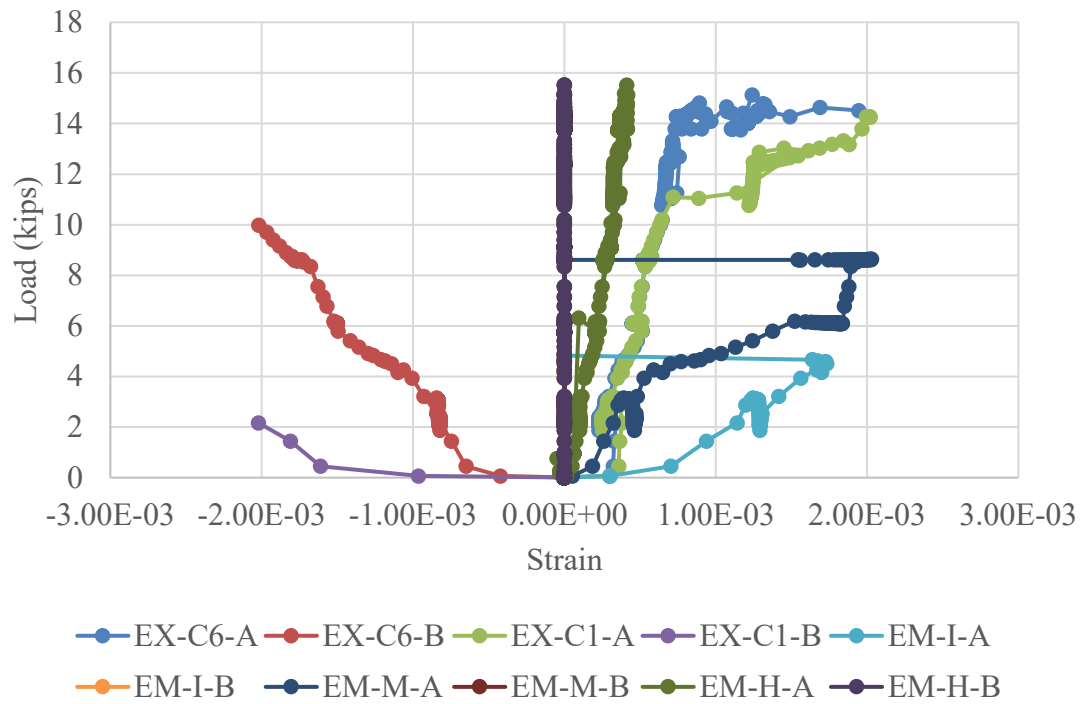


Figure 265. Specimen W-10-10 strain gauges strain measurements on the steel pile.

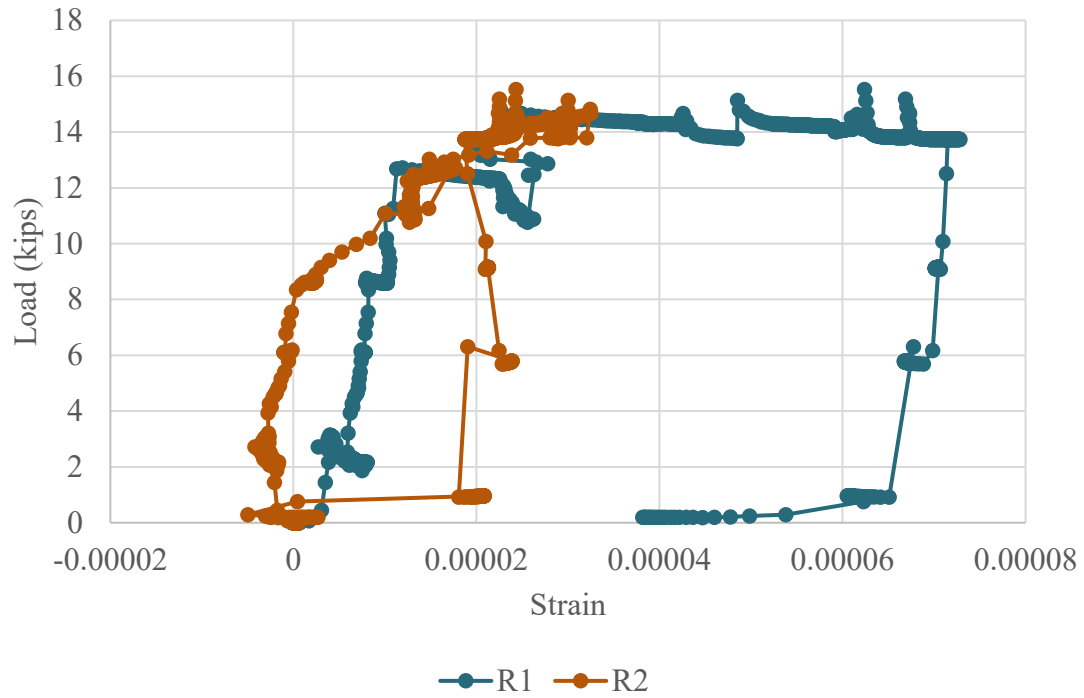


Figure 266. Specimen W-10-10 strain gauges strain measurements on steel reinforcement bars.

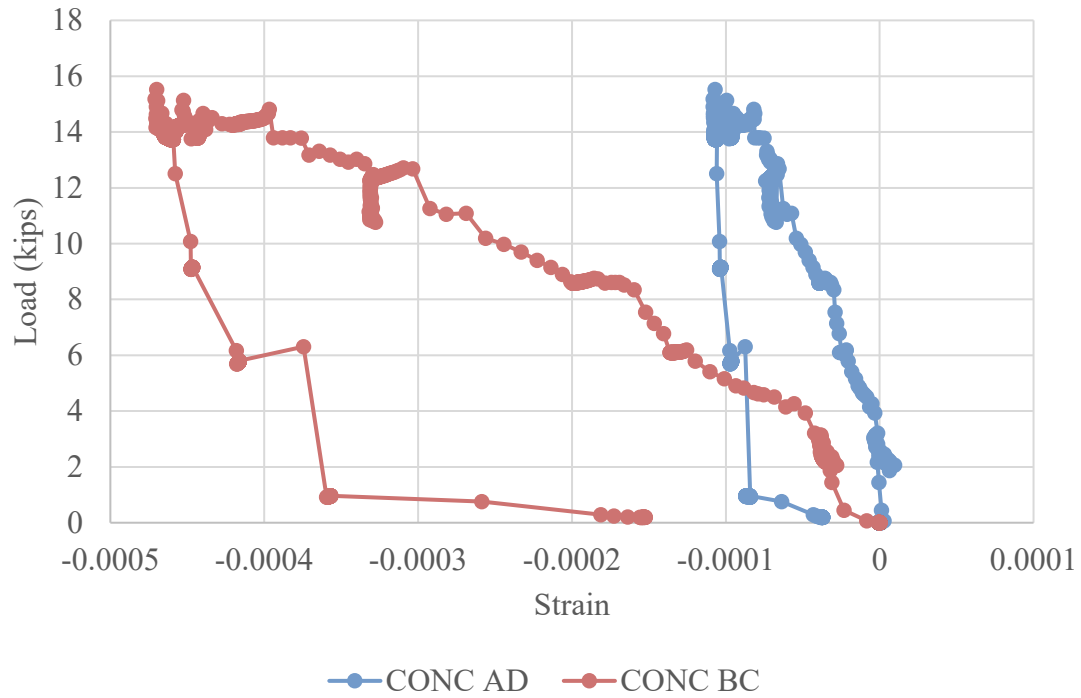


Figure 267. Specimen W-10-10 strain gauges strain measurements on concrete cap surface.

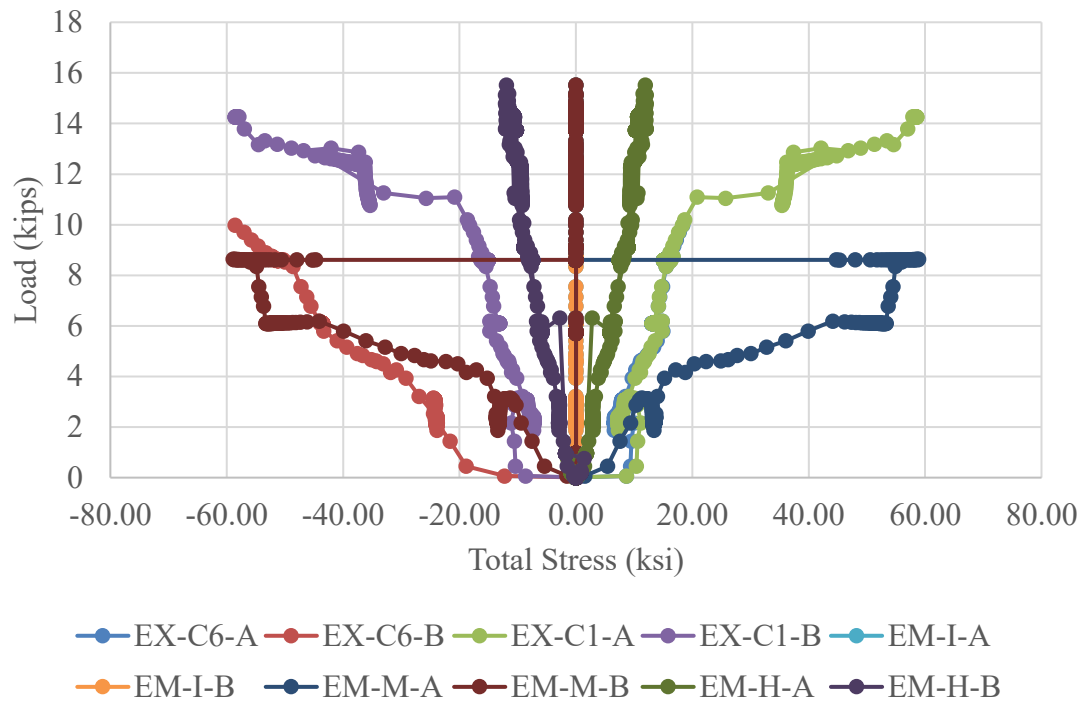


Figure 268. Specimen W-10-10 stress calculated from strain gauges on the steel pile.

3. Specimen W-10-7.5



Figure 269. Specimen W-10-7.5 (1st try) crack formation after target loads of 10.3 kips (applied load of 9.4 kips) (load direction: from right to left).



Figure 270. Specimen W-10-7.5 (2nd try) crack formation after target loads of 13.4 kips (applied load of 8 kips) (load direction: from right to left).

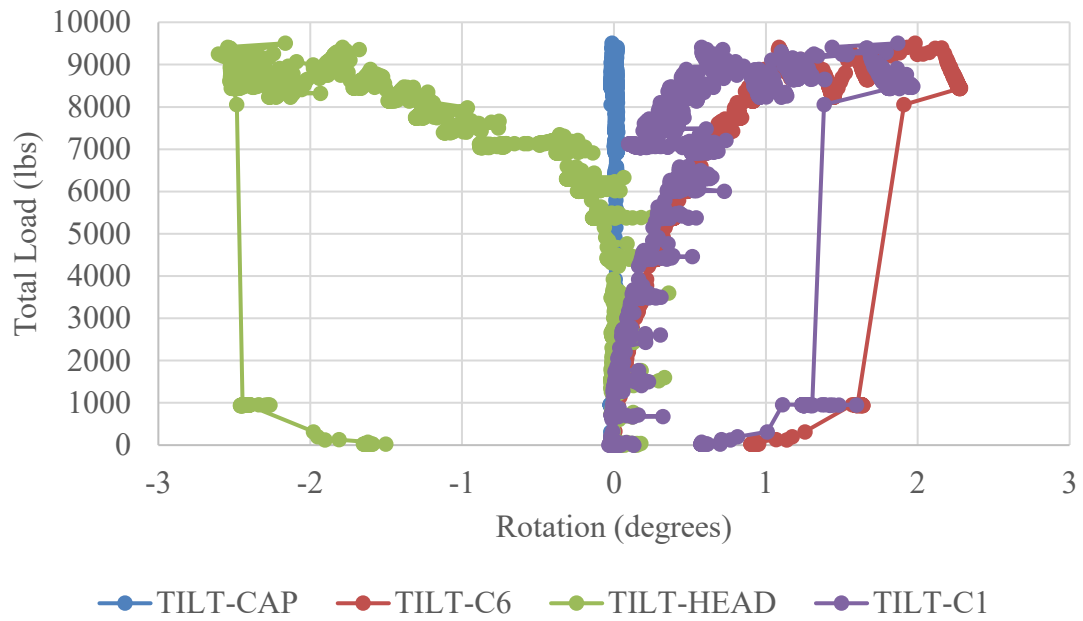


Figure 271. Specimen W-10-7.5 (1st try) , rotation measurements (positive rotation follows the load direction).

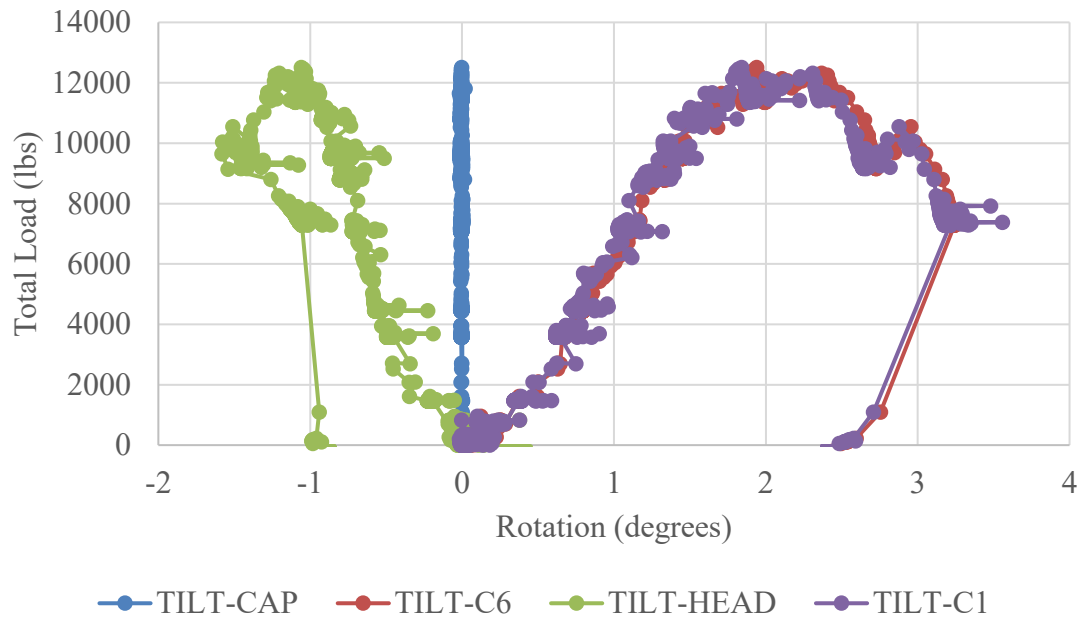


Figure 272. Specimen W-10-7.5 (2nd try) , rotation measurements (positive rotation follows the load direction).

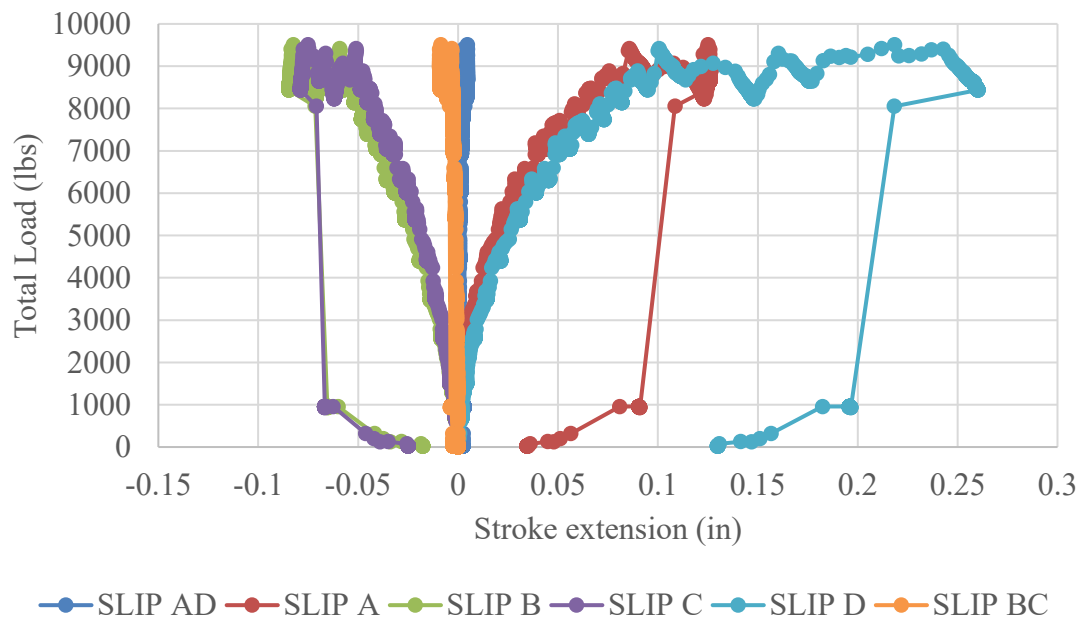


Figure 273. Specimen W-10-7.5 (1st try) , spring potentiometer measurements.

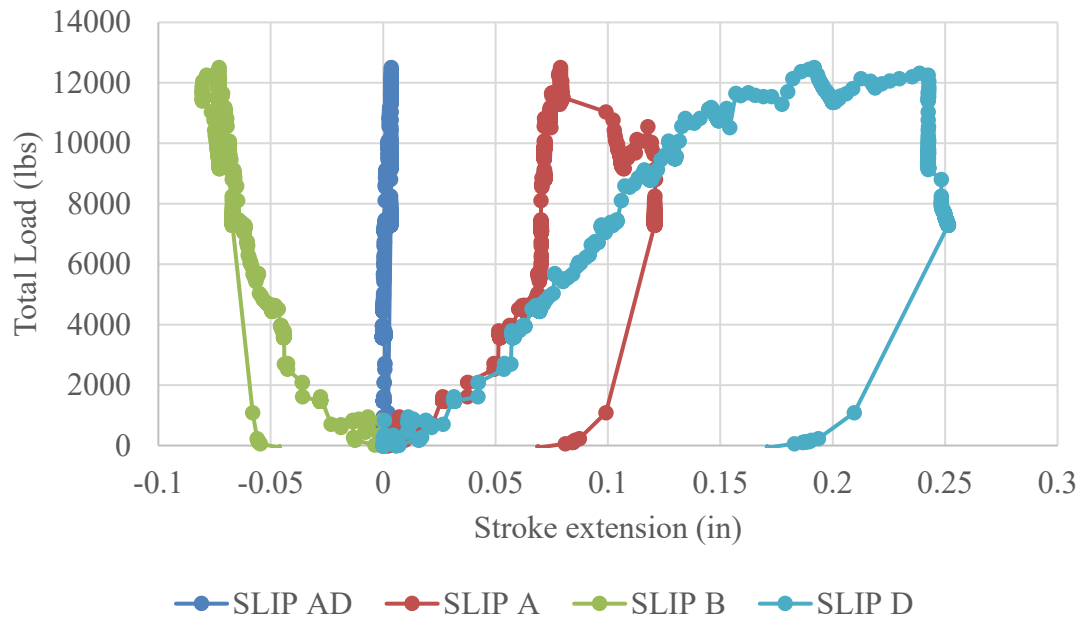


Figure 274. Specimen W-10-7.5 (2nd try) , spring potentiometer measurements.

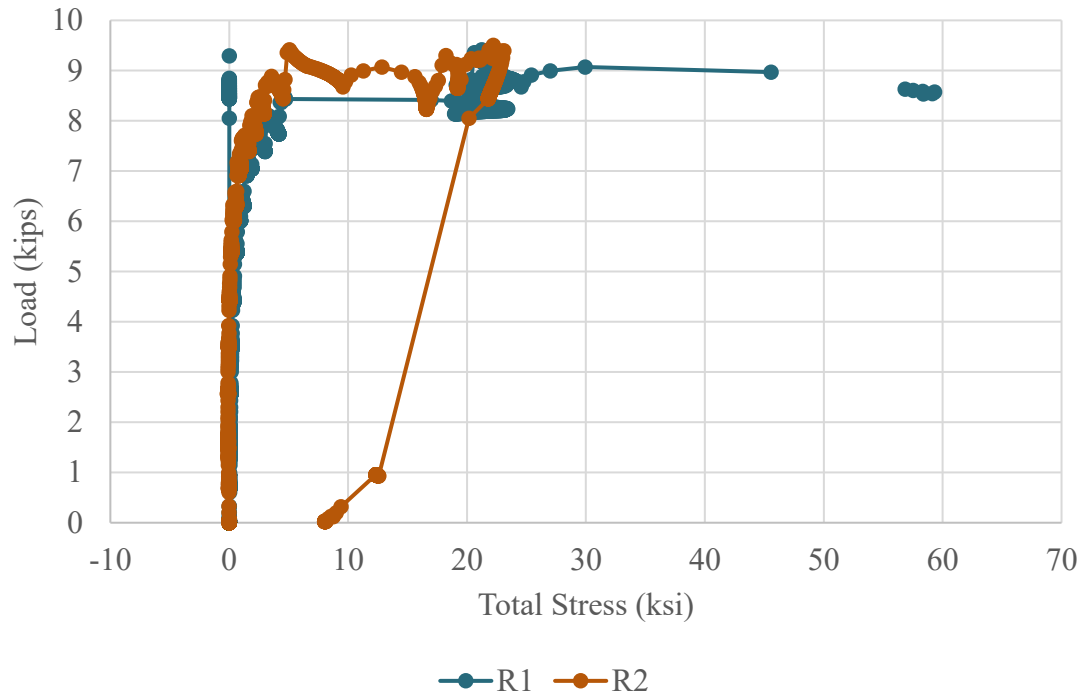


Figure 275. Specimen W-10-7.5 (1st try) , total stress measured by strain gauges R1 and R2 on steel reinforcement cage.

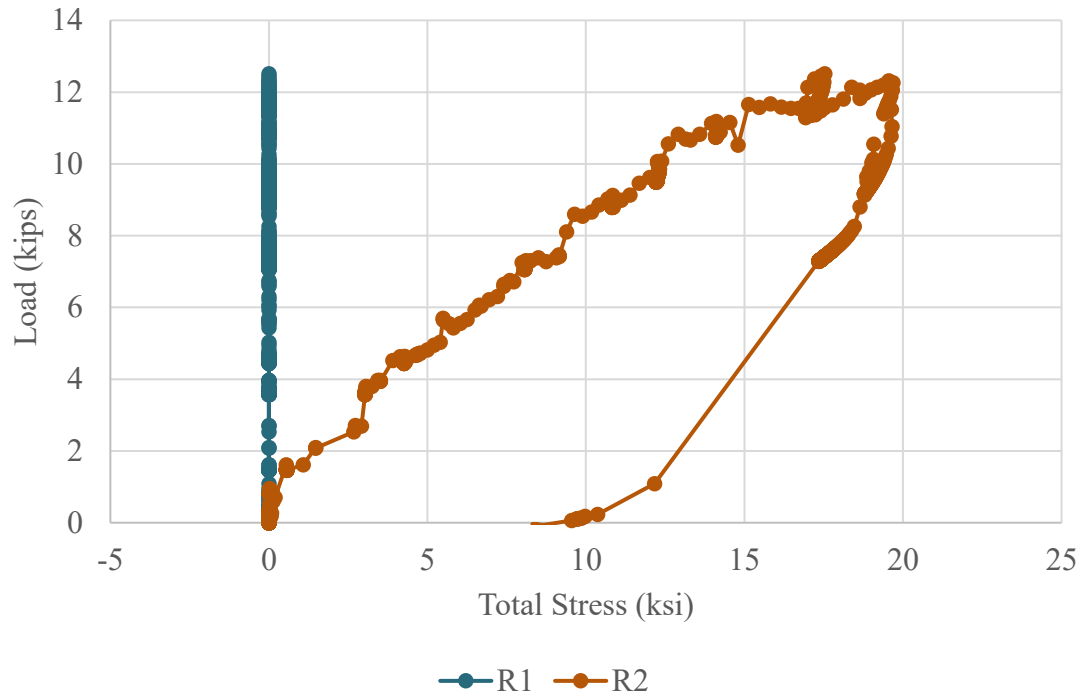


Figure 276. Specimen W-10-7.5 (2nd try) , total stress measured by strain gauges R1 and R2 on steel reinforcement cage.

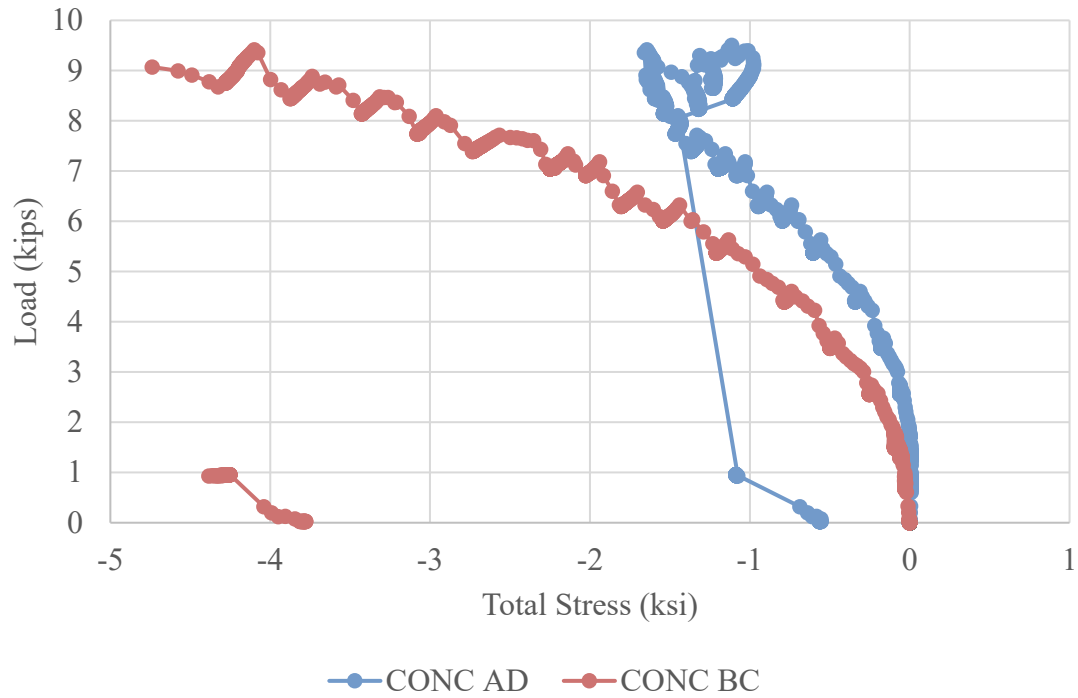


Figure 277. Specimen W-10-7.5 (1st try) , total stress measured by strain gauges CONC AD and CONC BC on the concrete cap surface.

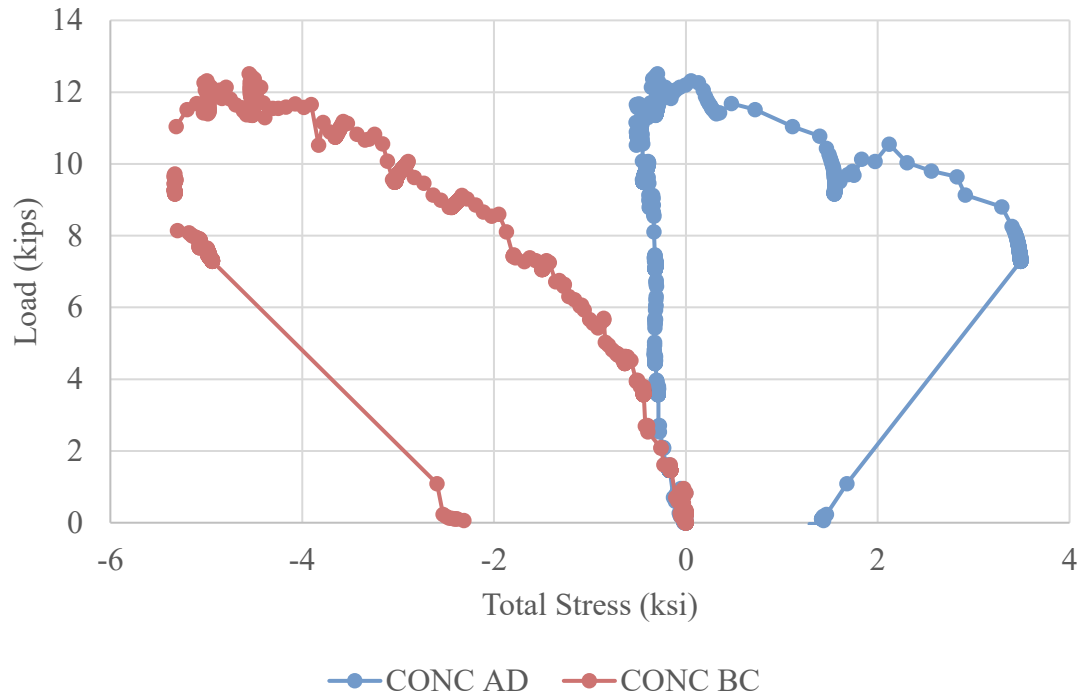


Figure 278. Specimen W-10-7.5 (2nd try) , total stress measured by strain gauges CONC AD and CONC BC on the concrete cap surface.

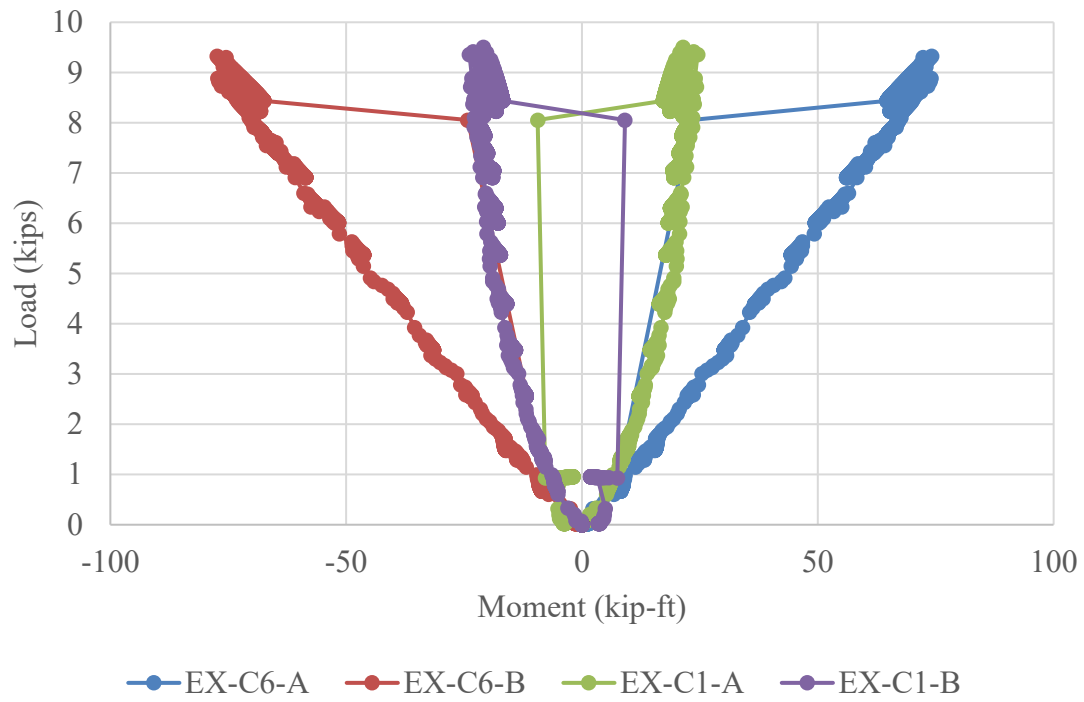


Figure 279. Specimen W-10-7.5 (1st try) , calculated bending moment for EX strain gauges.

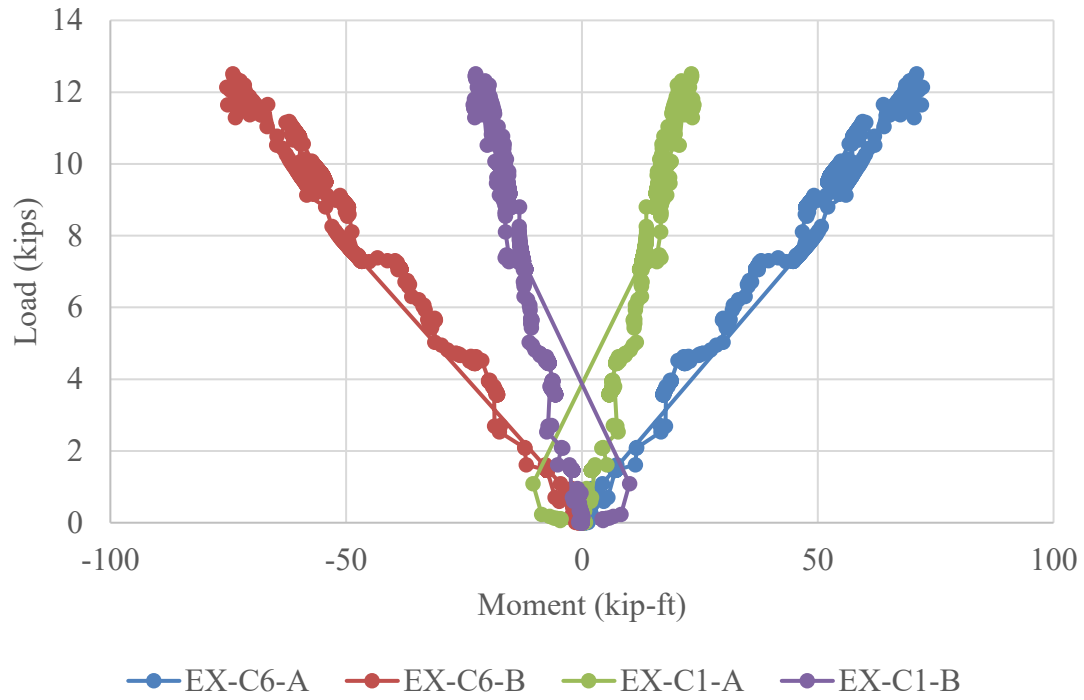


Figure 280. Specimen W-10-7.5 (2nd try) , calculated bending moment for EX strain gauges.

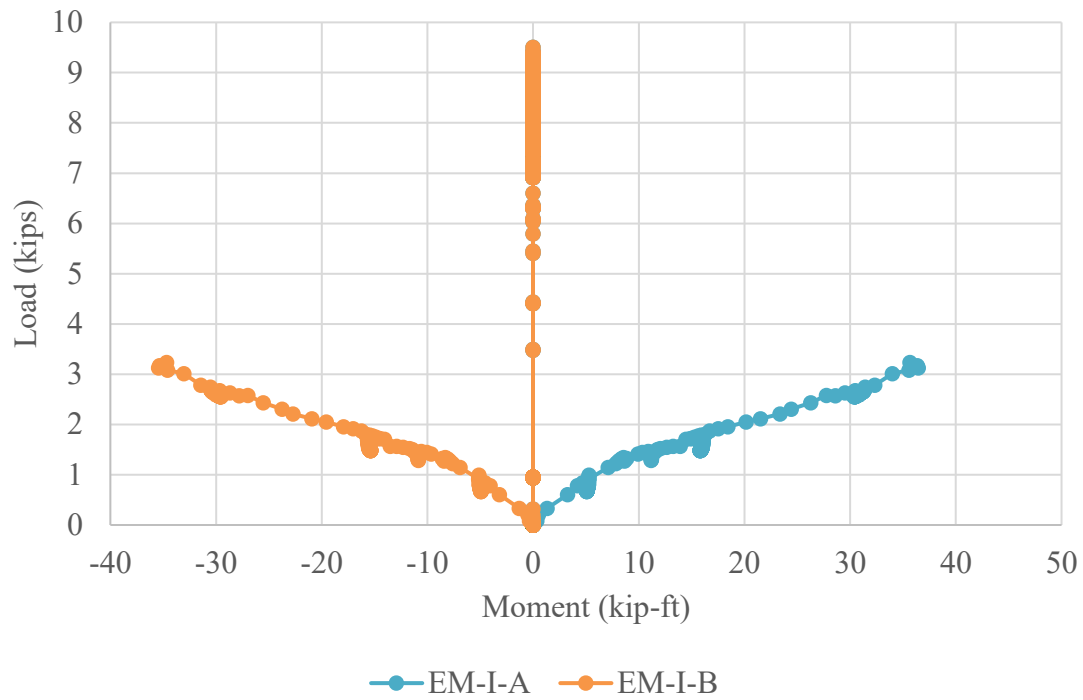


Figure 281. Specimen W-10-7.5 (1st try) , calculated bending moment for EM strain gauges.

In the embedded portion of the pile, all strain gauges (EM-I, or EM-H) malfunctioned in the 2nd try.

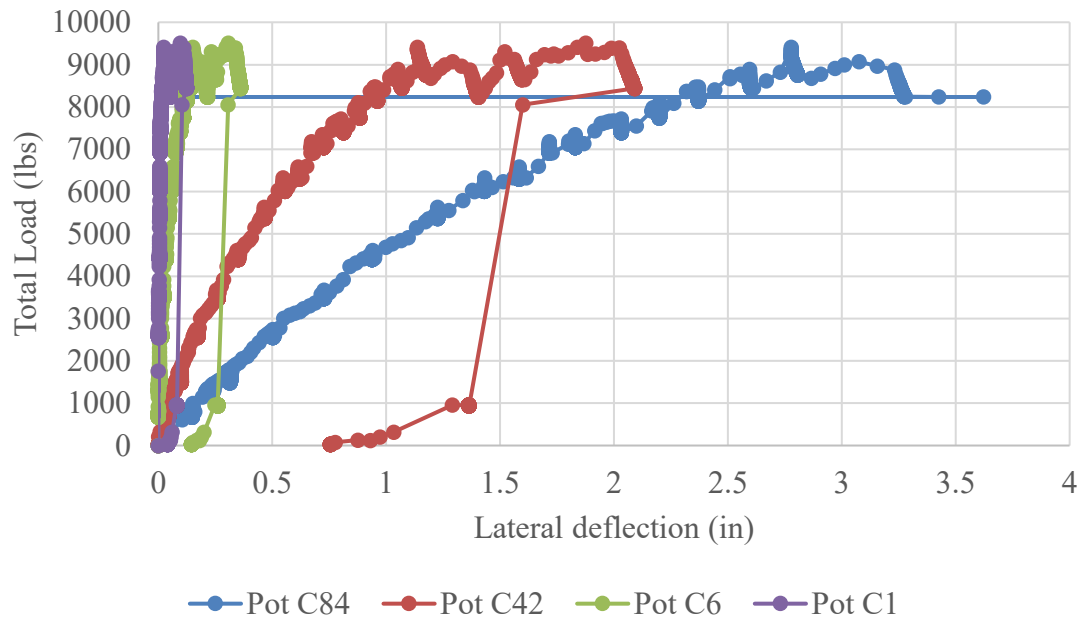


Figure 282. Specimen W-10-7.5 (1st try) wire potentiometers measurements at 1 in (C1), 6 in (C6), 42 in (C42) and 84 in (C84, load application point) above the cap surface.

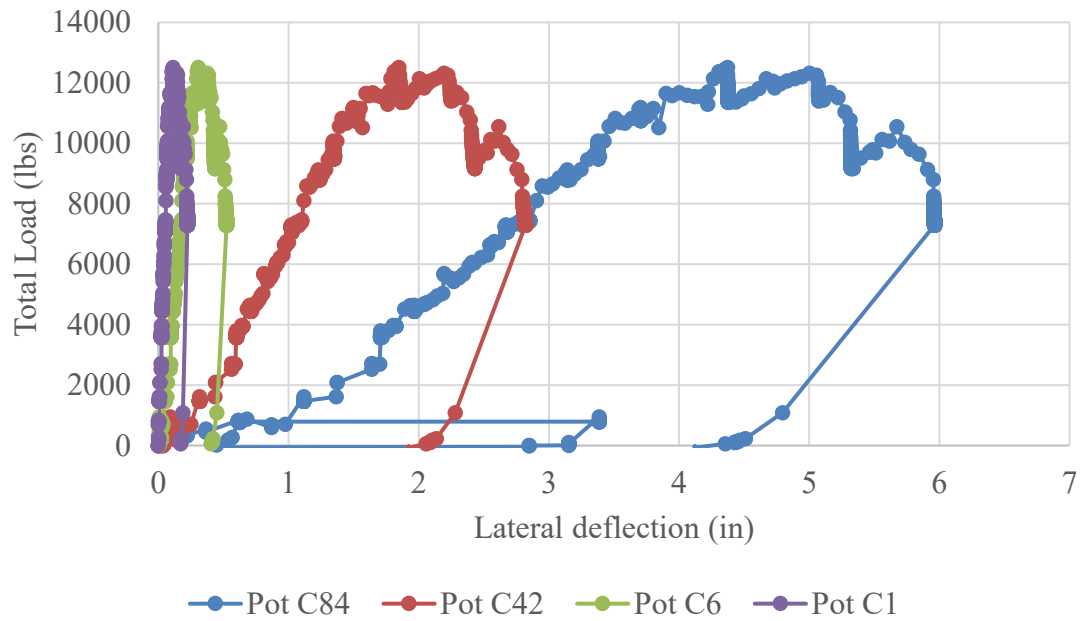


Figure 283. Specimen W-10-7.5 (2nd try) wire potentiometers measurements at 1 in (C1), 6 in (C6), 42 in (C42) and 84 in (C84, load application point) above the cap surface.

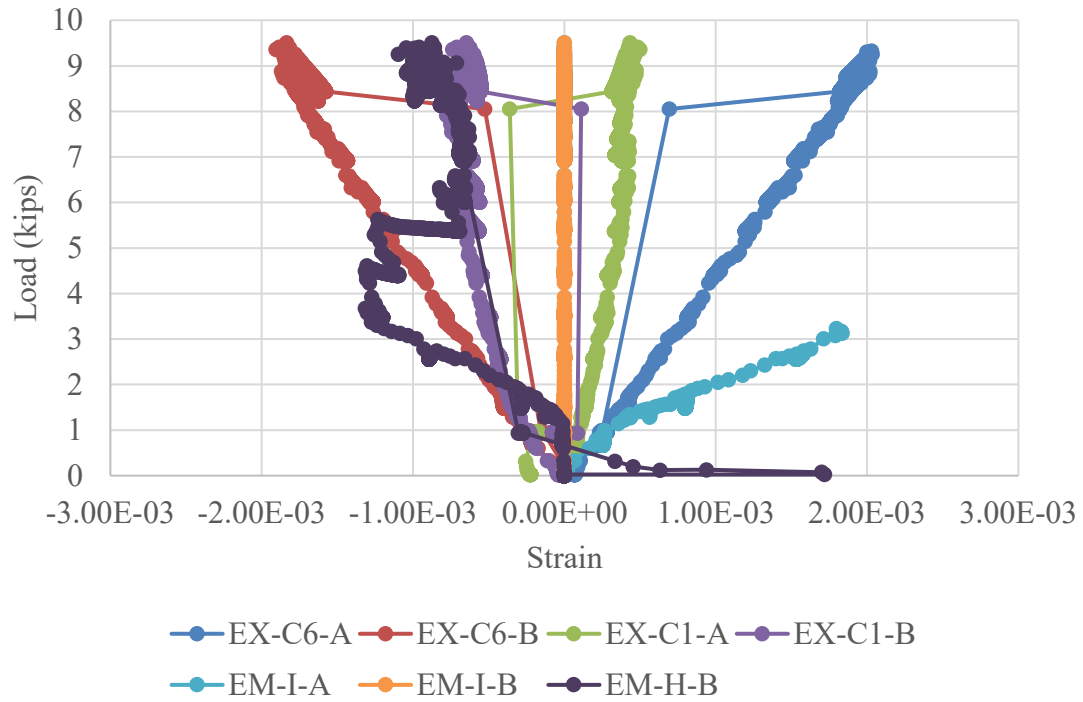


Figure 284. Specimen W-10-7.5 (1st try) strain gauges strain measurements on the steel pile.

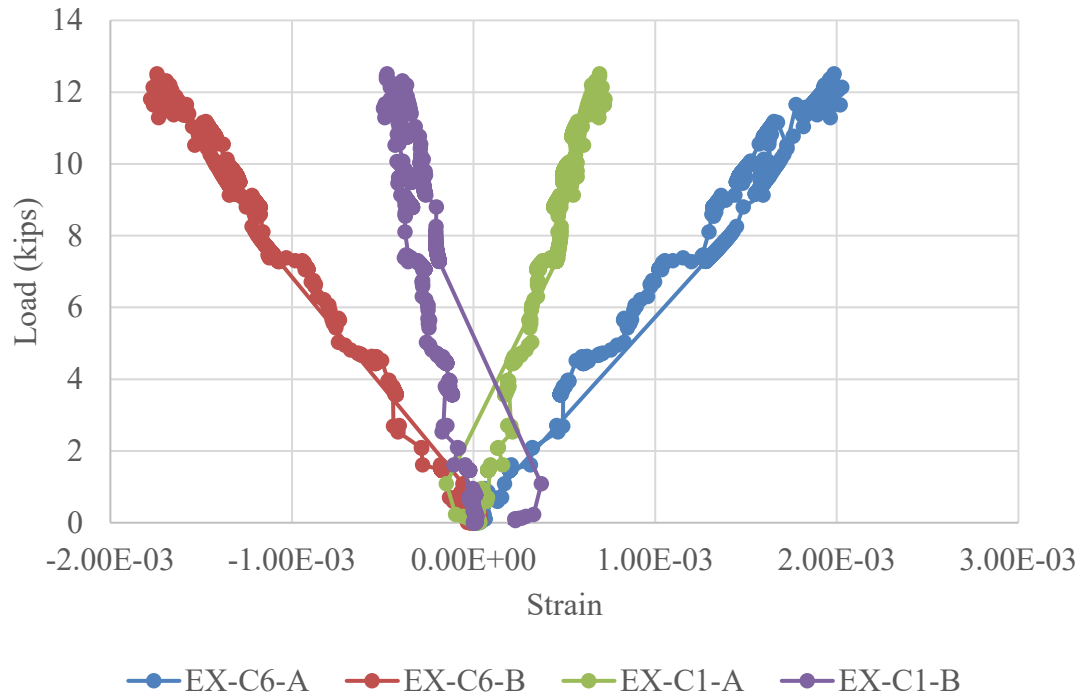


Figure 285. Specimen W-10-7.5 (2nd try) strain gauges strain measurements on the steel pile.

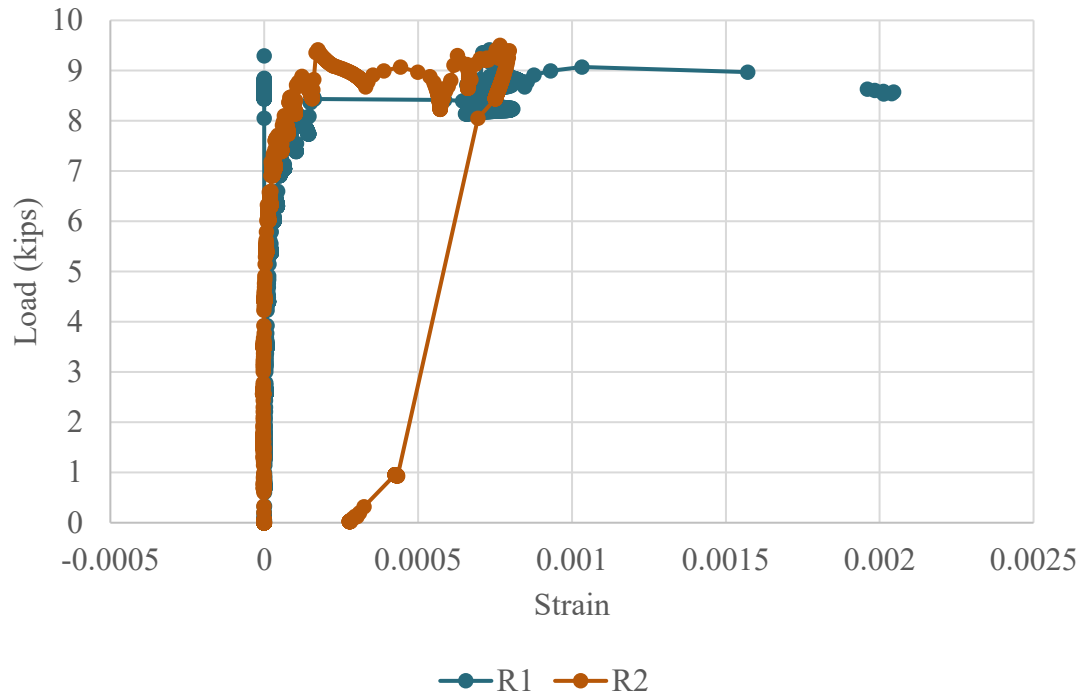


Figure 286. Specimen W-10-7.5 (1st try) strain gauges strain measurements on steel reinforcement bars.

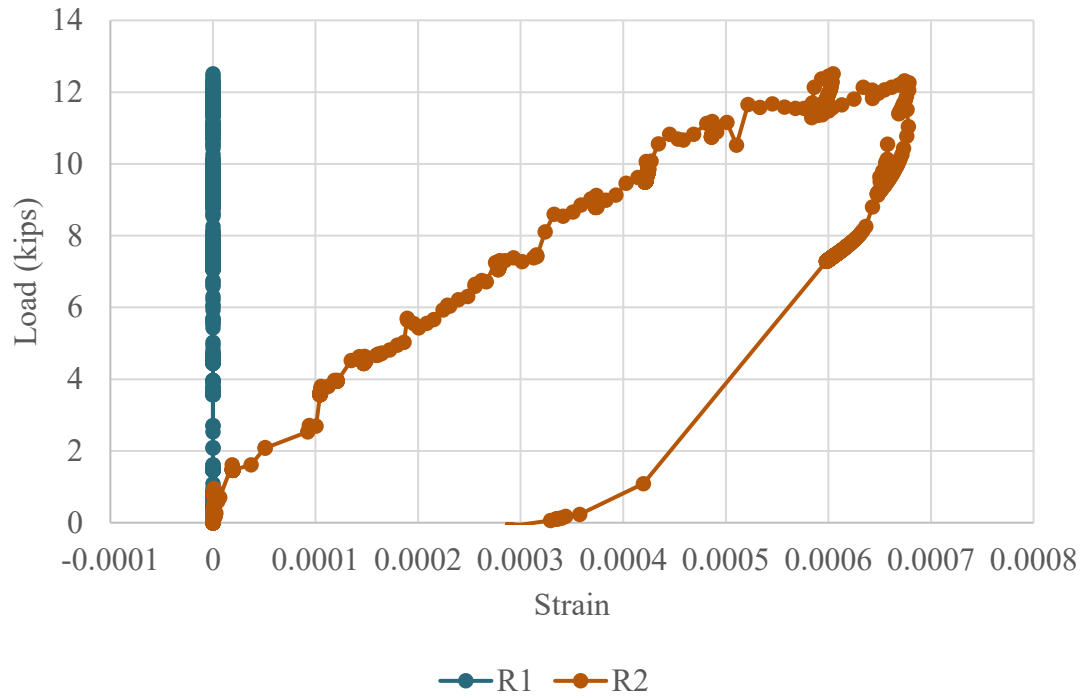


Figure 287. Specimen W-10-7.5 (2nd try) strain gauges strain measurements on steel reinforcement bars.

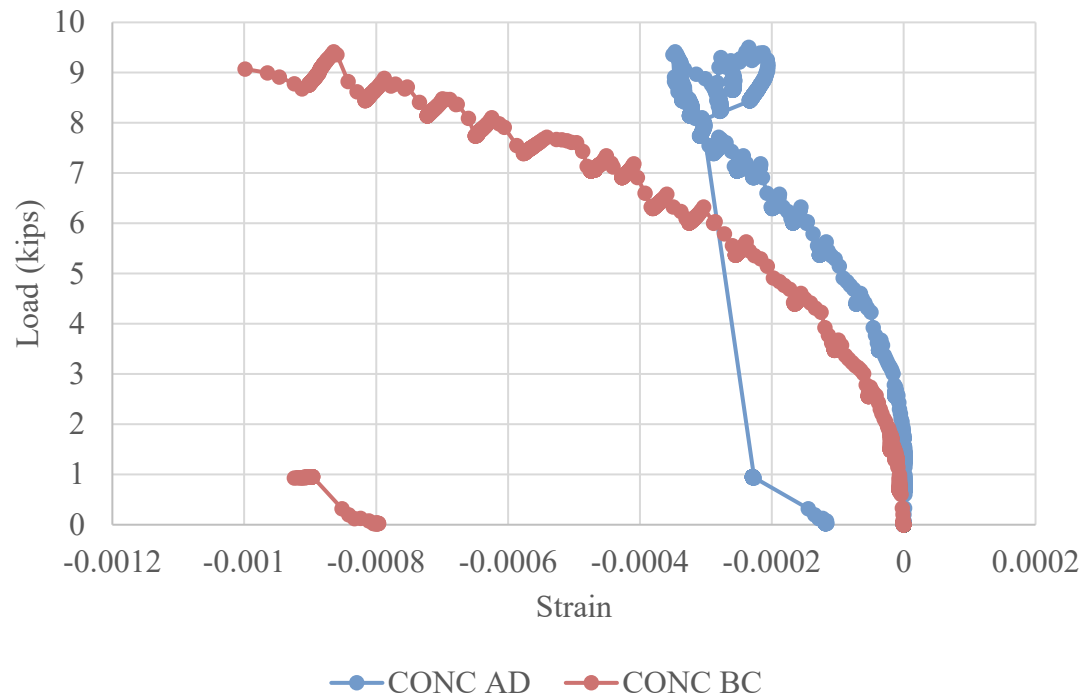


Figure 288. Specimen W-10-7.5 (1st try) strain gauges strain measurements on concrete cap surface.

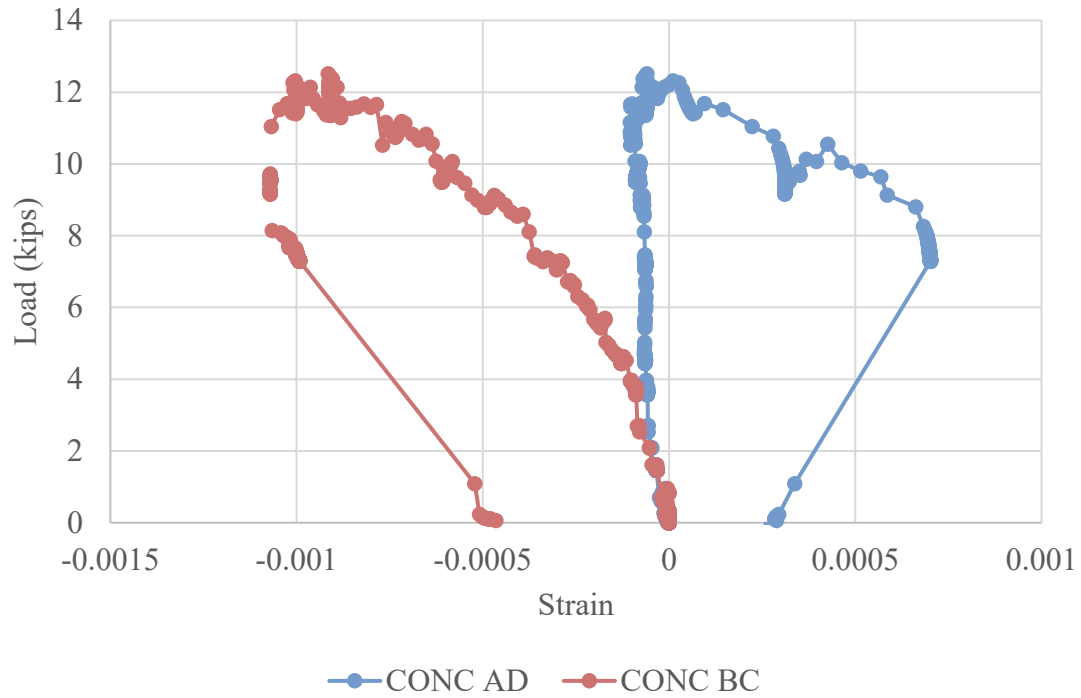


Figure 289. Specimen W-10-7.5 (2nd try) strain gauges strain measurements on concrete cap surface.

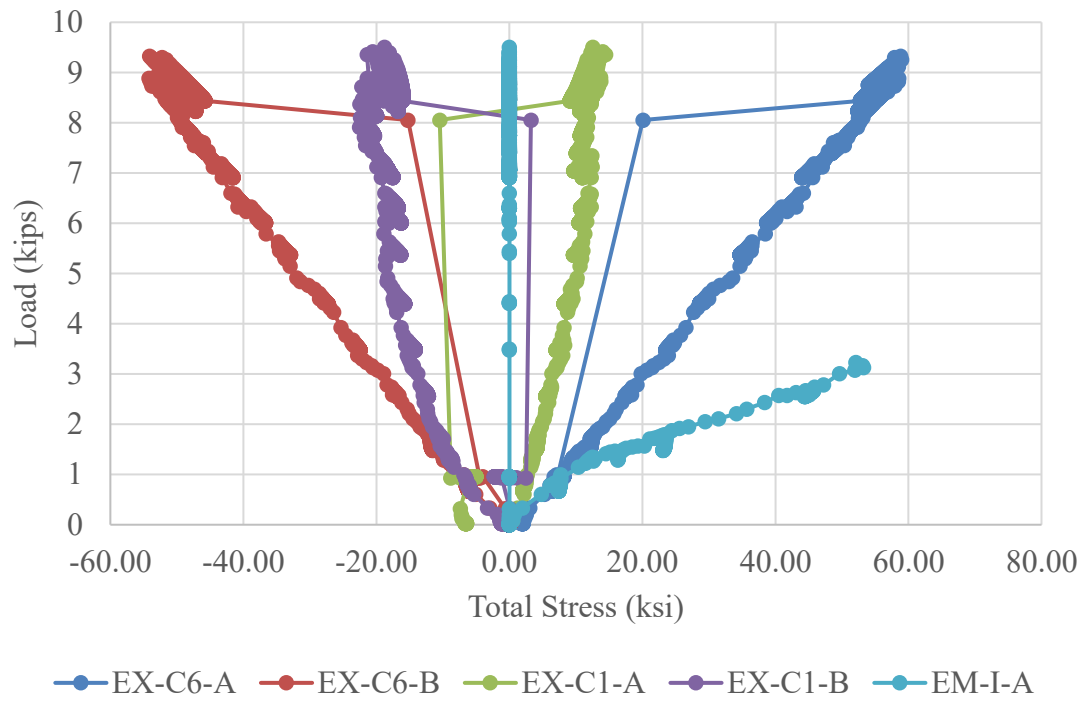


Figure 290. Specimen W-10-7.5 (1st try) stress calculated from strain gauges on the steel pile.

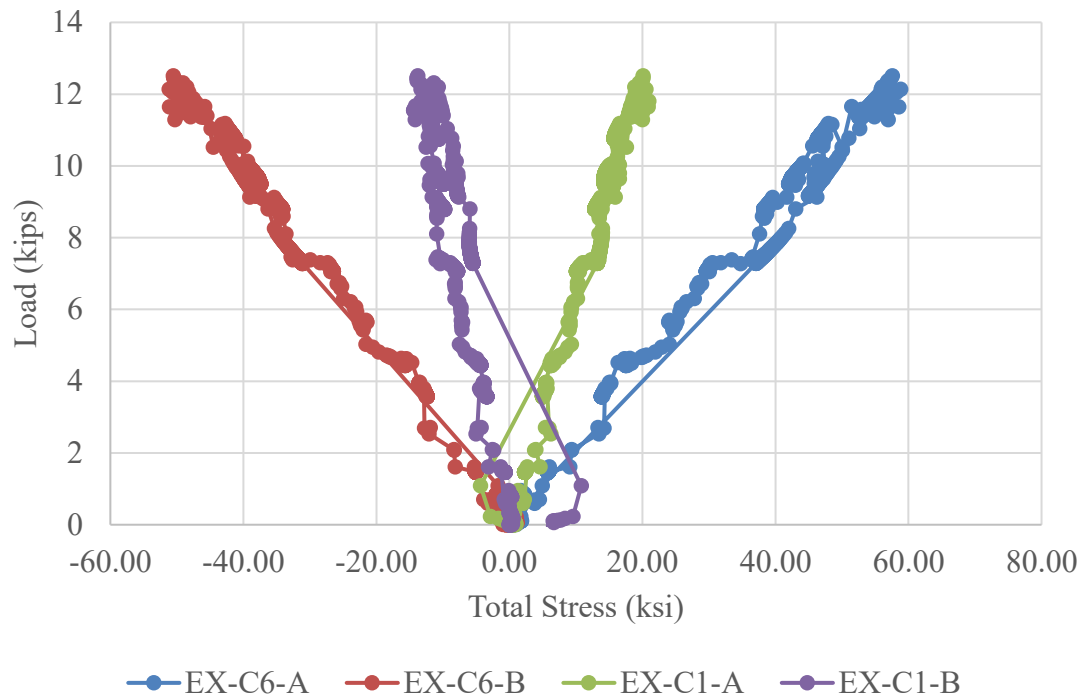


Figure 291. Specimen W-10-7.5 (2nd try) stress calculated from strain gauges on the steel pile.

4. Specimen W-10-7.5R

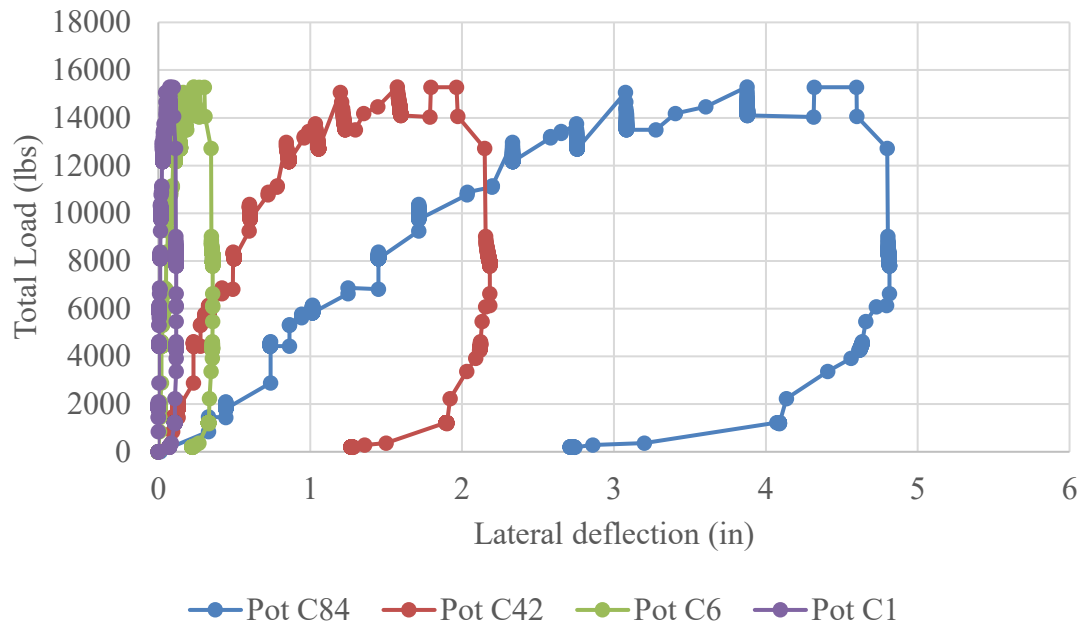


Figure 292. Specimen W-10-7.5R wire potentiometers measurements at 1 in (C1), 6 in (C6), 42 in (C42) and 84 in (C84, load application point) above the cap surface.

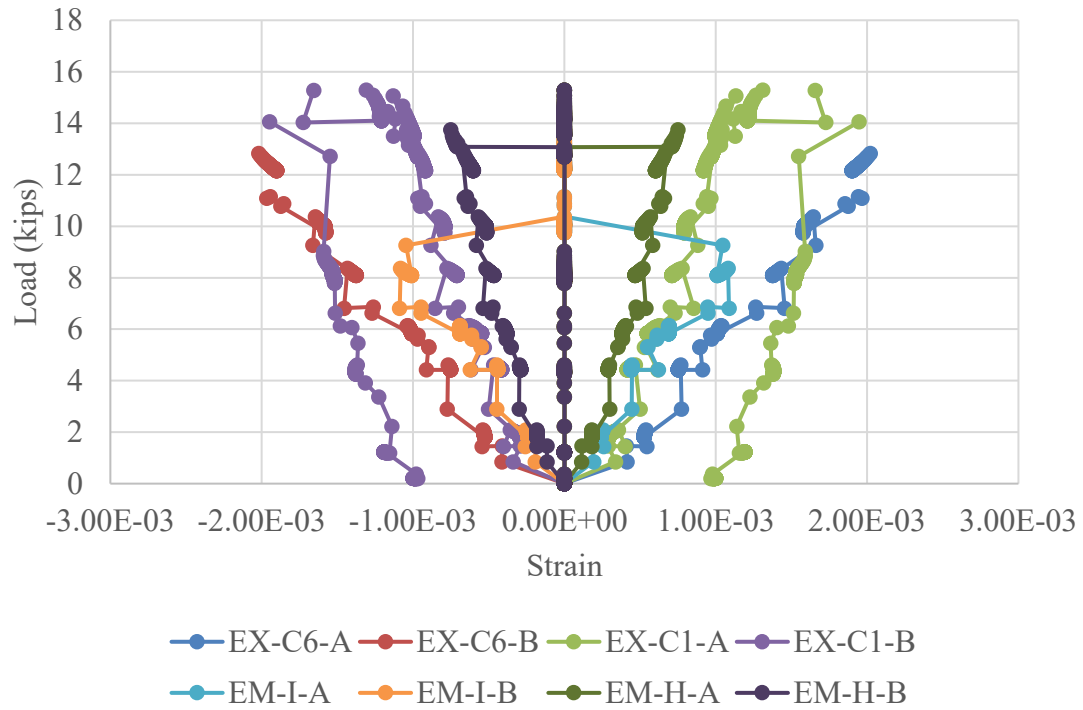


Figure 293. Specimen W-10-7.5R strain gauges strain measurements on the steel pile.

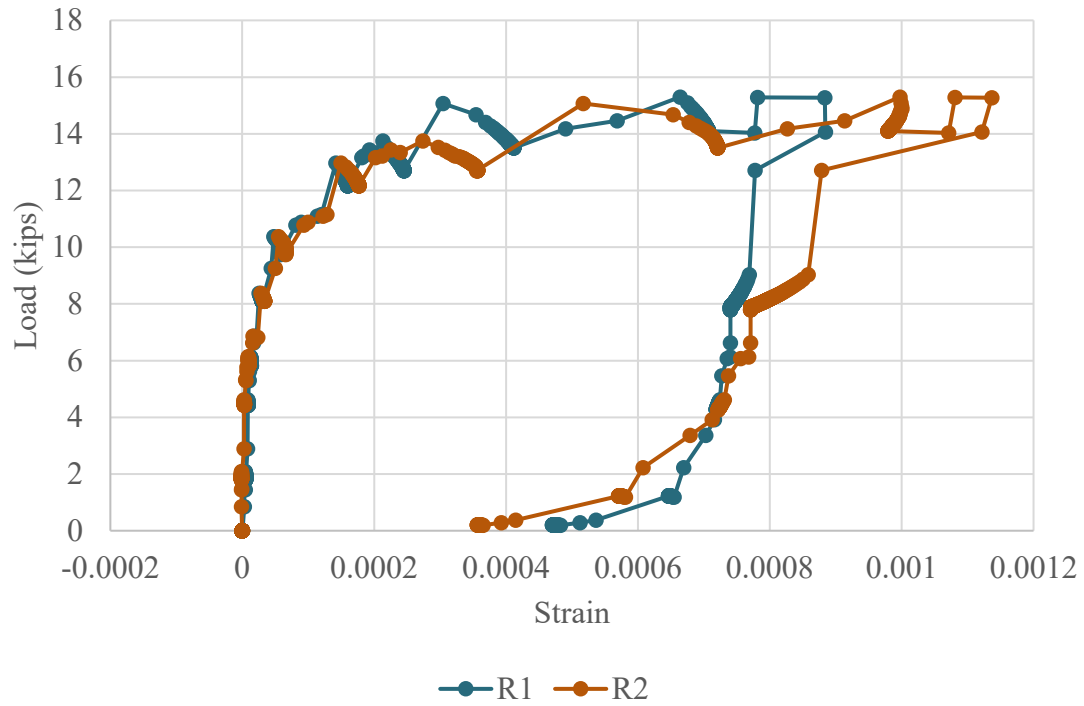


Figure 294. Specimen W-10-7.5R strain gauges strain measurements on steel reinforcement bars.

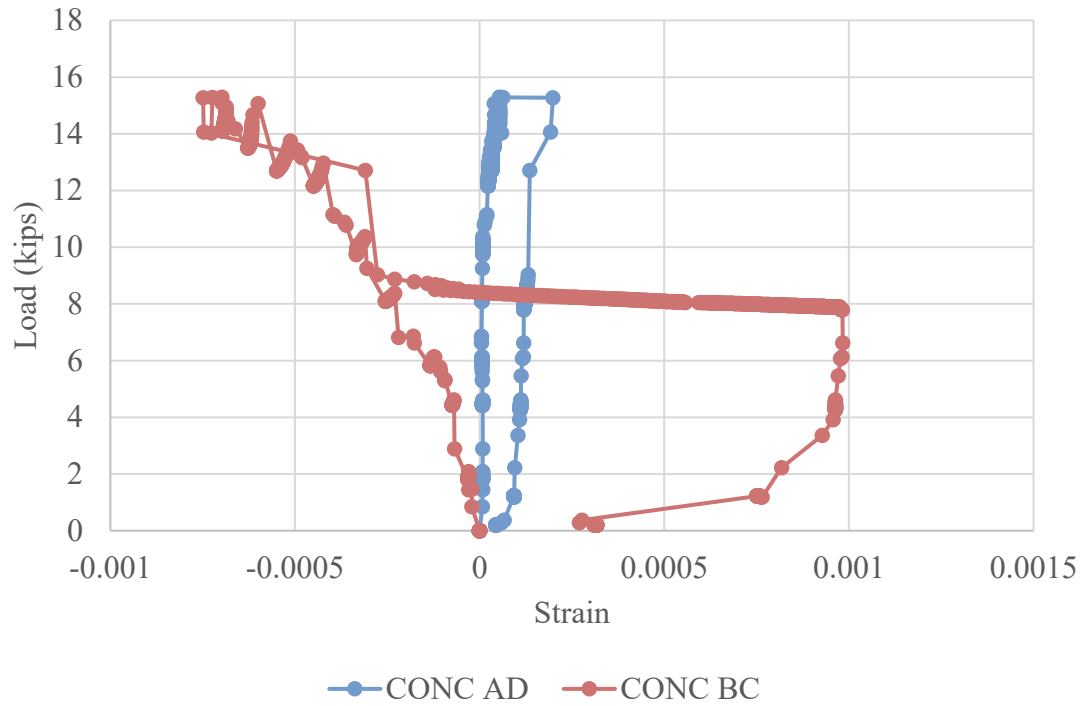


Figure 295. Specimen W-10-7.5R strain gauges strain measurements on concrete cap surface.

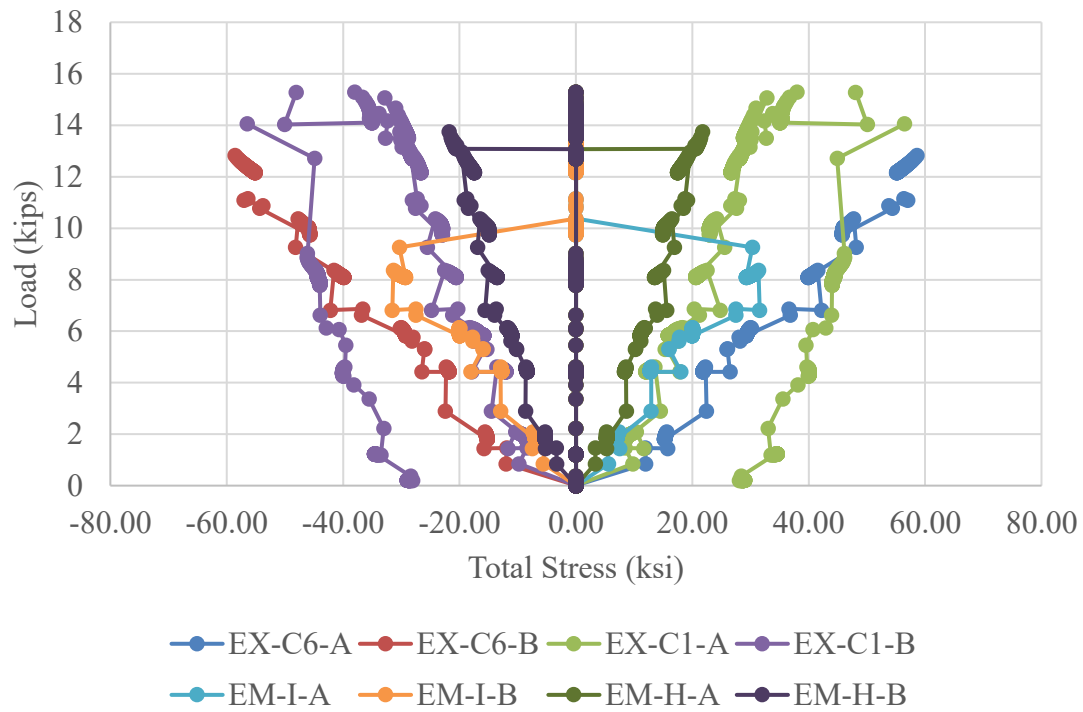


Figure 296. Specimen W-10-7.5R stress calculated from strain gauges on the steel pile.

5. Specimen W-10-5

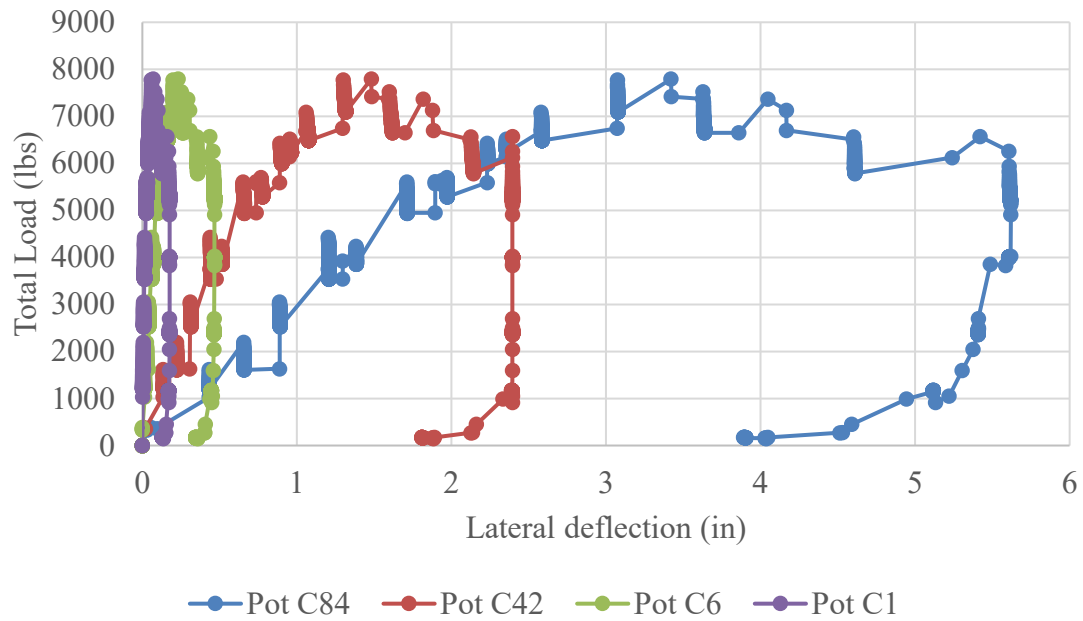


Figure 297. Specimen W-10-5 wire potentiometers measurements at 1 in (C1), 6 in (C6), 42 in (C42) and 84 in (C84, load application point) above the cap surface.

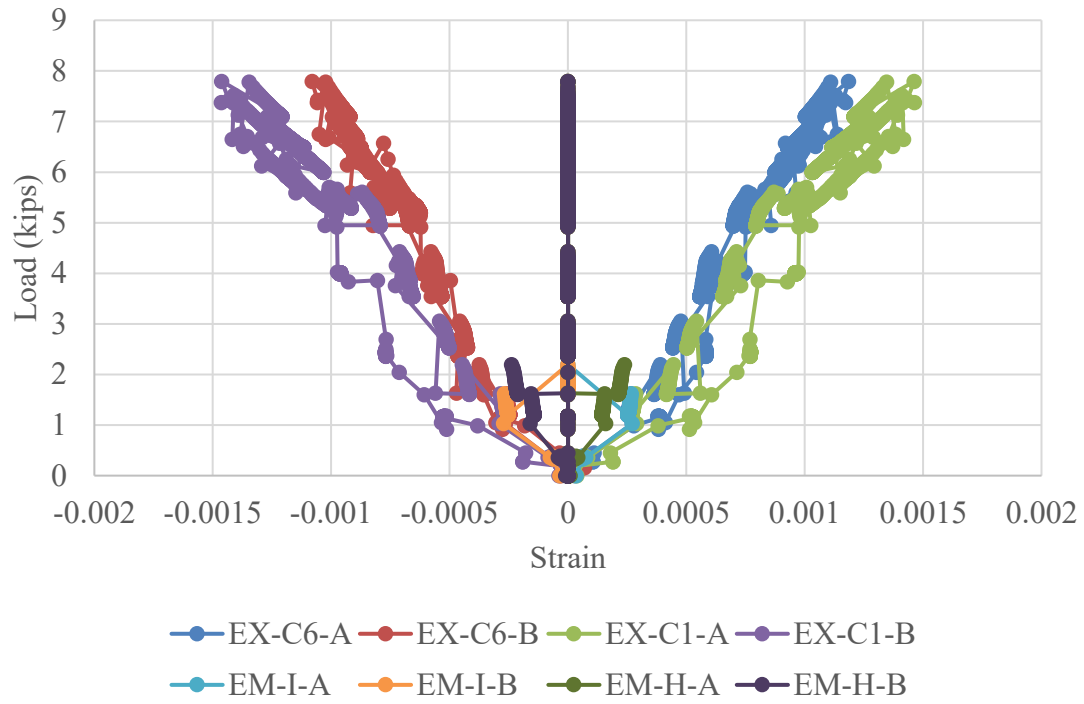


Figure 298. Specimen W-10-5 strain gauges strain measurements on the steel pile.

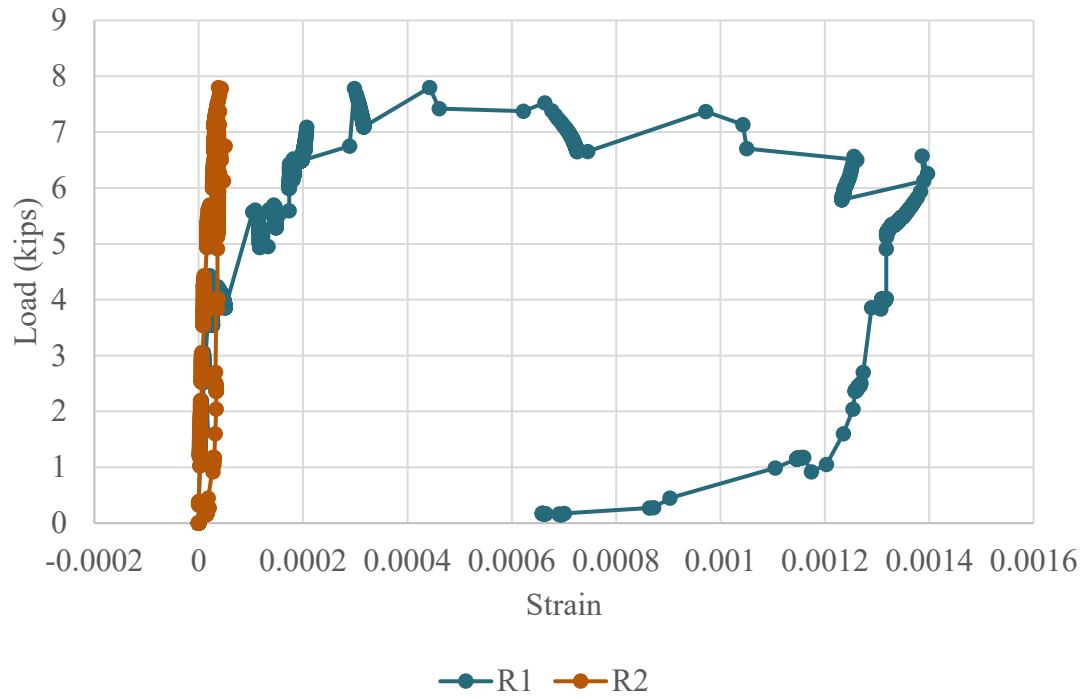


Figure 299. Specimen W-10-5 strain gauges strain measurements on steel reinforcement bars.

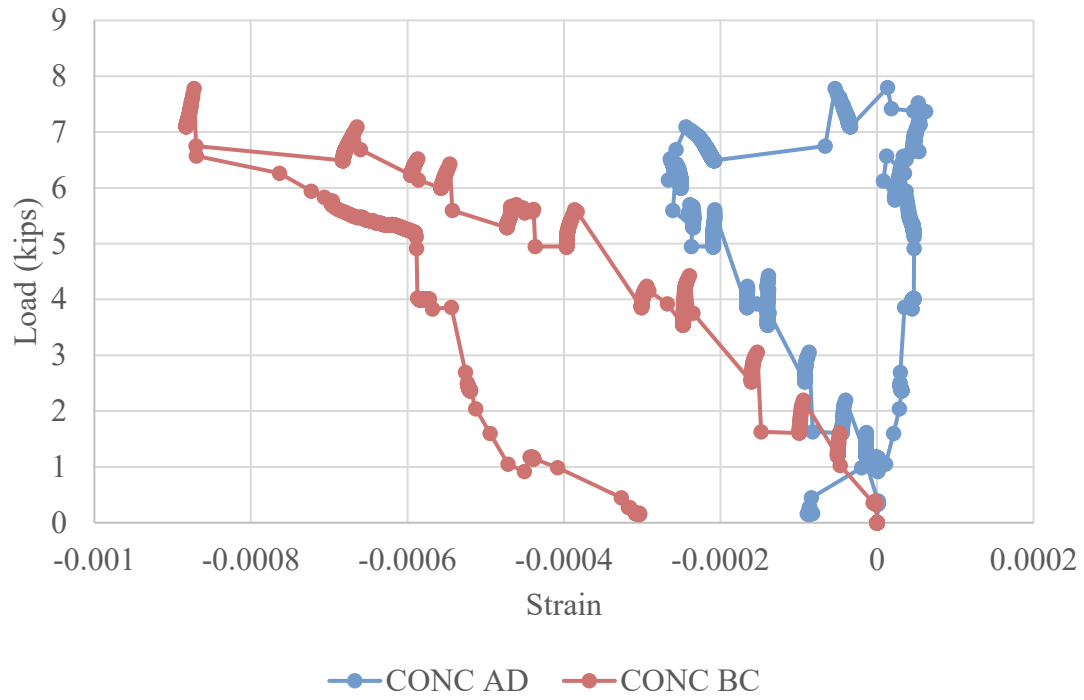


Figure 300. Specimen W-10-5 strain gauges strain measurements on concrete cap surface.

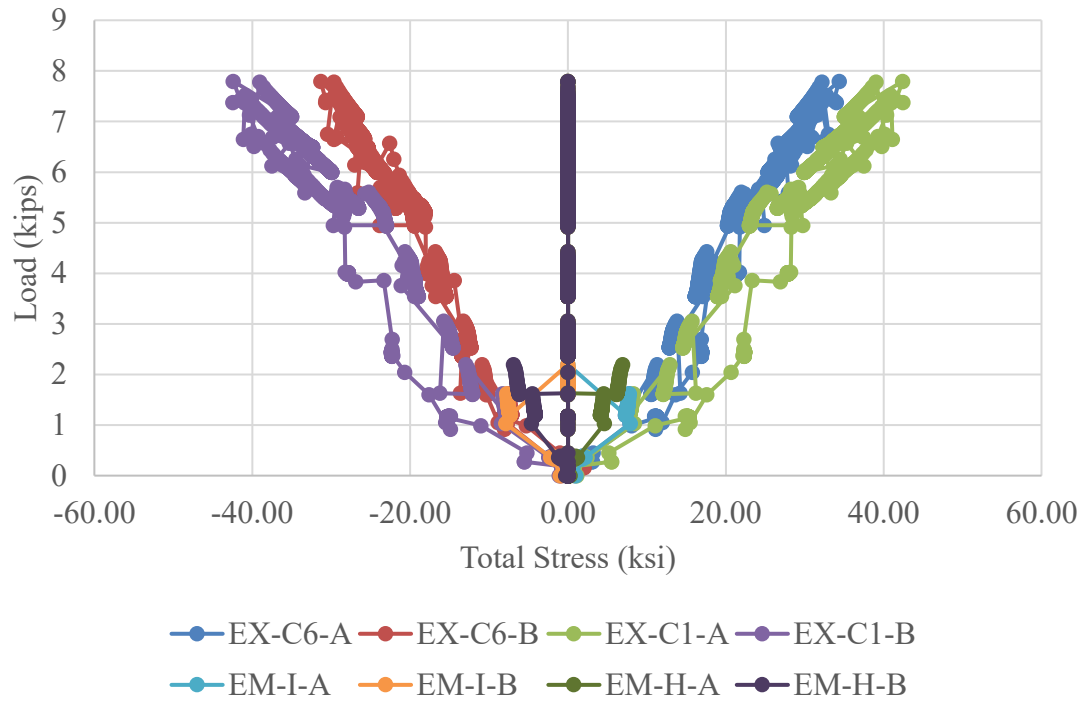


Figure 301. Specimen W-10-5 stress calculated from strain gauges on the steel pile.

6. Specimen W-12-12

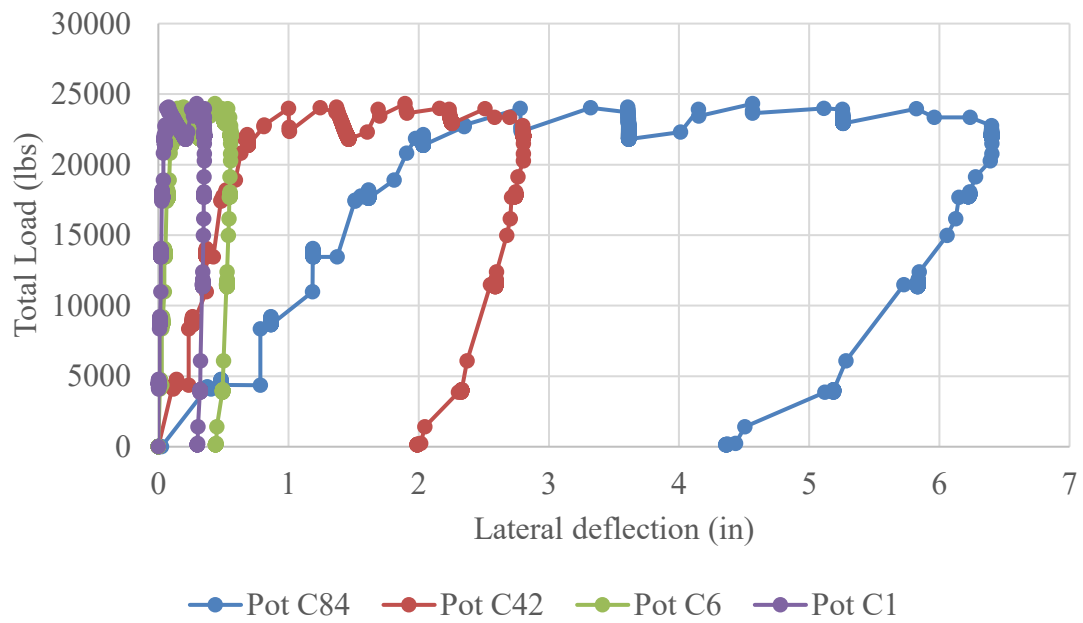


Figure 302. Specimen W-12-12 wire potentiometers measurements at 1 in (C1), 6 in (C6), 42 in (C42) and 84 in (C84, load application point) above the cap surface.

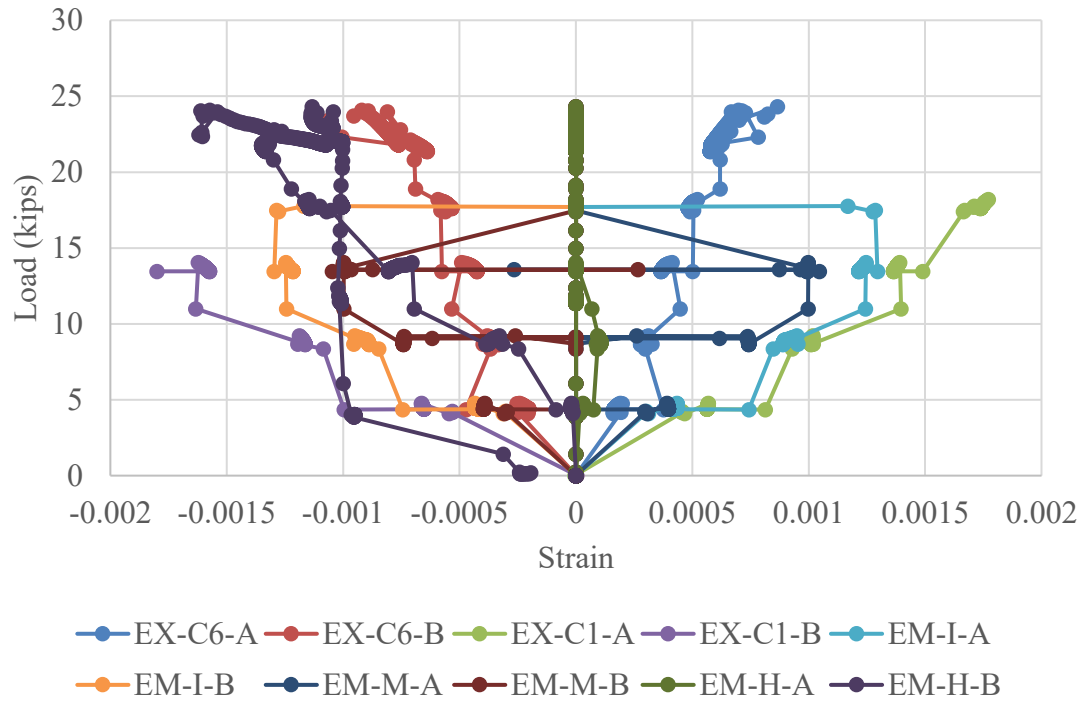


Figure 303. Specimen W-12-12 strain gauges strain measurements on the steel pile.

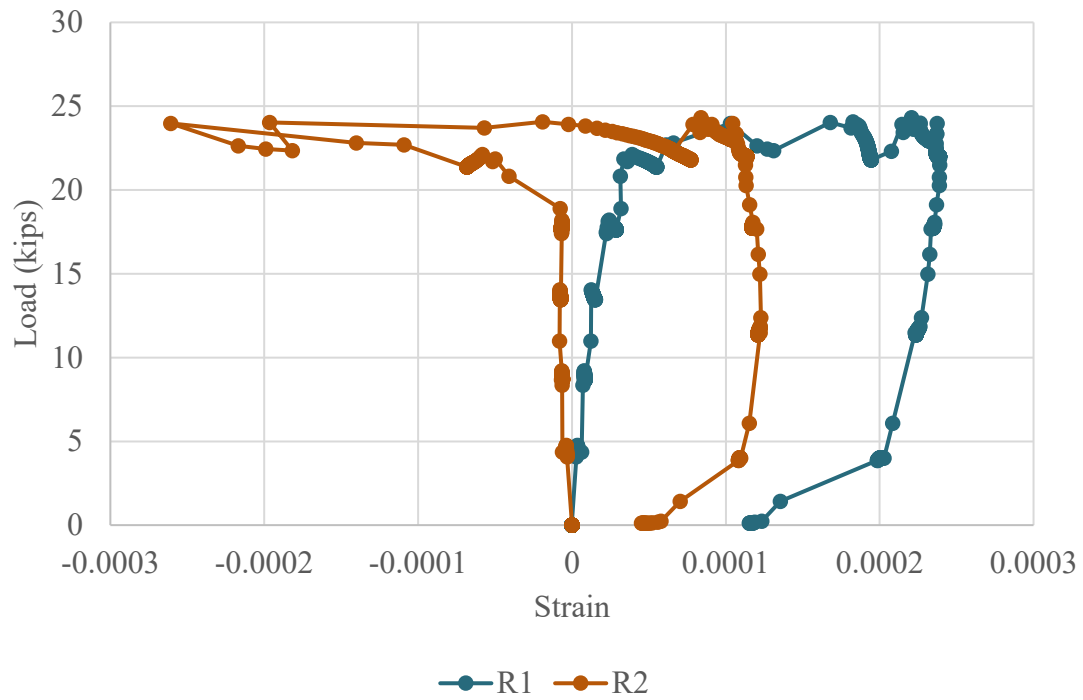


Figure 304. Specimen W-12-12 strain gauges strain measurements on steel reinforcement bars.

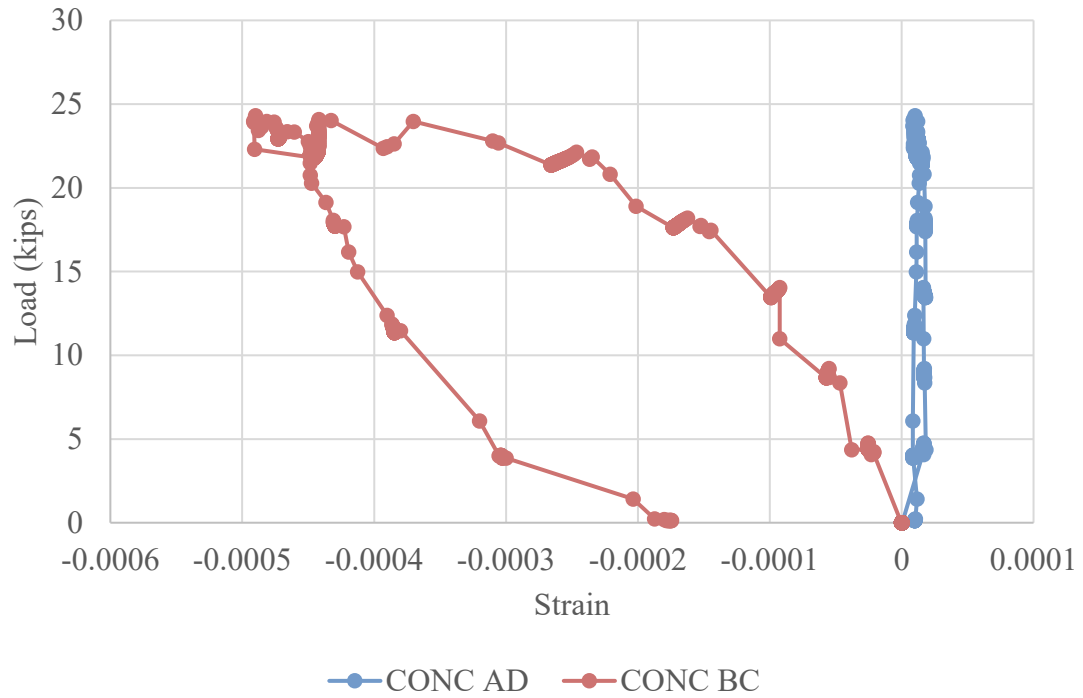


Figure 305. Specimen W-12-12 strain gauges strain measurements on concrete cap surface.

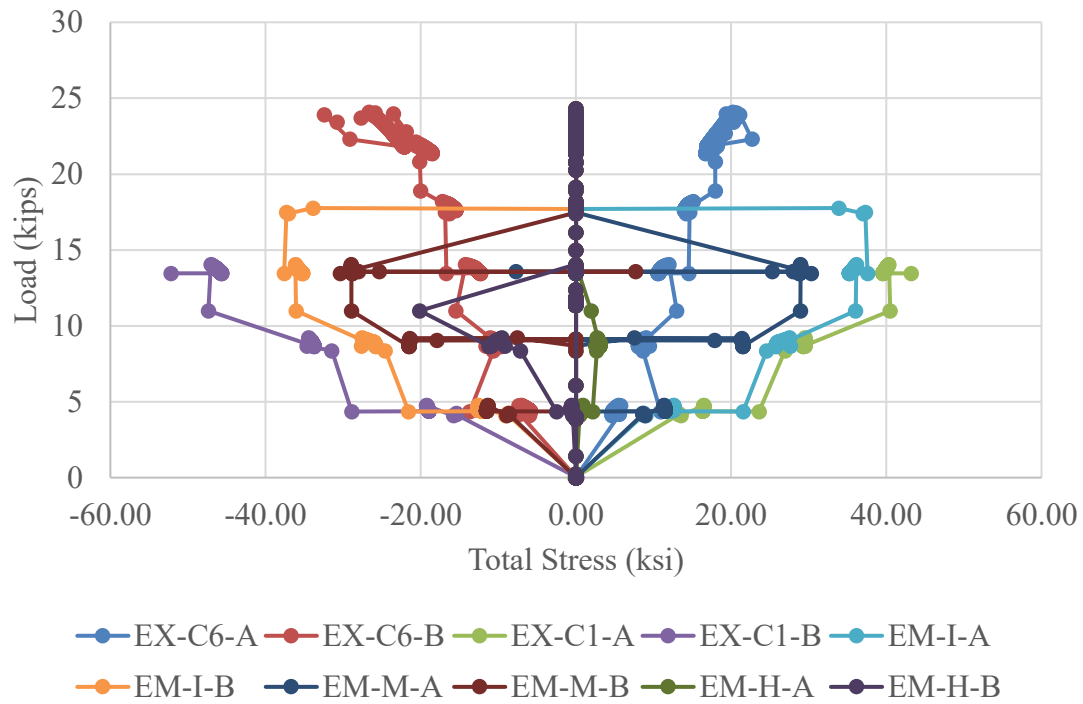


Figure 306. Specimen W-12-12 stress calculated from strain gauges on the steel pile.

7. Specimen W-12-6

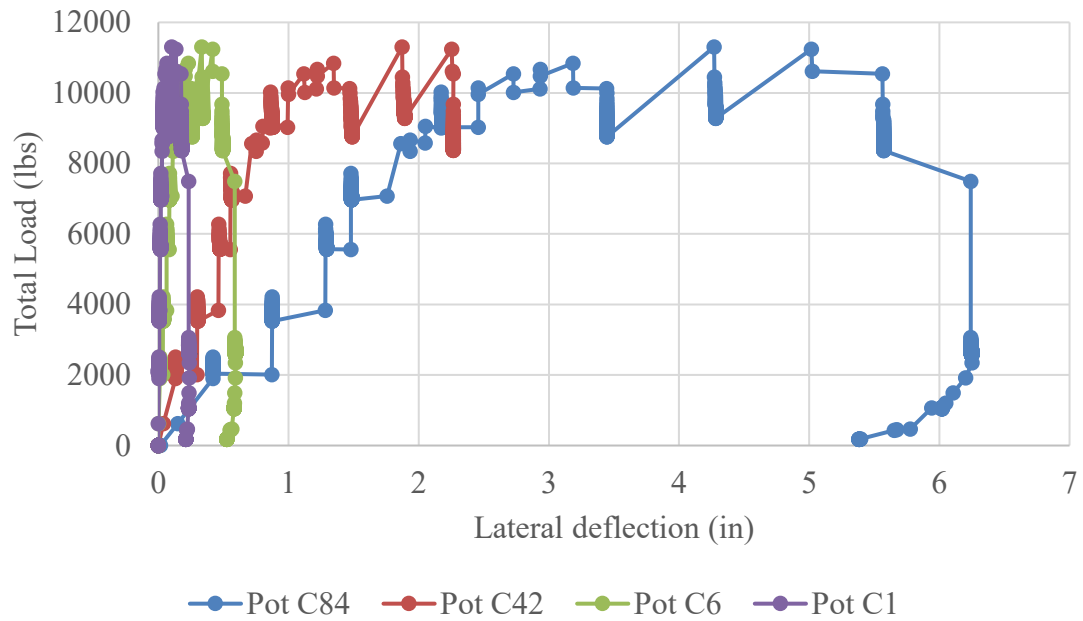


Figure 307. Specimen W-12-6 wire potentiometers measurements at 1 in (C1), 6 in (C6), 42 in (C42) and 84 in (C84, load application point) above the cap surface.

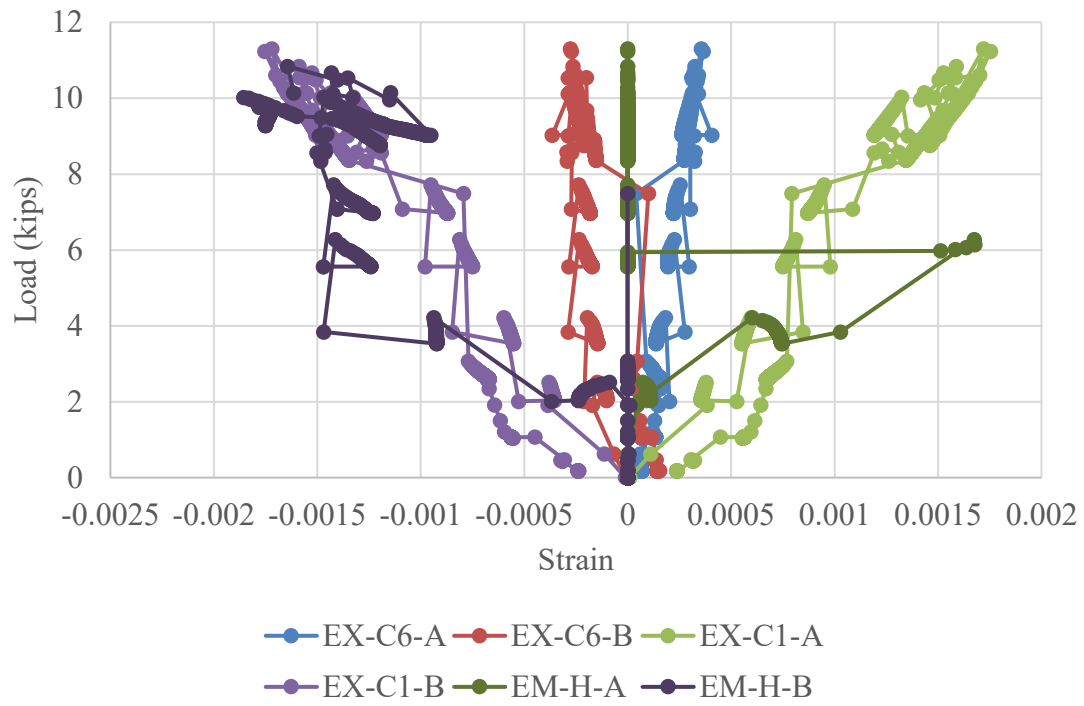


Figure 308. Specimen W-12-6 strain gauges strain measurements on the steel pile.

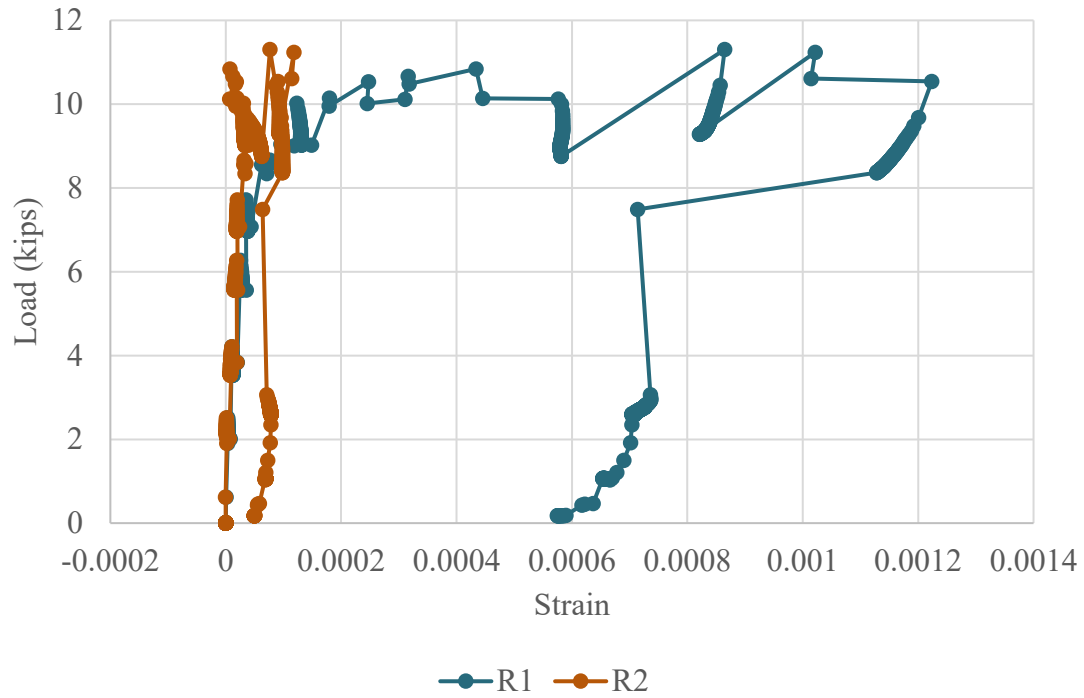


Figure 309. Specimen W-12-6 strain gauges strain measurements on steel reinforcement bars.

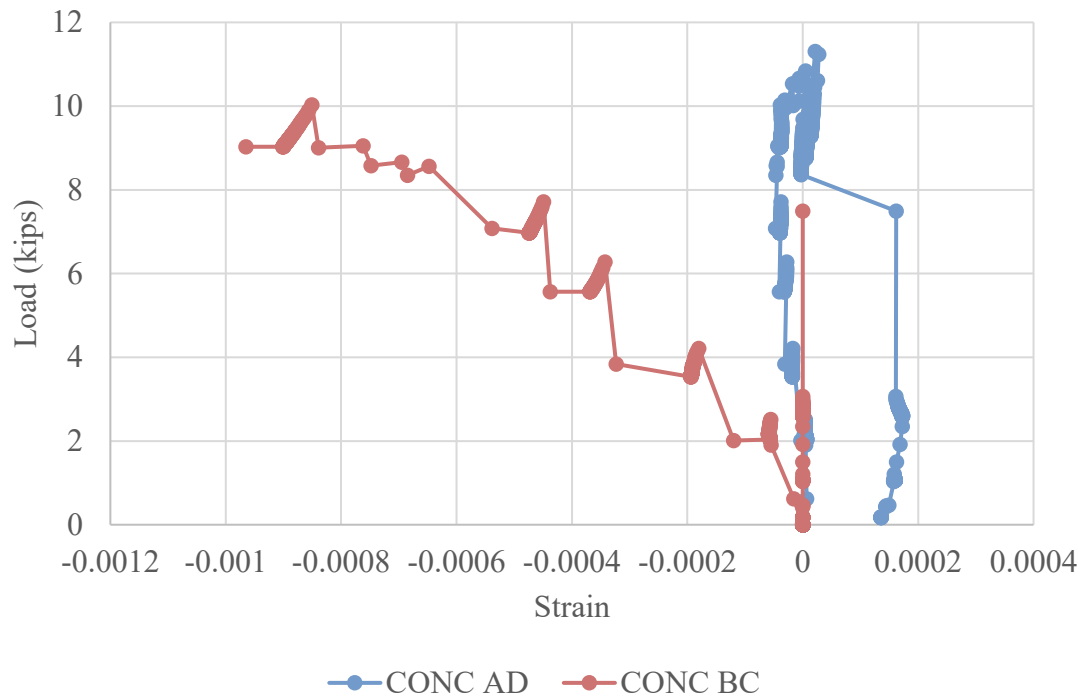


Figure 310. Specimen W-12-6 strain gauges strain measurements on concrete cap surface.

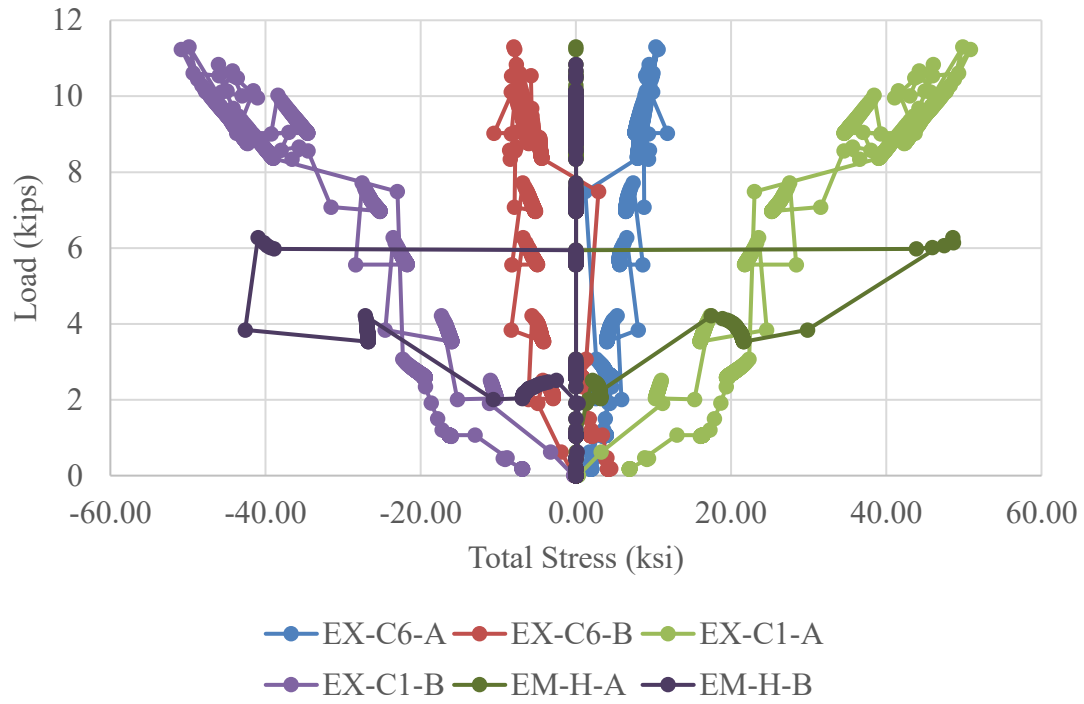


Figure 311. Specimen W-12-6 stress calculated from strain gauges on the steel pile.

8. Specimen S-10-10

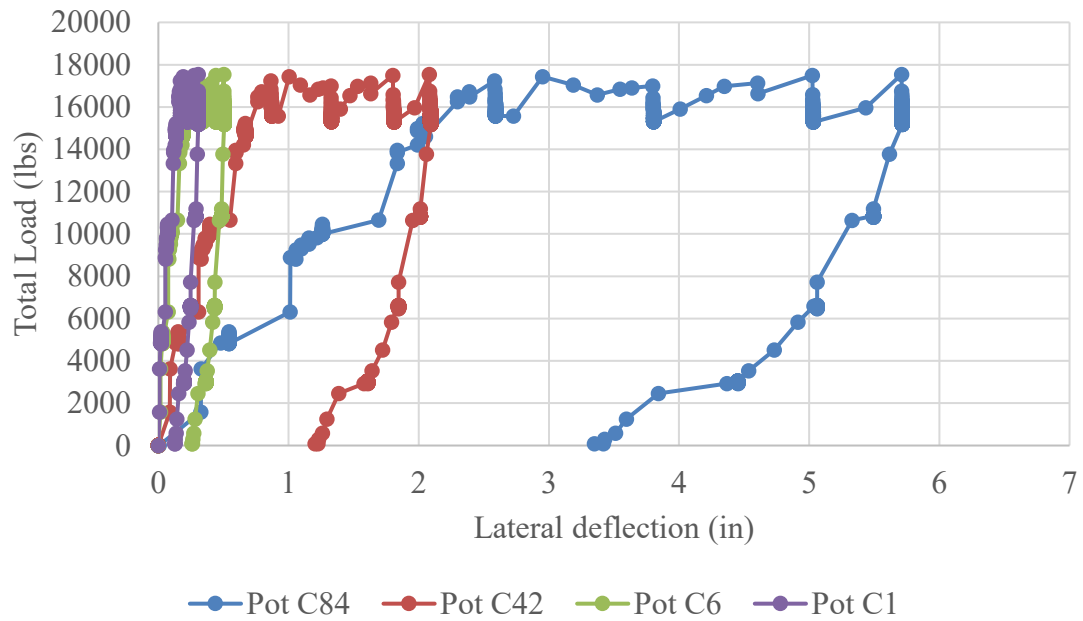


Figure 312. Specimen S-10-10 wire potentiometers measurements at 1 in (C1), 6 in (C6), 42 in (C42) and 84 in (C84, load application point) above the cap surface.

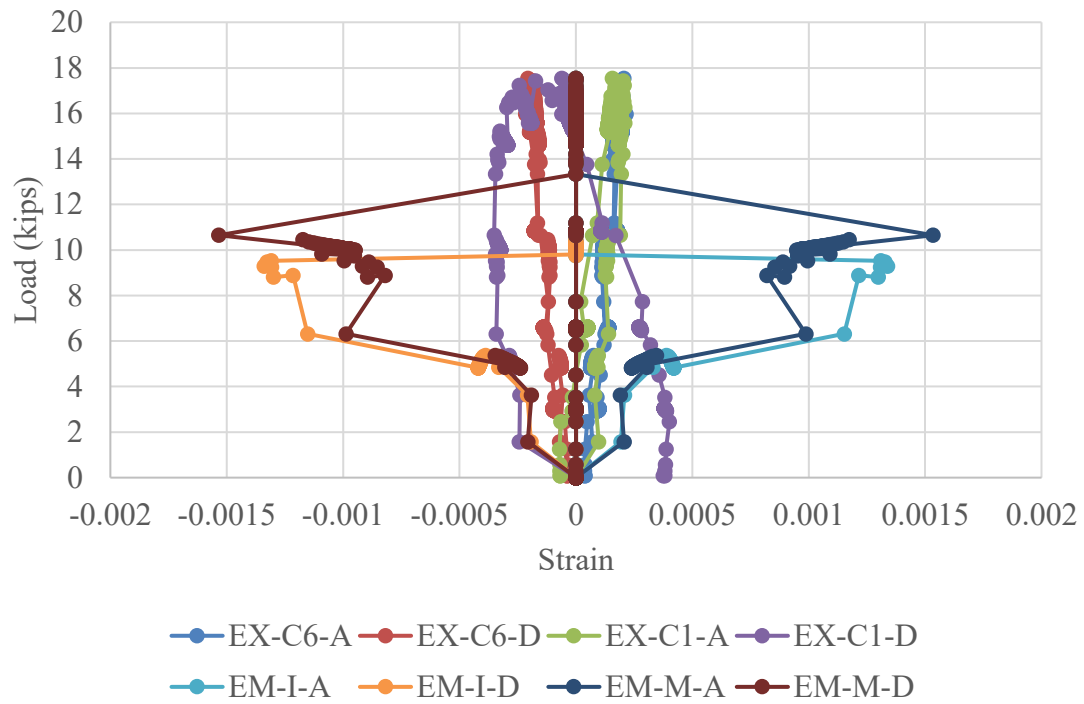


Figure 313. Specimen S-10-10 strain gauges strain measurements on the steel pile.

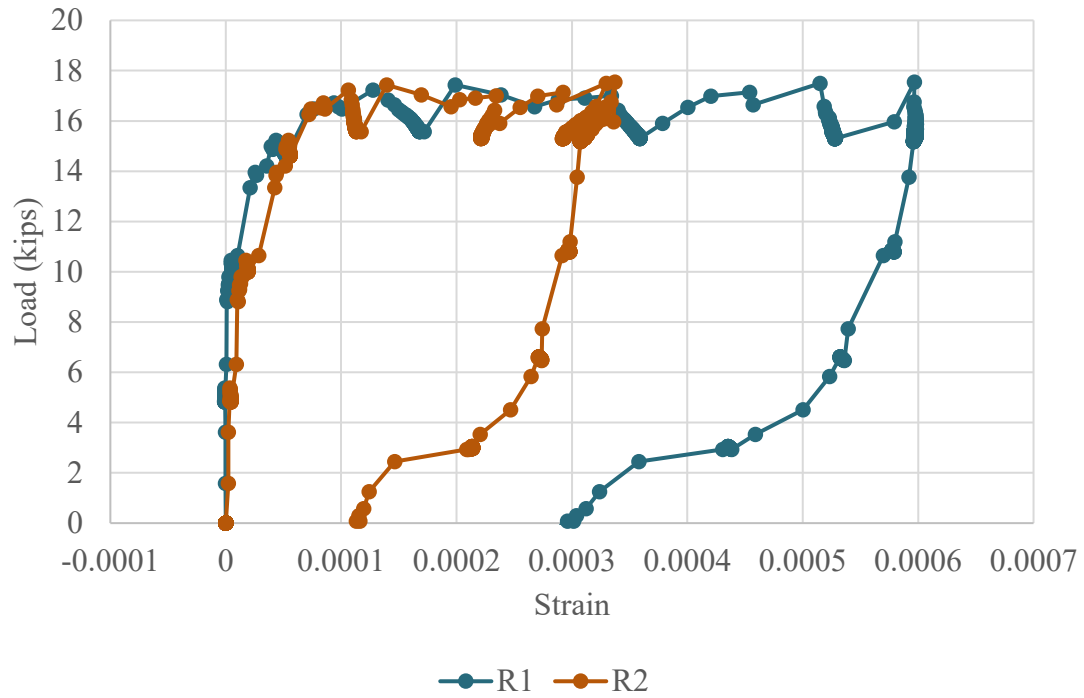


Figure 314. Specimen S-10-10 strain gauges strain measurements on steel reinforcement bars.

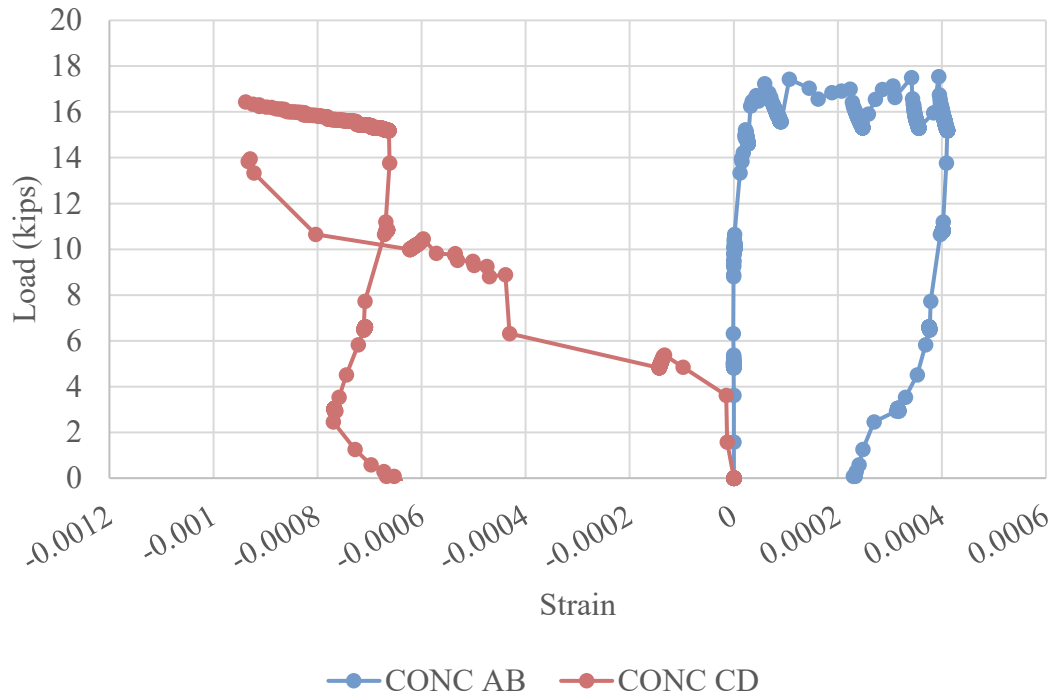


Figure 315. Specimen S-10-10 strain gauges strain measurements on concrete cap surface.

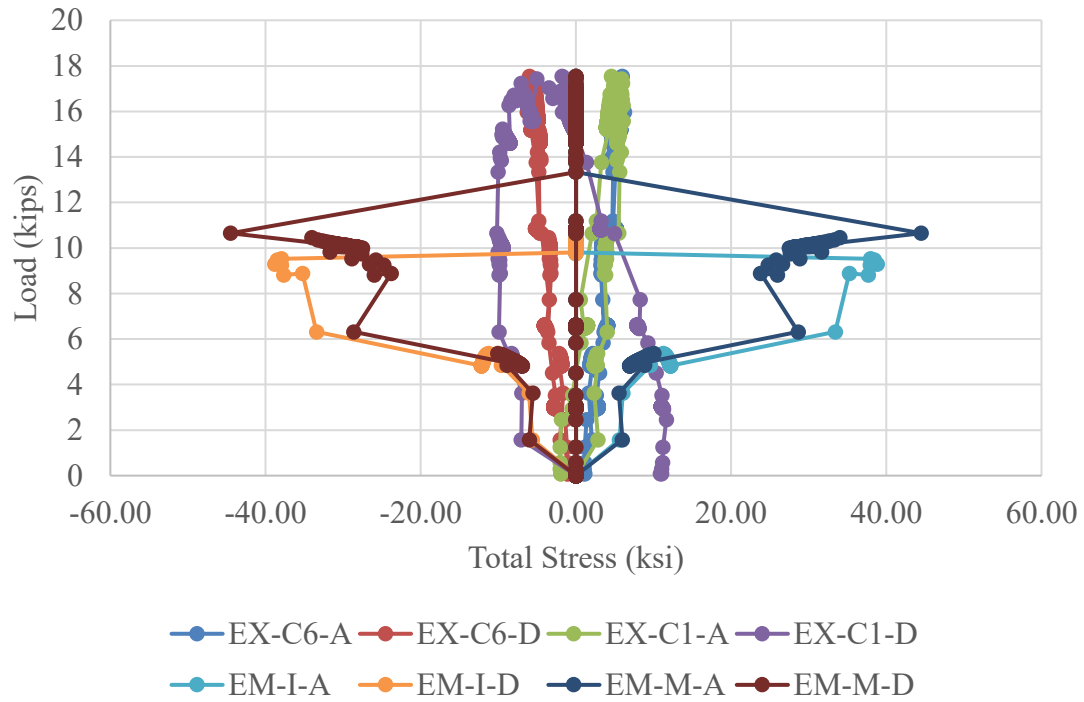


Figure 316. Specimen S-10-10 stress calculated from strain gauges on the steel pile.

Appendix C: MATLAB Code based on Hammett (2017)

This appendix presents the code presented by Hammett (2017), originally in MathCad, now modified and adapted for MATLAB.

```
%% Theoretical Moment-Rotation Curve for Embedded Steel Piles (Hammett, 2017)
% modified and adapted to MATLAB by Murilo Tarozzo, mht0028@auburn.edu, in
% 2026
%
% PURPOSE:
%   Computes the nonlinear moment-rotation relationship at the base of a steel
%   H-pile embedded in a plain-concrete bridge bent cap. Four independent
%   analysis methods are implemented and compared:
%   Method 1 – Link element, linear traction force peaking at the pile face
%   Method 2 – Link element, uniform traction force
%   Method 3 – Link element, linear traction force peaking at the idealized
%               beam section (distance L_pst from the pile face)
%   Method 4 – Rigid body rotation of the embedded pile segment
%
% THEORETICAL BACKGROUND:
%   When an applied moment rotates the embedded pile segment, the pile bears
%   against the concrete on opposite faces, forming a resisting couple. The
%   localized "pile bearing stresses" at the pile face dissipate over a length
%   L_pst (pile stress transformation length) into the "idealized bending
%   stresses" that would be predicted by ordinary frame analysis. The volume
%   of concrete between the pile face and the idealized section is called the
```

```
% Pile Stress Transfer Block (PSTB). Methods 1–3 model the PSTB as a set
% of 1-D link elements with different traction-force distributions. Method 4
% treats the embedded pile as a rigid body rotating about a neutral axis.
%
% UNITS: pounds (lb) and inches (in) throughout, unless otherwise stated.
%      Moments in lb-in; converted to kip-ft only for printed output.
%
% REFERENCE:
% Hammett, S. T. (2017). "Theoretical Moment-Rotation Curve for
% Steel Piles Embedded in Concrete." M.S. Thesis, Auburn University.
% Karthik, M. M. & Mander, J. B. (2011). "Stress-block parameters for
% unconfined and confined concrete based on a unified stress-strain model."
% ASCE J. Struct. Eng., 137(2), 270-273.

% INSTRUCTIONS:
% Change the parameters in sections 0.1 through 0.20 and then run the
% program (hit F5).

clear; clc; close all;

% Sections "0.1" through "0.20" contain only the input parameters for the
% main sections 1 through 20, respectively. If you need details about a
% specific section, go directly to to it.
%% =====
% SECTION 0.1 - INPUT PARAMETERS FOR STEEL H-PILE MATERIAL PROPERTIES
% --- Elastic properties ---
E_s = 29000000; % Steel elastic modulus (psi). Standard value for all grades
           % of structural steel; independent of yield strength.
f_y = 59000; % Yield stress (psi). ASTM A36 - used for HP10x42 because
           % this section was commonly specified in older ALDOT bridges
           % before the 13th edition AISC manual (2005) switched the
```

```

    % default to A572 Gr. 50. For HP12x53/HP14x89/HP18x204 use
    % f_y = 50,000 psi.
f_su = 80500; % Ultimate (tensile rupture) stress (psi). ASTM A36 minimum.
    % Governs the stress at  $\epsilon_{su}$  in the K&M hardening branch.
E_sh = 1160000; % Strain-hardening modulus (psi). Slope of the stress-strain
    % curve at the onset of strain hardening ( $\epsilon = \epsilon_{sh}$ ). Typically
    % 1%–4% of  $E_s$ . Here  $E_{sh}/E_s \approx 4\%$ , consistent with A36.
eps_sh = 0.014; % Strain at onset of strain hardening (in/in). End of the
    % yield plateau; after this point stress rises again toward  $f_{su}$ .
    % Typically approx. 6-10 times the yield strain ( $10 * f_y/E_s$ )
eps_su = 0.122; % Ultimate rupture strain (in/in). Defines the end of the
    % usable stress-strain range. Pile strains in this model
    % remain far below this limit.
    % Typically approx. 60-70 times the yield strain ( $60 * f_y/E_s$ )
G_s = 11000000; % Shear modulus (psi).  $G = E / [2(1+\nu)]$ ; with  $\nu = 0.30$ 
    % (Poisson's ratio for steel),  $G = 29e6/2.6 \approx 11.15 \times 10^6$  psi.
    % Used later for the Timoshenko (shear-deformable) beam
    % element stiffness matrix.

%% =====
% SECTION 0.2 - INPUT PARAMETERS FOR PILE CROSS-SECTION
% --- AISC steel manual geometry for HP Pile ---
Pile_Name = "HP10x42"; % Pile designation for plot titles. Update when changing
sections.
b_f = 10.1; % Flange width (in). Also the total pile depth for HP sections
    % (HP flanges  $\approx$  equal width and depth). For weak-axis bending
    % this is the critical dimension: extreme fiber at  $b_f/2 = 5.05$  in.
t_fw = 0.420; % Flange AND web thickness (in). For HP sections these are equal
    % by definition (near-square cross-section). Controls tributary
    % area of each fiber strip (width of material =  $t_{fw}$  per flange).
d = 9.70; % Total section depth (in). For HP sections  $d \approx b_f$ .

```

```

        % Used later as the pile bearing width against the concrete.
NoFbrsPile = 50; % Number of fibers in one half of the cross-section
        % (from flange tip to web face). Increasing this number
        % improves accuracy of the nonlinear integration but
        % has diminishing returns beyond ~30 fibers.

% --- Weak-axis section parameters (AISC manual) ---
I_y = 71.7; % Weak-axis moment of inertia (in4).
        %  $I_y = \int y^2 dA$  over the full cross-section.
S_y = 14.2; % Elastic section modulus (in3).  $S_y = I_y / (b_f/2)$ .
        % The moment  $M_{yield} = S_y * f_y$  causes first yield at the
        % extreme fiber (flange tip). This is the upper bound for
        % the applied moment range in this analysis.
Z_y = 21.8; % Plastic section modulus (in3).  $Z_y > S_y$  because full
        % plasticity develops a larger internal couple than elastic
        % first-yield. Shape factor =  $Z_y/S_y \approx 1.535$  for HP10x42
        % weak axis - fairly high compared to I-shapes in strong-axis
        % bending (~1.12), reflecting the wider, thinner flanges.
A_xs = 12.4; % Cross-sectional area (in2). Used as a check: sum(TribAreaPile)
        % should recover  $\approx A_{xs}$ .

% Discretization parameter for Extreme fiber strain sweep ( $0 \rightarrow \epsilon_{su}$ ) ---
% The outer loop in the fiber integration runs over NoPnts values of the
% extreme-fiber strain (at the flange tip,  $y = b_f/2$ ). The strain at any
% interior fiber is then proportional to its distance from the neutral axis
% (plane-sections-remain-plane hypothesis).
NoPnts = 500;

%% =====
% SECTION 0.3 - INPUT PARAMETERS FOR SHAPE FACTORS
% No input parameter.

```



```
%% =====
% SECTION 0.4 - INPUT PARAMETERS FOR CONCRETE MATERIAL PROPERTIES
f_co = 7619; % Specified compressive strength (psi). PRIMARY SCENARIO INPUT.
           % Change to 5,000 or 10,000 psi to reproduce other Hammett cases.
           % All derived concrete parameters below scale with f_co.

f_c1 = 1740; % Stress at the end of the post-peak softening branch (psi).
           % Taken as a fixed value in K&M (2011) for unconfined
           % concrete regardless of f_co; represents residual capacity
           % on the descending branch before linear crush to zero.

eps_c1 = 0.0036; % Strain at the end of the post-peak softening branch (in/in).
                % At this strain the concrete stress has fallen from f_co
                % to f_c1 along the descending branch.

%% =====
% SECTION 0.5 - INPUT PARAMETERS FOR UNREINFORCED CONCRETE CAP
% BEAM
h_cap = 36; % Cap section height (in). Standard ALDOT cap dimension.
b_cap = 36; % Cap section width (in). Standard ALDOT cap dimension.
p_e = 10; % Pile embedment depth (in). Distance from bottom of cap to
          % the top of the embedded pile segment. This is also h_RB
          % (rigid body height) used in Method 4.
NoFbrsCap = 51; % Number of fibers in the cap cross-section. MUST BE ODD so
                % that the midpoint fiber lands exactly at y = p_e.
% --- Bottom-fiber tensile strain sweep for cap moment curve ---
% The cap moment is computed at NoPnts_cap points from zero to eps_ct.
% The tension limit eps_ct is enforced: the cap cannot carry tension beyond
% the modulus of rupture (Harries & Petrou 2001 recommendation adopted by
% Hammett 2017 as the limiting condition for cap integrity).
```

```
NoPnts_cap = 21; % 21 points is sufficient because the
                  % cap curve is nearly linear up to eps_ct

%% =====
% SECTION 0.6 - INPUT PARAMETERS FOR ELEMENT STIFFNESS MATRICES
% --- Timoshenko element stiffness matrix (force method) ---
% V_ShapeFactor ( $\kappa$ ): shear correction factor for the pile cross-section.
%  $\kappa = 1.2$  is the standard value for rectangular and H-pile sections
% under shear in the plane of the flanges (weak-axis bending direction).
% It accounts for the non-uniform shear stress distribution.
V_ShapeFactor = 1.2;

%% =====
% SECTION 0.7 - INPUT PARAMETERS FOR GLOBAL STIFFNESS MATRIX
(ELEMENTS) & L_pst
% Flag controlling which element stiffness to use:
% 0  $\rightarrow$  Euler-Bernoulli (no shear deformation)
% 1  $\rightarrow$  Timoshenko (with shear deformation) - recommended for short elements
ShearDeformations = 1;

%% =====
% SECTION 0.8 - INPUT PARAMETERS FOR CONCRETE BEARING SPRINGS &
COMPLETE GLOBAL K
% No input parameters.

%% =====
% SECTION 0.9 - INPUT PARAMETERS FOR LOAD CASES, DISPLACEMENTS &
BEARING FORCES
NoPileForceEvaluationPoints = 11; % 11 data points from zero to M_yield
% (tenth points: 0, 0.1, 0.2, ..., 1.0  $\times$  M_yield)
```

```
%% =====
% SECTION 0.10 - INPUT PARAMETERS FOR CAP BEAM MOMENTS (MBeam
Routine)
CapBendingScenario = 1; % See Hammett (2017) for explanation
    % 1: equal moments both sides (represents interior pile)
    % 2: moment on left side only (Exterior pile where
    % the cantilevered cap on the right carries no
    % frame moment.)
    % 3: moment on right side only (same as 2 but
    % mirrored)
    % 4: No beam moments on either side (typical of
    % piles embedded in large concrete elements
    % (walls, mat foundations) where frame action is
    % absent.
Span = 96; % Ex: Center-to-center pile spacing (in). 8 ft × 12 in/ft = 96 in.
    % Moments are calculated over a half-span (Span/2 = 48 in)
    % assuming an inflection point at the midspan of the cap beam.

%% =====
% SECTION 0.11 - INPUT PARAMETERS FOR PSTB STRAINS & NEUTRAL AXIS
% No input parameters.

%% =====
% SECTION 0.12 - INPUT PARAMETERS FOR PSTB CAP MOMENT VERIFICATION
& TRIB AREAS
% No input parameters.

%% =====
% SECTION 0.13 - INPUT PARAMETERS FOR CAP FIBER FORCES AT PSTB
BOUNDARIES
% No input parameters.
```

```
%% =====  
% SECTION 0.14 - INPUT PARAMETERS FOR METHOD 1: ITERATIVE NLA WITH  
% LINEAR TRACTION  
%          FORCE, PEAK AT PILE FACE (NLAMethod1)  
% No input parameters.
```

```
%% =====  
% SECTION 0.15 - INPUT PARAMETERS FOR METHOD 2: ITERATIVE NLA WITH  
% UNIFORM TRACTION  
%          FORCE (NLAMethod2)  
% No input parameters.
```

```
%% =====  
% SECTION 0.16 - INPUT PARAMETERS FOR METHOD 3: ITERATIVE NLA WITH  
% LINEAR TRACTION  
%          FORCE, PEAK AT IDEALIZED BEAM SECTION (NLAMethod3)  
% No input parameters.
```

```
%% =====  
% SECTION 0.17 - INPUT PARAMETERS FOR ROTATION VECTORS, MOMENT  
% VECTORS, NEUTRAL  
%          AXIS & RIGID BODY HEIGHT (Methods 1, 2, 3)  
% No input parameters.
```

```
%% =====  
% SECTION 0.18 - INPUT PARAMETERS FOR METHOD 4: RIGID BODY BEHAVIOR  
% (AnalysisRB)  
NAPrecision = 501;      % number of trial NA locations in the search grid.  
                    % Hammett uses 501; finer grids improve accuracy  
                    % at the cost of memory and compute time.
```

```
StrainPrecision = 1500;      % number of trial  $\epsilon_{bf}$  values from 0 to  $\epsilon_{co}$ .
                             % Hammett uses 1500. The strain step is  $\epsilon_{co}/1499$ .

%% =====
% SECTION 0.19 - INPUT PARAMETERS FOR METHOD 4: ROTATION, MOMENT
VECTORS,
%           FINAL PLOT & MOMENT CAPACITY (CapacityRB)
% --- CapacityRB: pure moment capacity at  $\epsilon_{bf} = \epsilon_{co}$  ---
% Set the bottom-face strain to  $\epsilon_{co}$  (peak concrete strain) and search
% only for the NA location that balances moments. This gives the theoretical
% flexural strength of the connection (Hammett 2017, Section 3.9).
NAPrecision_Cap = 101;      % 101 trial NA locations for capacity search

%% =====
% SECTION 0.20 - INPUT PARAMETERS FOR ROTATIONAL STIFFNESS SUMMARY
% No input parameters.

%=====
=====
%=====
=====
%=====
=====
%=====
=====
%=====
=====
% MAIN PROGRAM
%% =====
% SECTION 1 - STEEL H-PILE MATERIAL PROPERTIES
%
```

```
% Nonlinear stress-strain curve for the pile steel using the
% unified model of Karthik & Mander (2011). The model is
% continuous and differentiable across four regimes:
% (1) elastic  $0 \leq \epsilon \leq \epsilon_y$ 
% (2) yield plateau  $\epsilon_y < \epsilon \leq \epsilon_{sh}$ 
% (3) strain hardening  $\epsilon_{sh} < \epsilon \leq \epsilon_{su}$ 
% (4) fracture at  $\epsilon_{su}$ 
%
% An elastic-perfectly-plastic (EPP) reference curve is also
% produced. For this problem, pile strains rarely exceed  $\epsilon_{sh}$ 
% because the limiting condition is first yield of the extreme
% fiber ( $\epsilon_{extreme} = \epsilon_y$  at the moment capacity threshold).
%% =====

% --- Karthik & Mander (2011) shape exponent P ---
% P controls the curvature of the strain-hardening branch. It is derived from
% the boundary condition that the slope of the stress-strain curve at  $\epsilon = \epsilon_{sh}$ 
% equals  $E_{sh}$ . The K&M closed-form result is:
%  $P = E_{sh} * (\epsilon_{su} - \epsilon_{sh}) / (f_{su} - f_y)$ 
% A larger P produces a flatter hardening branch (less curvature at  $\epsilon_{sh}$ ).
P = E_sh * (eps_su - eps_sh) / (f_su - f_y);

% --- Yield strain ---
% The extreme fiber of the pile cross-section first yields when its strain
% reaches  $\epsilon_y$ . This is the key threshold used throughout the analysis:
% the applied moment is swept from 0 to  $M_{yield}$  (the moment that causes
%  $\epsilon_{extreme} = \epsilon_y$ ), and the code flags results where concrete stress
% exceeds  $f_{co}$  to exclude them.
eps_y = f_y / E_s; % yield strain (in/in) =  $f_y / E_s$  (Hooke's law)

fprintf('--- Steel Material Parameters ---\n')
```

```

fprintf('P    = %.4f (K&M shape exponent)\n', P)
fprintf('eps_y = %.5f (yield strain)\n',    eps_y)

% --- Strain vector for plotting (0 → eps_su) ---
% 500 points give a smooth curve through the elastic, plateau, and hardening
% branches without excessive memory use.
es = linspace(0, eps_su, 500);

% --- Nonlinear stress (Karthik & Mander 2011) ---
% The unified equation combines two additive terms:
% Term 1: elastic-to-yield transition
%   E_s*ε / (1 + |E_s*ε/f_y|^20)^0.05
%   → at small ε: denominator ≈ 1 → linear (slope E_s)
%   → as ε → ε_y: the ^20 exponent drives a sharp but smooth transition
%       to yield, avoiding the abrupt kink of EPP
% Term 2: strain-hardening branch
%   (f_su - f_y) * [1 - |eps_su - ε|^P / (|eps_su - eps_sh|^(20P) + |eps_su - ε|^(20P))]^0.05
%   → at ε = eps_sh: Term 2 ≈ 0 (no additional stress yet)
%   → at ε = eps_su: Term 2 → (f_su - f_y) (adds full hardening increment)
% Both terms are symmetric in sign, so the function works for tension and
% compression without modification (consistent with sign convention:
% tension positive throughout the pile section).
fs_NL = E_s.*es ./ (1 + abs(E_s.*es./f_y).^20).^0.05 ...
        + (f_su - f_y) .* (1 - abs(eps_su - es).^P ...
        ./ (abs(eps_su - eps_sh).^(20*P) + abs(eps_su - es).^(20*P)).^0.05);

% --- Elastic-perfectly-plastic (EPP) reference ---
% Bilinear: slope = E_s up to f_y, then perfectly flat.
% The min() enforces the plateau without a conditional branch.
fs_EPP = min(E_s .* es, f_y * ones(size(es)));

```

```
% --- Plot: steel stress-strain comparison ---
figure
plot(es, fs_NL/1000, 'b-', 'LineWidth', 2); hold on
plot(es, fs_EPP/1000, 'r--', 'LineWidth', 1.5)
xline(eps_y, 'k:', 'LineWidth', 1, 'Label', '\epsilon_y')
xline(eps_sh, 'k:', 'LineWidth', 1, 'Label', '\epsilon_{sh}')
yline(f_y/1000, 'k--', 'LineWidth', 1, 'Label', 'f_y')
yline(f_su/1000, 'k--', 'LineWidth', 1, 'Label', 'f_{su}')
xlabel('Strain \epsilon_s (in/in)'); ylabel('Stress f_s (ksi)')
title('Steel Stress-Strain - Karthik & Mander (2011)')
legend('Nonlinear (K&M 2011)', 'Elastic-Perfectly Plastic', 'Location', 'northwest')
grid on

%% =====
% SECTION 2 - PILE CROSS-SECTION
%
% The cross-section is analyzed as a fiber section. Fibers
% are horizontal strips arranged from the flange tip (extreme
% fiber,  $y = b_f/2$ ) inward to the web face ( $y = t_{fw}/2$ ). Only
% one side (half cross-section) is discretized; contributions
% from the symmetric half are accounted for by  $\times 2$ .
%
% WEAK-AXIS BENDING GEOMETRY:
% Neutral axis: vertical through the web centerline ( $y = 0$ ).
% "y" here = horizontal distance from the neutral axis.
% Extreme fiber at  $y = b_f/2$  (tip of either flange).
% Flange width governs the lever arm; web contributes little.
%
% HP SECTIONS: Equal-thickness flanges and web ( $t_{fw}$ ).
% Unlike W-shapes, HP flanges and webs have the same nominal
```



```
% thickness, simplifying the fiber area calculation.
%% =====

% --- Uniform fiber spacing (half cross-section) ---
% Fibers span from the flange tip ( $b_f/2$ ) to the web face ( $t_{fw}/2$ ).
% The range excludes the web interior; the web area is lumped into
% the last fiber (fiber 50) to keep the fiber count manageable.
% dy = spacing between consecutive fiber centroids (in).
dy = ( $b_f/2 - t_{fw}/2$ ) / (NoFbrsPile - 1);

% --- Fiber centroid locations (distance from neutral axis, in) ---
% FbrLocPile(i) =  $y_i$ , the horizontal distance from the neutral axis (web
% centerline) to the centroid of fiber i.
% i = 1: flange tip     $y = b_f/2$     (extreme fiber, max strain)
% i = 2...49: interior flange  $y = b_f/2 - dy*(i-1)$ 
% i = 50: web face     $y = t_{fw}/2$     (innermost fiber, min strain)
FbrLocPile = zeros(NoFbrsPile, 1);
for i = 1:NoFbrsPile
    if i < NoFbrsPile
        FbrLocPile(i) =  $b_f/2 - dy*(i-1)$ ; % counts inward from flange tip
    else
        FbrLocPile(i) =  $t_{fw}/2$ ;          % web-face fiber pinned at  $t_{fw}/2$ 
    end
end

% --- Tributary areas ( $\text{in}^2$ ) ---
% Each fiber represents a horizontal strip with width  $t_{fw}$  (flange thickness)
% and height dy (fiber spacing). The  $\times 2$  accounts for both the top and bottom
% flanges, which are symmetric about the strong axis and at equal distances
% from the weak-axis neutral axis.
%
```

```
% Fiber 1 (flange tip): half-tributary strip (end fiber) × 2 flanges
% Fibers 2–49 (interior): full-tributary strip × 2 flanges
% Fiber 50 (web face): half-tributary strip × 2 flanges
%      PLUS the web area lumped at this location.
%      Web contribution per side: (t_fw/4) * d
%      (the web extends from the web centerline to the flange face;
%      half of this, t_fw/4 * d, is lumped at the innermost fiber
%      on each side; the ×2 for both sides is already embedded in
%      the formula (t_fw/2)*d/2 × 2 = (t_fw/4)*d × 2... but the
%      code writes (t_fw/4)*d and the ×2 for flanges applies to
%      the flange strip term only; the web term is written once for
%      both web halves.)
TribAreaPile = zeros(NoFbrsPile, 1);
for i = 1:NoFbrsPile
    if i == 1
        % Flange tip: half-width tributary strip, both flanges
        TribAreaPile(i) = 0.5 * dy * t_fw * 2;
    elseif i < NoFbrsPile
        % Interior flange fiber: full-width strip, both flanges
        TribAreaPile(i) = dy * t_fw * 2;
    else
        % Web-face fiber: half-width flange strip + web area lumped here
        % Web area = (depth d) × (web thickness t_fw/2 from centerline to face,
        % per half-web) × 2 half-webs = t_fw * d / 2... simplified as (t_fw/4)*d
        % after accounting for the factor-of-2 already applied to flanges.
        TribAreaPile(i) = 0.5*dy*t_fw*2 + (t_fw/4)*d;
    end
end

% --- Extreme fiber strain sweep (0 → eps_su) ---
% The outer loop in the fiber integration runs over NoPnts values of the
```

```
% extreme-fiber strain (at the flange tip,  $y = b_f/2$ ). The strain at any
% interior fiber is then proportional to its distance from the neutral axis
% (plane-sections-remain-plane hypothesis).
StrainPile = linspace(0, eps_su, NoPnts)'; % column vector, in/in

% --- Nonlinear moment vs. extreme fiber strain (fiber integration) ---
%  $M(\epsilon_{\text{extreme}}) = 2 \times \sum_j [f_s(\epsilon_j) \times A_j \times y_j]$ 
% where  $\epsilon_j = \epsilon_{\text{extreme}} \times (y_j / (b_f/2))$  [linear strain profile]
%  $f_s = \text{steel\_stress}()$  from K&M (2011)
%  $A_j = \text{TribAreaPile}(j)$  [tributary area]
%  $y_j = \text{FbrLocPile}(j)$  [moment arm]
%  $\times 2 =$  both symmetric halves of the cross-section
% Sign convention: M positive when it causes compression at the flange tip
% in the direction of the applied moment (consistent throughout).
PileMomentNL = zeros(NoPnts, 1);
for i = 1:NoPnts
    M = 0;
    for j = 1:NoFbrsPile
        % Linear strain profile:  $\epsilon_j / \epsilon_{\text{extreme}} = y_j / (b_f/2)$ 
        eps_fib = StrainPile(i) * FbrLocPile(j) / (b_f/2);
        % Stress from K&M model (handles both elastic and post-yield)
        M = M + steel_stress(eps_fib, E_s, f_y, f_su, eps_sh, eps_su, P) ...
            * TribAreaPile(j) * FbrLocPile(j);
    end
    PileMomentNL(i) = 2 * M; % factor of 2: both halves of section
end

% --- AISC reference moments ---
%  $M_{\text{yield}}$ : moment at which the extreme fiber (flange tip) first reaches  $f_y$ .
%  $= S_y * f_y$ . This is the maximum moment evaluated in the analysis.
%  $M_{\text{plastic}}$ : moment if ALL fibers are simultaneously at  $f_y$  (full plasticity).
```

```
%
    = Z_y * f_y. Not reached here because the pile is limited to M_yield.
M_yield = S_y * f_y; % (lb-in) first-yield moment, governs the load range
M_plastic = Z_y * f_y; % (lb-in) full plastic moment for reference

% --- EPP bilinear curve (3 points: origin, yield, ultimate) ---
% Used as a visual reference on the moment-strain plot. The EPP pile has
% the same initial stiffness (E_s) and the same M_yield, but then remains
% flat at Z_y*f_y instead of exhibiting the nonlinear hardening of K&M.
StrainPileLinear = [0; f_y/E_s; eps_su]; % (in/in) extreme fiber strain
PileMomLinear = [0; S_y*f_y; Z_y*f_y]; % (lb-in) corresponding moment

% --- Plot: pile moment vs. extreme fiber strain ---
figure
plot(StrainPile, PileMomentNL/12000, 'b-', 'LineWidth', 2); hold on
plot(StrainPileLinear, PileMomLinear/12000, 'r--', 'LineWidth', 1.5)
yline(M_yield/12000, 'k:', 'LineWidth', 1.2, 'Label', 'M_y', 'LabelHorizontalAlignment',
'left')
yline(M_plastic/12000, 'k:', 'LineWidth', 1.2, 'Label', 'M_p', 'LabelHorizontalAlignment',
'left')
xline(eps_y, 'm:', 'LineWidth', 1, 'Label', '\epsilon_y')
xline(eps_sh, 'm:', 'LineWidth', 1, 'Label', '\epsilon_{sh}')
xlabel('Extreme Fiber Strain \epsilon_{bot} (in/in)')
ylabel('Weak-Axis Moment M (kip-ft)')
title(Pile_Name + " - Pile Moment vs. Extreme Fiber Strain (Weak Axis)")
legend('Nonlinear (K&M 2011)', 'Elastic-Perfectly Plastic', 'Location', 'northwest')
grid on

%% =====
% SECTION 3 - SHAPE FACTORS
%
% The shape factor is Z/S, the ratio of plastic to elastic
```

```
% section modulus. Here it is computed from the fiber model
% at three reference strain levels, providing a check that
% the fiber discretization recovers the AISC tabulated values.
%
% Three shape factors are reported:
% ShapeFactor_NoSH: extreme fiber at eps_sh (onset of hardening)
%           → all fibers still at or just reaching f_y
%           → numerically close to  $Z_y * f_y / (S_y * f_y) = Z_y/S_y$ 
% ShapeFactor_WithSH: extreme fiber at eps_su (full hardening reached)
%           → fibers near the tip carry stress > f_y due to hardening
%           → shape factor exceeds  $Z_y/S_y$ 
%% =====

% --- Yield moment from nonlinear fiber integration (eps_extreme = eps_y) ---
% M at first yield = moment when the extreme fiber just reaches f_y.
% All interior fibers are still elastic. This should match  $S_y * f_y$  closely.
M_Yield_NL = 0;
for j = 1:NoFbrsPile
    eps_fib = eps_y * FbrLocPile(j) / (b_f/2); % linear strain profile at eps_y
    M_Yield_NL = M_Yield_NL + steel_stress(eps_fib, E_s, f_y, f_su, eps_sh, eps_su, P)
    ...
    * TribAreaPile(j) * FbrLocPile(j);
end
M_Yield_NL = 2 * M_Yield_NL; % both halves

% --- Plastic moment without strain hardening (eps_extreme = eps_sh) ---
% At eps_sh the yield plateau is fully developed but hardening has not yet begun.
% Every fiber along the flanges carries approximately f_y; the resulting moment
% is close to  $Z_y * f_y$ . Any deviation from  $Z_y * f_y$  reflects the transition
% smoothing in the K&M model (not a perfect EPP step).
M_Plastic_NL = 0;
```

```

for j = 1:NoFbrsPile
    eps_fib    = eps_sh * FbrLocPile(j) / (b_f/2);
    M_Plastic_NL = M_Plastic_NL + steel_stress(eps_fib, E_s, f_y, f_su, eps_sh, eps_su,
P) ...
                * TribAreaPile(j) * FbrLocPile(j);
end
M_Plastic_NL = 2 * M_Plastic_NL;

% --- Plastic moment with full strain hardening (eps_extreme = eps_su) ---
% At eps_su fibers near the extreme tip carry stress approaching f_su.
% The resulting moment exceeds Z_y * f_y and provides an upper bound on
% the pile section capacity if fracture is not the controlling limit state.
M_PlasticSH_NL = 0;
for j = 1:NoFbrsPile
    eps_fib    = eps_su * FbrLocPile(j) / (b_f/2);
    M_PlasticSH_NL = M_PlasticSH_NL + steel_stress(eps_fib, E_s, f_y, f_su, eps_sh,
eps_su, P) ...
                * TribAreaPile(j) * FbrLocPile(j);
end
M_PlasticSH_NL = 2 * M_PlasticSH_NL;

% Shape factor = M_plastic / M_yield (dimensionless)
% For a perfectly rectangular cross-section, shape factor = 1.50.
% For I-shapes in strong-axis bending, ≈ 1.10–1.15.
% For HP10x42 weak axis (wide flat flanges), ≈ 1.535, consistent with Z_y/S_y.
ShapeFactor_NoSH = M_Plastic_NL / M_Yield_NL; % no hardening
ShapeFactor_WithSH = M_PlasticSH_NL / M_Yield_NL; % with full hardening

fprintf('\n--- Shape Factors (Nonlinear Fiber Model) ---\n')
fprintf('M_Yield_NL      = %9.0f lb-in (%.1f kip-ft)\n', M_Yield_NL,  M_Yield_NL/12000)

```

```
fprintf('M_Plastic_NL          = %9.0f lb-in   (%.1f kip-ft)\n', M_Plastic_NL,
M_Plastic_NL/12000)
fprintf('M_PlasticSH_NL       = %9.0f lb-in   (%.1f kip-ft)\n', M_PlasticSH_NL,
M_PlasticSH_NL/12000)
fprintf('ShapeFactor_NoSH    = %.3f   (cf. Z_y/S_y = %.3f)\n', ShapeFactor_NoSH,
Z_y/S_y)
fprintf('ShapeFactor_WithSH = %.3f\n', ShapeFactor_WithSH)
```

```
%% =====
```

```
% SECTION 4 - CONCRETE MATERIAL PROPERTIES
```

```
%
```

```
% Karthik & Mander (2011) four-branch model for UNCONFINED
```

```
% concrete. The four branches, in order of increasing strain:
```

```
% Branch 1: tension softening ( $\epsilon < 0$ , up to  $-\epsilon_{ct}$ )
```

```
% Branch 2: pre-peak compression ( $0 \leq \epsilon \leq \epsilon_{co}$ )
```

```
% Branch 3: post-peak softening ( $\epsilon_{co} < \epsilon \leq \epsilon_{c1}$ )
```

```
% Branch 4: linear crush to zero ( $\epsilon_{c1} < \epsilon \leq \epsilon_{sp}$ )
```

```
% Beyond  $\epsilon_{sp}$ : stress = 0 (spalling/crush)
```

```
%
```

```
% Sign convention in concrete_stress(): positive  $\epsilon$  = compression.
```

```
% The PSTB concrete is unconfined (no stirrups or ties in typical
```

```
% ALDOT bent caps near the pile connection).
```

```
%
```

```
% The limiting condition in the cap analysis is:
```

```
% Compression:  $f_c \leq f_{co}$  (no post-peak allowed in Methods 1–3)
```

```
% Tension:  $f_t \leq f_{ct}$  (modulus of rupture, cracking limit)
```

```
%% =====
```

```
% --- Derived concrete parameters (Karthik & Mander 2011) ---
```

```
eps_co = 0.0015 + f_co/1e7; % Strain at peak compressive stress f_co (in/in).
```

```
    % K&M (2011) express this as a linear function of f_co.
```

```
    % For f_co = 3,000 psi: eps_co = 0.0015 + 0.0003 = 0.0018.
```

```
% For f_co = 5,000 psi: eps_co = 0.0015 + 0.0005 = 0.0020.

eps_sp = 0.012 - 0.0000007*f_co;
    % Spalling strain (in/in). Beyond this strain the cover
    % concrete is assumed to have spalled and carries zero stress.
    % For f_co = 3,000 psi: eps_sp = 0.012 - 0.0021 = 0.0099.

f_ct = 7.5 * sqrt(f_co); % Modulus of rupture (tensile cracking stress, psi).
    % ACI 318-style expression: f_ct = 7.5√f_c.
    % For f_co = 3,000 psi: f_ct ≈ 411 psi.
    % This is the tension limit enforced in the cap analysis.

eps_ct = 0.1 * eps_co; % Strain at modulus of rupture (in/in). K&M (2011) set
    % the tension cracking strain at 10% of eps_co.

E_c = 60000 * sqrt(f_co); % Initial tangent modulus of concrete (psi).
    % ACI 363 expression (more accurate than ACI 318's 57,000√f_c
    % at higher strengths). For f_co = 3,000 psi: E_c ≈ 3,286,000 psi.

n_c = E_c * eps_co / f_co; % Compressive curve shape parameter (dimensionless).
    % K&M (2011): n_c = E_c * eps_co / f_co.
    % Controls the sharpness of the pre-peak branch;
    % n_c > 1 for normal-strength concrete.

n_t = E_c * eps_ct / f_ct; % Tension softening shape parameter (dimensionless).
    % Analogous to n_c but for the tension branch.

fprintf('\n--- Concrete Properties (f_co = %g psi) ---\n', f_co)
fprintf('E_c = %.0f psi\n', E_c)
fprintf('eps_co = %.6f\n', eps_co)
fprintf('eps_c1 = %.6f\n', eps_c1)
```



```

fprintf('eps_sp = %.6f\n',    eps_sp)
fprintf('f_ct  = %.1f psi\n', f_ct)
fprintf('eps_ct = %.6f\n',    eps_ct)
fprintf('n_c   = %.4f\n',    n_c)
fprintf('n_t   = %.4f\n',    n_t)

% Strain vector: slightly into tension (-1.5*eps_ct) to spalling (eps_sp)
% The tension side is included to show the cracking branch on the plot.
ec_vec = linspace(-eps_ct*1.5, eps_sp, 600);
fc_vec = arrayfun(@(e) concrete_stress(e, f_co, f_c1, f_ct, ...
    eps_co, eps_c1, eps_sp, eps_ct, n_c, n_t), ec_vec);

% --- Plot: concrete stress-strain ---
figure
plot(ec_vec, fc_vec/1000, 'b-', 'LineWidth', 2); hold on
xline(0, 'k:', 'LineWidth', 1)
xline(eps_co, 'k:', 'LineWidth', 1, 'Label', '\epsilon_{co}')
xline(eps_c1, 'k:', 'LineWidth', 1, 'Label', '\epsilon_{c1}')
xline(eps_sp, 'k:', 'LineWidth', 1, 'Label', '\epsilon_{sp}')
yline(f_co/1000, 'k--', 'LineWidth', 1, 'Label', 'f_{co}', 'LabelHorizontalAlignment', 'left')
yline(-f_ct/1000, 'k--', 'LineWidth', 1, 'Label', '-f_{ct}', 'LabelHorizontalAlignment', 'left')
xlabel('Strain \epsilon_c (in/in)'); ylabel('Stress f_c (ksi)')
title('Concrete Stress-Strain - Karthik & Mander (2011)')
grid on

%% =====
% SECTION 5 - UNREINFORCED CONCRETE CAP BEAM
%
% Computes the moment-bottom-fiber-strain relationship for the
% UNREINFORCED cap section. This is used to translate idealized
% beam moments (from frame analysis) into fiber strains and

```

```
% neutral axis positions at the four PSTB boundary planes
% (Sections 11–13).
%
% FOr example: If the cap is 36 in × 36 in. The fiber discretization uses an
% ODD number of fibers (NoFbrsCap = 51) so that a fiber falls
% exactly on the pile-embedment depth boundary (y = p_e = 12 in)
% and on the top and bottom faces. The midpoint fiber (fiber 26)
% sits at y = p_e.
%
% The neutral axis position (YbarCap) is found iteratively for
% each value of bottom-fiber tensile strain StrainCap(i):
% given a trial ybar (distance from bottom to NA), compute the
% strain profile, integrate stresses, sum net force → if Fnet ≠ 0,
% shift ybar toward equilibrium. Repeat until Fnet < 0.001 lb.
%
% Sign convention: bottom fiber is in tension (positive strain),
% top fiber is in compression (positive stress in concrete_stress).
% Therefore the strain at fiber j is:
%   ec_j = eps_b * (FbrLocCap(j) - ybar) / ybar
% where FbrLocCap(j) is measured from the BOTTOM of the cap:
%   j=1 → y = 0 (bottom face), ec_j = -eps_b (tension, negative for concrete)
%   j=NoFbrsCap → y = h_cap (top face), ec_j > 0 (compression)
%% =====

% Index of the midpoint fiber (at the pile top / embedment boundary)
mid = (NoFbrsCap + 1) / 2; % = 26 for NoFbrsCap = 51

% Fiber spacing below and above the embedment boundary:
% Bottom half (fibers 1..mid): p_e inches divided into (mid-1) intervals
% Top half (fibers mid..end): (h_cap - p_e) inches into (mid-1) intervals
dy_bot = p_e / (0.5*(NoFbrsCap - 1)); % spacing in lower zone (in)
```

```
dy_top = (h_cap - p_e) / (0.5 * (NoFbrsCap - 1)); % spacing in upper zone (in)

% --- Fiber centroid locations measured from the bottom face of the cap (in) ---
% Fibers 1..(mid-1): evenly spaced from y=0 upward at increments of dy_bot
% Fiber mid: at y = p_e (pile top / embedment boundary)
% Fibers (mid+1)..NoFbrsCap: evenly spaced from p_e to h_cap at dy_top
FbrLocCap = zeros(NoFbrsCap, 1);
for i = 1:NoFbrsCap
    if i < mid
        FbrLocCap(i) = dy_bot * (i - 1); % lower zone: y increases upward
    else
        FbrLocCap(i) = h_cap - dy_top * (NoFbrsCap - i); % upper zone: counted from top
    end
end

% --- Tributary areas for each cap fiber (in2) ---
% Each fiber represents a horizontal strip of width b_cap and height dy_bot
% (lower zone) or dy_top (upper zone). End fibers get half-width strips.
% The midpoint fiber spans half of dy_bot below and half of dy_top above.
TribAreaCap = zeros(NoFbrsCap, 1);
for i = 1:NoFbrsCap
    if i == 1
        TribAreaCap(i) = 0.5 * dy_bot * b_cap; % bottom face (half strip)
    elseif i < mid
        TribAreaCap(i) = dy_bot * b_cap; % interior, lower zone
    elseif i == mid
        TribAreaCap(i) = (h_cap / (NoFbrsCap - 1)) * b_cap; % midpoint: average spacing
    elseif i < NoFbrsCap
        TribAreaCap(i) = dy_top * b_cap; % interior, upper zone
    else
        TribAreaCap(i) = 0.5 * dy_top * b_cap; % top face (half strip)
```

```
end
end

% --- Bottom-fiber tensile strain sweep for cap moment curve ---
% The cap moment is computed at NoPnts_cap points from zero to eps_ct.
% The tension limit eps_ct is enforced: the cap cannot carry tension beyond
% the modulus of rupture (Harries & Petrou 2001 recommendation adopted by
% Hammett 2017 as the limiting condition for cap integrity).
StrainCap = linspace(0, eps_ct, NoPnts_cap)'; % bottom-fiber tensile strain (in/in)

% --- Neutral axis search for each bottom-fiber strain ---
% YbarCap(i) = distance from the bottom face to the neutral axis (in) when
% the bottom-fiber strain is StrainCap(i). Iteration adjusts ybar by
%  $\Delta ybar = Fnet / Fgross$  (weighted average stress correction)
% This converges because Fgross (sum of |stress × area|) provides a
% normalizing denominator that scales the step size to the current stress level.
YbarCap = zeros(NoPnts_cap, 1);
for i = 1:NoPnts_cap
    eps_b = StrainCap(i); % target bottom-fiber strain (tension, in/in)
    if eps_b == 0
        % Zero strain → no curvature → NA at geometric centroid (h_cap/2)
        YbarCap(i) = h_cap / 2;
        continue
    end
    ybar = h_cap / 2; % initial guess: NA at mid-height
    Fnet = 1; % initialize to force at least one iteration
    iter = 0;
    while abs(Fnet) >= 0.001 && iter < 5000
        Fnet = 0; Fgross = 0;
        for j = 1:NoFbrsCap
            % Strain at fiber j: linear profile referenced to ybar.
```

```

% Sign: FbrLocCap(j) < ybar → tension → ec_j < 0 (in concrete convention)
% FbrLocCap(j) > ybar → compression → ec_j > 0
ec_j = eps_b * (FbrLocCap(j) - ybar) / ybar;
fc_j = concrete_stress(ec_j, f_co, f_c1, f_ct, eps_co, eps_c1, eps_sp, eps_ct, n_c,
n_t);

Fnet = Fnet + fc_j * TribAreaCap(j); % net axial force (lb); must be 0 at NA
Fgross = Fgross + abs(fc_j) * TribAreaCap(j); % |force| for step normalization
end
% Shift ybar proportional to force imbalance
if Fgross > 0; ybar = ybar + Fnet/Fgross; end
iter = iter + 1;
end
YbarCap(i) = ybar; % converged NA position (in from bottom)
end

% --- Cap moment at each bottom-fiber strain (fiber integration) ---
%  $M = \sum_j [f_c(\epsilon_j) \times A_j \times (y_j - ybar)]$ 
% Moment arm = (FbrLocCap(j) - YbarCap(i)): distance from NA to fiber j.
% Compression fibers (above NA) add positive moment; tension fibers (below)
% add negative contribution. Net M is the restoring moment resisting cap bending.
CapMomNL = zeros(NoPnts_cap, 1);
for i = 1:NoPnts_cap
    M = 0;
    for j = 1:NoFbrsCap
        ec_j = StrainCap(i) * (FbrLocCap(j) - YbarCap(i)) / YbarCap(i);
        fc_j = concrete_stress(ec_j, f_co, f_c1, f_ct, eps_co, eps_c1, eps_sp, eps_ct, n_c,
n_t);
        M = M + fc_j * TribAreaCap(j) * (FbrLocCap(j) - YbarCap(i));
    end
    CapMomNL(i) = M; % (lb-in)
end
end

```

```
% --- Linear (elastic) reference cracking moment ---
% For a homogeneous, uncracked rectangular section under pure bending:
%  $M_{cr} = (b \cdot h^2 / 6) \cdot f_{ct}$  (elastic section modulus  $\times$  modulus of rupture)
% This is the moment at which the bottom fiber just reaches  $f_{ct}$  in the
% linear-elastic model, for comparison with the nonlinear result.
StrainCapLinear = [0; eps_ct]; % (in/in)
CapMomLinear = [0; b_cap * h_cap^2 / 6 * f_ct]; % (lb-in)

% Ratio of nonlinear cap moment to linear cracking moment at eps_ct.
% Should be close to 1.0; deviations reflect the nonlinearity in concrete_stress.
CapCapacityRatio = CapMomNL(end) / CapMomLinear(2);

fprintf('\n--- Cap Beam Moment Capacity ---\n')
fprintf('Linear cracking moment = %.0f lb-in (%.1f kip-ft)\n', CapMomLinear(2),
CapMomLinear(2)/12000)
fprintf('Nonlinear cap moment = %.0f lb-in (%.1f kip-ft)\n', CapMomNL(end),
CapMomNL(end)/12000)
fprintf('CapCapacityRatio = %.4f\n', CapCapacityRatio)

% --- Plot: cap moment vs. bottom fiber strain ---
figure
plot(StrainCap, CapMomNL, 'r-', 'LineWidth', 2); hold on
plot(StrainCapLinear, CapMomLinear, 'b--', 'LineWidth', 1.5)
xlabel('Bottom Fiber Strain (in/in)'); ylabel('Unreinforced Cap Moment (lb-in)')
title('Unreinforced Cap Moment vs. Extreme Bottom (Tension) Fiber Strain')
legend('Nonlinear', 'Linear Stress-Strain Curve', 'Location', 'northwest')
grid on

%% =====
% SECTION 6 - ELEMENT STIFFNESS MATRICES
```

```
%
% The embedded pile segment is modelled as a series of
% Timoshenko (shear-deformable) beam elements. Each element
% has 4 DOFs: [v_i, θ_i, v_j, θ_j] (transverse displacement
% and rotation at the near and far nodes). Two formulations
% are provided and cross-checked:
%
% k_ElementNoShear: Euler-Bernoulli (classical) beam element.
% Neglects shear deformation; valid when L/d >> 1.
%
% k_ElementWithShear: Timoshenko beam element derived via the
% force method (flexibility matrix inversion). Accounts for
% shear deformation through the shear area A_v.
%
% For short embedment lengths (L_PE is small), shear deformation
% is non-negligible and the Timoshenko form should be used.
%
% beta_s = shear-to-bending flexibility ratio. When beta_s << 1,
% Timoshenko → Euler-Bernoulli.
%% =====

% Number of pile elements in the embedment zone:
% The pile is discretized using the same fiber layout as the cap.
% The cap has NoFbrsCap = 51 fibers → 50 inter-fiber intervals.
% Only the LOWER half (embedment zone, 0 to p_e) is modelled: 25 elements.
NoPileElements = (NoFbrsCap - 1) / 2; % = 25 elements

% Total degrees of freedom:
% Each element contributes 2 new DOFs at its far node (v + θ).
% The base node (bottom of cap) has 2 DOFs (node 1).
% Plus 2 DOFs at the top-of-pile node (node NoPileElements+1).
```

```
% Total: 2*(NoPileElements+1) = 2*26 = 52 DOFs.
DOFS = 2 * NoPileElements + 2; % = 52

% Length of each pile element (in):
% p_e (12 in) divided equally among (NoFbrsCap-1)/2 = 25 elements.
L_PE = p_e / (0.5 * (NoFbrsCap - 1)); % = 12/25 = 0.48 in per element

fprintf('\n--- Stiffness Model Geometry ---\n')
fprintf('NoPileElements = %d, DOFS = %d, L_PE = %.4f in\n', NoPileElements, DOFS,
L_PE)

% Bending stiffness of the pile (lb-in2):
% EI = E_s * I_y, where I_y is the weak-axis moment of inertia.
EI = E_s * I_y;

% --- Euler-Bernoulli element stiffness matrix (4×4) ---
% Standard beam stiffness matrix; rows/cols ordered [v_i, θ_i, v_j, θ_j].
k_ElementNoShear = EI * ...
    [ 12/L_PE^3, 6/L_PE^2, -12/L_PE^3, 6/L_PE^2;
      6/L_PE^2, 4/L_PE, -6/L_PE^2, 2/L_PE;
     -12/L_PE^3, -6/L_PE^2, 12/L_PE^3, -6/L_PE^2;
      6/L_PE^2, 2/L_PE, -6/L_PE^2, 4/L_PE ];

% --- Timoshenko element stiffness matrix (force method) ---
% V_ShapeFactor (κ): shear correction factor for the pile cross-section.
% κ = 1.2 is the standard value for rectangular and H-pile sections
% under shear in the plane of the flanges (weak-axis bending direction).
% It accounts for the non-uniform shear stress distribution.
% V_ShapeFactor defined in the input section 0.6

% Effective shear area A_v (in2):
```



```
% For weak-axis bending of an H-pile, shear is resisted by the two flanges.
% Nominal shear area = 2 * b_f * t_fw.
% Effective (reduced) shear area = (2 * b_f * t_fw) / κ.
A_v = (2 * b_f * t_fw) / V_ShapeFactor;

% Flexibility matrix d for a fixed-free beam element (force method):
% Applied forces: [V, M] at the free end (far node j).
% Resulting deformations: [v_j, θ_j] at the free end.
% d(1,1) = L³/(3EI) + L/(G*A_v) → shear + bending contribution to tip deflection
% d(1,2) = d(2,1) = L²/(2EI) → cross-term (moment → tip deflection / shear → rotation)
% d(2,2) = L/EI → pure bending contribution to tip rotation
d_ReducedShear = [ L_PE^3/(3*EI) + L_PE/(G_s*A_v), L_PE^2/(2*EI);
                  L_PE^2/(2*EI), L_PE/EI ];

% Reduced stiffness matrix (2×2) at the free end: k_reduced = inv(d)
k_ReducedShear = inv(d_ReducedShear);

% Transformation matrix 'a' relates the 4-DOF element vector [v_i,θ_i,v_j,θ_j]
% to the 2-DOF free-end deformation vector [v_j - v_i - θ_i*L, θ_j - θ_i]
% (relative transverse displacement and relative rotation at the far node).
a = [-1, -L_PE, 1, 0; % relative transverse displacement at j
     0, -1, 0, 1]; % relative rotation at j

% Full 4×4 Timoshenko element stiffness matrix via congruent transformation:
% k_element = a' * k_reduced * a
k_ElementWithShear = a' * k_ReducedShear * a;

% --- Cross-check: closed-form Timoshenko stiffness matrix ---
% This is the well-known explicit form with shear correction factor beta_s.
% beta_s = 12*EI / (G*A_shear*L²)
% When beta_s = 0 (rigid in shear), this collapses to k_ElementNoShear.
```

```

beta_s = (12 * EI * V_ShapeFactor) / (G_s * (2*b_f*t_fw) * L_PE^2);
k_Check = EI / (L_PE^3 * (1 + beta_s)) * ...
    [ 12,          6*L_PE,          -12,          6*L_PE;
      6*L_PE, L_PE^2*(4+beta_s), -6*L_PE, L_PE^2*(2-beta_s);
     -12,          -6*L_PE,          12,          -6*L_PE;
      6*L_PE, L_PE^2*(2-beta_s), -6*L_PE, L_PE^2*(4+beta_s)];

% Maximum entry-wise absolute difference between the two Timoshenko formulations.
% Should be at machine precision (~1e-10) for correct derivation.
residual = max(max(abs(k_ElementWithShear - k_Check))) / max(max(abs(k_Check)));
fprintf('\n--- Stiffness Matrix Verification: residual = %.2e ---\n', residual)
if residual > 1e-8
    error('Stiffness matrix mismatch - check k_ElementWithShear formulation.')
else
    fprintf('CHECK PASSED.\n')
end

%% =====
% SECTION 7 - GLOBAL STIFFNESS MATRIX (PILE ELEMENTS) & L_pst
%
% Assembles the global stiffness matrix for the embedded pile
% segment alone (without concrete bearing springs). Standard
% direct stiffness assembly: each element's 4x4 k_elem is
% added to the 4 global DOFs corresponding to that element's
% two nodes.
%
% Nodes are numbered from the bottom of the cap (node 1, DOFs 1-2)
% to the top of the pile (node 26, DOFs 51-52).
%
% L_pst: pile stress transformation length (in). This is the
% distance from the pile face to the point where the PSTB

```

```
% 45°-planes intersect the sides of the cap beam. For a cap
% wider than the pile depth d, the 45° planes spread outward
% at half the width difference per side:
% L_pst = (b_cap - d) / 2
% This is the length of every link element (spring) in the model.
%% =====

% Flag controlling which element stiffness to use:
% 0 → Euler-Bernoulli (no shear deformation)
% 1 → Timoshenko (with shear deformation) - recommended for short elements
% ShearDeformations parameter defined in section 0.7
if ShearDeformations == 0
    k_elem = k_ElementNoShear;
else
    k_elem = k_ElementWithShear;
end

% Initialize the global element stiffness matrix (DOFS × DOFS)
K_GEO = zeros(DOFS, DOFS);

% Assembly loop: element h connects node h (DOFs h, h+1) to node h+2 (DOFs h+2,
h+3).
% The odd index step (1, 3, 5, ...) aligns with DOF pairs: node 1 → DOFs 1-2,
% node 2 → DOFs 3-4, etc. Each iteration adds the 4×4 k_elem to the appropriate
% 4×4 sub-block of K_GEO.
for h = 1:2:(DOFS - 3)
    K_GEO(h:h+3, h:h+3) = K_GEO(h:h+3, h:h+3) + k_elem;
end

K_GlobalElementsOnly = K_GEO; % saved for later (springs added separately)
```

```
% Pile stress transformation length:
% The PSTB extends outward from the pile face at 45° angles. For a cap of
% width b_cap = 36 in and a pile of depth d = 9.7 in:
% L_pst = (36 - 9.7)/2 = 13.15 in
% This is the length of each link element (spring) connecting a pile face
% node to the corresponding idealized beam section node.
L_pst = 0.5 * (b_cap - d);

fprintf('\nK_GlobalElementsOnly assembled (%d x %d), L_pst = %.4f in\n', DOFS, DOFS,
L_pst)

%% =====
% SECTION 8 - CONCRETE BEARING SPRINGS & COMPLETE GLOBAL K
%
% The concrete bearing resistance is modelled as transverse
% (compression-only) springs at each node of the embedded pile.
% Each spring represents a link element of length L_pst with:
% Area = d × tributary length at that node (d × L_PE or d × L_PE/2)
% Modulus = E_c (initial elastic; updated to tangent modulus each iteration)
% Stiffness k_spring = (A × E_c) / L_pst (axial spring formula)
%
% Sign convention: positive spring force = pile pushes into concrete
% (compression in the concrete = positive in concrete_stress convention).
%
% End nodes (bottom of cap = node 1, top of pile = node 26) receive
% half the tributary length (L_PE/2) compared to interior nodes (L_PE),
% consistent with the trapezoidal integration rule.
%
% Note: springs act only in the translational DOFs (odd-numbered DOFs).
% Rotational DOFs (even-numbered) have no spring support - the concrete
% is assumed to provide no moment resistance at the element nodes.
```

```
%
% K_GlobalComplete = K_GlobalElementsOnly + K_GlobalSpringsOnly
% This is the stiffness matrix used in the first iteration of Methods 1–3.
% Subsequent iterations replace K_GlobalSpringsOnly with updated spring
% stiffnesses based on the tangent modulus at the spring midpoint.
%% =====

K_GSO = zeros(DOFS, DOFS); % Global spring stiffness matrix (initially all zeros)

for h = 1:2:(DOFS - 1)
    % h = odd DOF index = translational DOF at node (h+1)/2
    if h == 1 || h == DOFS - 1
        % End nodes: tributary length = L_PE/2 (half strip)
        K_GSO(h,h) = d * (L_PE/2) * E_c / L_pst;
    else
        % Interior nodes: tributary length = L_PE (full strip)
        K_GSO(h,h) = d * L_PE * E_c / L_pst;
    end
end

K_GlobalSpringsOnly = K_GSO; % saved as initial (elastic) spring matrix

% Column vector of initial spring stiffnesses for each translational DOF.
% Used in the reaction-force comparison within the NLA iteration loops.
K_GlobalSpringsTransOnly = zeros(DOFS/2, 1);
for h = 1:DOFS/2
    K_GlobalSpringsTransOnly(h) = K_GlobalSpringsOnly(2*h-1, 2*h-1);
end

% Initial complete global stiffness matrix (elastic springs, no iteration yet)
K_GlobalComplete = K_GlobalElementsOnly + K_GlobalSpringsOnly;
```

```
fprintf('Springs: end k=%.2f lb/in, interior k=%.2f lb/in\n', K_GSO(1,1), K_GSO(3,3))

%% =====
% SECTION 9 - LOAD CASES, DISPLACEMENTS & BEARING FORCES
%
% Applied loads correspond to the pile moment and shear at the
% bottom face of the cap. The applied moment ranges from zero
% to M_yield_NL (the nonlinear yield moment from the fiber model),
% and the corresponding shear is derived from portal frame analysis.
%
% Portal frame assumption (Hammett 2017, Section 3.4):
% Bent: 20 ft tall, 4 vertical piles at 8 ft spacing, pinned at base.
% Inflection points at beam midpoints. Equal shear in each pile.
% Shear at pile top = V, moment at pile base = V × half-column height.
% Inverting: M_base = V × (10 ft × 12 in/ft) → V = M / 120 in
%
% The load vector F_Pile has size DOFS × NoPileForceEvaluationPoints:
% F_Pile(1, j) = V_j (shear, lb) → applied at DOF 1 (bottom node, translation)
% F_Pile(2, j) = M_j (moment, lb-in) → applied at DOF 2 (bottom node, rotation)
% F_Pile(3..DOFS, j) = 0 → no external load on interior DOFs
%
% The negative sign on MBaseCapMax reflects sign convention:
% The pile pushes to the right at the bottom (positive shear),
% which induces a clockwise (negative) moment at the base.
%
% Linear displacement solution:
% Delta_Global = K_GlobalComplete \ F_Pile
% This gives the elastic (first-iteration) displacement vector.
% Methods 1–3 iterate from here, updating spring stiffnesses.
%
```

```
% Bearing forces at each node:
% P_PileFace = K_spring × Delta_trans
% Positive = pile pushes into concrete (compression side).
%% =====

% NoPileForceEvaluationPoints defined in section 0.9

% Shear range at bottom face of cap (lb):
% Portal analysis:  $V = M / (\text{column height} / 2) = M / (10 \times 12 / 2) = M / 60$ .
% But Hammett uses 4 piles sharing the shear →  $V_{\text{per pile}} = M / (4 \times 30 \text{ in})...$ 
% In the code:  $V_{\text{BaseCapMax}} = 4 * M_{\text{Yield\_NL}} / (10 * 12)$ . This represents the total
% lateral force divided by 4 piles, with the column height = 10 ft × 12 in/ft.
VBaseCapMin = 0;
VBaseCapMax = 4 * M_Yield_NL / (10 * 12); % max shear at yield (lb)

% Moment range at bottom face of cap (lb-in):
% Negative because the moment is clockwise (hogging) at the pile base
% for a rightward lateral load in the portal frame model.
MBaseCapMin = 0;
MBaseCapMax = -M_Yield_NL; % at max load:  $M = -M_{\text{yield\_NL}}$  (lb-in)

% Assemble load matrix: columns are load cases j=1..11
F_Pile = zeros(DOFS, NoPileForceEvaluationPoints);
for j = 1:NoPileForceEvaluationPoints
    frac = (j - 1) / (NoPileForceEvaluationPoints - 1); % 0, 0.1, 0.2, ..., 1.0
    F_Pile(1, j) = (VBaseCapMax - VBaseCapMin) * frac; % shear (lb)
    F_Pile(2, j) = (MBaseCapMax - MBaseCapMin) * frac; % moment (lb-in)
end

% Elastic displacement solution (used as reference; iterated in Methods 1–3)
Delta_Global = K_GlobalComplete \ F_Pile; % (DOFS × NoPileForceEvaluationPoints)
```

```
% Extract only the translational DOFs (1, 3, 5, ... → rows 1:2:DOFS)
Delta_GlobalTransOnly = Delta_Global(1:2:end, :);

% Initial bearing force at each translational DOF (lb):
% P = k_spring × displacement. Positive = compression.
P_PileFace = K_GlobalSpringsTransOnly .* Delta_GlobalTransOnly;

fprintf('\nLoad cases: V_max=%.2f lb, M_max=%.0f lb-in\n', VBaseCapMax,
MBaseCapMax)

%% =====
% SECTION 10 - CAP BEAM MOMENTS (MBeam Routine)
%
% Computes the idealized beam moments at the four PSTB boundary
% sections (two per side of the pile: at the pile face and at
% the end of L_pst). These moments are used to determine the
% idealized bending stresses that the concrete cap carries at
% those sections. The stresses are then used as boundary
% conditions for the link element reactions in Methods 1–3.
%
% Sign convention for MBeam:
% Positive = tension on the bottom fiber of the cap (sagging).
% Negative = tension on the top fiber (hogging).
% All beam moments here are sagging (positive) under the assumed
% lateral load direction.
%
% The four CapBendingScenarios reflect different pile positions:
% Case 1: equal moments on both sides (interior pile, symmetric)
% Case 2: moment on left side only (exterior pile, right cap end free)
% Case 3: moment on right side only (exterior pile, left cap end free)
```



```
% Case 4: no beam moments (isolated pile in large concrete element)
%
% MTotal at the connection =  $V \times h_{\text{cap}} + |M_{\text{base}}|$ 
% (shear  $\times$  cap height + applied base moment = total cap moment)
%
% The moments at each of the four boundary sections are computed
% by linear interpolation along a simply-supported span of half
% the center-to-center spacing (Span/2 = 48 in):
%  $M_{\text{at}_x} = M_{\text{Left}} \times (\text{Span}/2 - x) / (\text{Span}/2)$ 
% where  $x$  = distance from the left support (pile face) to the section.
%% =====

% CapBendingScenario defined in section 0.10
% Span defined in section 0.10

% M_Beam(1,j): left side, at the left PILE FACE ( $x = b_f/2$  from pile centerline)
% M_Beam(2,j): left side, at the left IDEALIZED SECTION ( $x = b_f/2 + L_{\text{pst}}$ )
% M_Beam(3,j): right side, at the right PILE FACE
% M_Beam(4,j): right side, at the right IDEALIZED SECTION
M_Beam = zeros(4, NoPileForceEvaluationPoints);
for j = 1:NoPileForceEvaluationPoints
    % Total moment demand at the connection for load case j:
    % MCapTotal =  $V \times h_{\text{cap}} + |M_{\text{base}}|$ 
    % F_Pile(2,j) is negative (hogging), so -F_Pile(2,j) recovers the magnitude.
    MCapTotal = F_Pile(1,j) *  $h_{\text{cap}}$  - F_Pile(2,j);

    % Distribute MCapTotal to the left and right cap segments
    if CapBendingScenario == 1; MLeft = (1/2)*MCapTotal; % symmetric split
    elseif CapBendingScenario == 2; MLeft = MCapTotal; % all to the left
    else; MLeft = 0; % none to the left
end
```

```

% Linear interpolation along the left half-span:
% At x = b_f/2 (pile face): M = MLeft * (Span/2 - b_f/2) / (Span/2)
% At x = b_f/2 + L_pst (idealized section): M further reduced
M_Beam(1,j) = MLeft * (Span/2 - b_f/2) / (Span/2); % left pile face
M_Beam(2,j) = MLeft * (Span/2 - b_f/2 - L_pst) / (Span/2); % left idealized section

if CapBendingScenario == 1; MRight = MCapTotal - MLeft; % symmetric
elseif CapBendingScenario == 3; MRight = MCapTotal; % all to the right
else; MRight = 0;
end

M_Beam(3,j) = MRight * (Span/2 - b_f/2) / (Span/2); % right pile face
M_Beam(4,j) = MRight * (Span/2 - b_f/2 - L_pst) / (Span/2); % right idealized section
end

fprintf('\nMBeam Scenario %d: M1_max=%.0f, M2_max=%.0f lb-in\n', ...
        CapBendingScenario, M_Beam(1,end), M_Beam(2,end))

%% =====
% SECTION 11 - PSTB STRAINS & NEUTRAL AXIS
%
% Converts the four sets of beam moments (M_Beam rows 1–4) into
% corresponding bottom-fiber tensile strains (PSTBStrains) and
% neutral axis positions (PSTBYbarCap) using the nonlinear cap
% moment-strain curve computed in Section 5.
%
% The conversion is a 1-D table lookup with linear interpolation:
% Given M_Beam(i,j), find eps_b such that CapMomNL(eps_b) = M_Beam(i,j).
% Then re-run the NA iteration to find YbarCap at that eps_b.
%
```

```
% These PSTB strains and NA positions define the idealized bending
% stress distribution at each PSTB boundary, which in turn sets the
% required link element reactions (cap fiber forces) used in the
% iterative NLA of Methods 1–3.
%
% A warning is issued if any beam moment exceeds the cap cracking
% capacity ( $M > \text{CapMomNL}(\text{end})$ , i.e.,  $\text{eps}_b > \text{eps}_{ct}$ ). In that case,
% the cap section is insufficient and  $h_{\text{cap}}$  must be increased.
%% =====

% PSTBStrains(i,j) = bottom-fiber tensile strain (in/in) in the cap at
% boundary section i, load case j. Rows: 1=left pile face, 2=left idealized,
% 3=right pile face, 4=right idealized.
PSTBStrains = zeros(4, NoPileForceEvaluationPoints);
for i = 1:4
    for j = 1:NoPileForceEvaluationPoints
        eps_ij = 0;
        % Walk through CapMomNL table until the bracketing interval is found,
        % then linearly interpolate to get the strain corresponding to M_Beam(i,j).
        for k = 1:NoPnts_cap - 1
            if CapMomNL(k) <= M_Beam(i,j)
                eps_ij = StrainCap(k) + ...
                    (StrainCap(k+1) - StrainCap(k)) / ...
                    (CapMomNL(k+1) - CapMomNL(k)) * ...
                    (M_Beam(i,j) - CapMomNL(k));
            end
        end
        PSTBStrains(i,j) = eps_ij;
    end
end
```

```
% For each PSTB boundary, find the neutral axis position corresponding
% to PSTBStrains(i,j) using the same iterative NA search as Section 5.
PSTBYbarCap = zeros(4, NoPileForceEvaluationPoints);
cap_exceeded = false;
for i = 1:4
    for j = 1:NoPileForceEvaluationPoints
        eps_b = PSTBStrains(i,j);
        if eps_b > eps_ct
            warning('Tension strain exceeded at [%d,%d]: %.6f > %.6f. Increase cap.', ...
                i, j, eps_b, eps_ct)
            cap_exceeded = true;
        end
        if eps_b == 0
            PSTBYbarCap(i,j) = h_cap / 2; % no curvature → NA at mid-height
            continue
        end
        ybar = h_cap / 2;
        Fnet = 1;
        iter = 0;
        while abs(Fnet) >= 0.001 && iter < 5000
            Fnet = 0; Fgross = 0;
            for k = 1:NoFbrsCap
                ec_k = eps_b * (FbrLocCap(k) - ybar) / ybar;
                fc_k = concrete_stress(ec_k, f_co, f_c1, f_ct, eps_co, eps_c1, eps_sp, eps_ct,
n_c, n_t);
                Fnet = Fnet + fc_k * TribAreaCap(k);
                Fgross = Fgross + abs(fc_k) * TribAreaCap(k);
            end
            if Fgross > 0; ybar = ybar + Fnet/Fgross; end
            iter = iter + 1;
        end
    end
end
```

```

        PSTBYbarCap(i,j) = ybar;
    end
end
if ~cap_exceeded; fprintf('\nCap section check PASSED.\n'); end

%% =====
% SECTION 12 - PSTB CAP MOMENT VERIFICATION & TRIB AREAS
%
% Verification: re-integrates stresses at each PSTB boundary
% using the converged NA position (PSTBYbarCap) and strain
% (PSTBStrains) to recover the moment PSTBCapMomNL. Compares
% against M_Beam to confirm the table lookup + NA iteration
% converged correctly.
%
% Acceptance criteria (consistent with Hammett Appendix C):
% < 1% of M_Yield_NL → CHECK PASSED
% 1–5% → WARNING (increase NoPnts_cap)
% > 5% → ERROR (must increase NoPnts_cap before proceeding)
%
% TribAreaPSTB: tributary areas of the PSTB bearing face (in2).
% The bearing face is the pile face against the concrete: width = d
% (pile depth), discretized into N_PSTB fibers over the embedment
% depth p_e. These areas are used to convert nodal spring forces
% (lb) to bearing stresses (psi) in the NLA iteration loops.
%% =====

% Re-integrate moment at each PSTB boundary to verify accuracy
PSTBCapMomNL = zeros(4, NoPileForceEvaluationPoints);
for i = 1:4
    for j = 1:NoPileForceEvaluationPoints
        YBC = PSTBYbarCap(i,j); % converged NA position (in from bottom)
    end
end

```

```

M = 0;
for k = 1:NoFbrsCap
    ec_k = PSTBStrains(i,j) * (FbrLocCap(k) - YBC) / YBC;
    fc_k = concrete_stress(ec_k, f_co, f_c1, f_ct, ...
        eps_co, eps_c1, eps_sp, eps_ct, n_c, n_t);
    M = M + fc_k * TribAreaCap(k) * (FbrLocCap(k) - YBC);
end
PSTBCapMomNL(i,j) = M;
end
end

% Residual: difference between target beam moment and recovered moment
CheckCapMomNL = M_Beam - PSTBCapMomNL; % should be ≈ 0 everywhere
max_abs_error = max(abs(CheckCapMomNL(:)));
threshold_warn = 0.01 * M_Yield_NL; % 1% tolerance
threshold_reject = 0.05 * M_Yield_NL; % 5% hard limit

fprintf('\n--- PSTBCapMomNL Accuracy Check ---\n')
fprintf(' Max absolute residual = %.1f lb-in\n', max_abs_error)
fprintf(' M_Yield_NL = %.0f lb-in\n', M_Yield_NL)
fprintf(' Relative error = %.3f%%\n', 100*max_abs_error/M_Yield_NL)
if max_abs_error > threshold_reject
    error(['CheckCapMomNL: moment recovery error (%.1f lb-in = %.2f%% of\n',
        M_Yield_NL) ' ...
        'exceeds 5%% threshold. Increase NoPnts_cap in Section 5 and re-run.'], ...
        max_abs_error, 100*max_abs_error/M_Yield_NL)
elseif max_abs_error > threshold_warn
    warning(['CheckCapMomNL: moment recovery error (%.1f lb-in = %.2f%% of\n',
        M_Yield_NL) ' ...
        'exceeds 1%% threshold. Consider increasing NoPnts_cap in Section 5.'], ...
        max_abs_error, 100*max_abs_error/M_Yield_NL)

```

```

else
    fprintf(' CHECK PASSED: error within 1%% of M_Yield_NL.\n')
end

% --- Tributary areas of the PSTB pile bearing face ---
% N_PSTB fibers over the embedment depth p_e, each with width d (pile depth).
% Used to convert nodal spring forces (lb) to stresses (psi) at each node.
% N_PSTB = (NoFbrsCap+1)/2 = 26 → same count as pile nodes (NoPileElements+1).
N_PSTB = (NoFbrsCap + 1) / 2; % number of fibers on bearing face
TribAreaPSTB = zeros(N_PSTB, 1);
for i = 1:N_PSTB
    if i == 1 || i == N_PSTB
        TribAreaPSTB(i) = 0.5 * dy_bot * d; % end fibers: half strip
    else
        TribAreaPSTB(i) = dy_bot * d; % interior fibers: full strip
    end
end

fprintf('\n--- TribAreaPSTB (pile bearing face) ---\n')
fprintf(' N_PSTB = %d fibers (embedment zone, y=0 to y=p_e)\n', N_PSTB)
fprintf(' End fiber area = %.4f in^2 (i=1 and i=%d)\n', TribAreaPSTB(1), N_PSTB)
fprintf(' Interior area = %.4f in^2 (i=2 to i=%d)\n', TribAreaPSTB(2), N_PSTB-1)
fprintf(' Total PSTB area = %.4f in^2 (should ≈ p_e × d = %.4f)\n', ...
    sum(TribAreaPSTB), p_e*d)

%% =====
% SECTION 13 - CAP FIBER FORCES AT PSTB BOUNDARIES
%
% Converts the idealized bending stress distribution at each
% PSTB boundary into equivalent nodal forces (lb) on the link
% elements. These nodal forces are the "required link element

```

```
% reactions" that Methods 1–3 iterate toward.
%
% Physical meaning:
% The cap beam carries moment M_Beam(i,j) at the PSTB boundary.
% This produces a stress distribution fc(y) across the section.
% The nodal force at pile node k = fc(y_k) × TribAreaPSTB(k).
% This is the force that the surrounding cap concrete must supply
% to the link element at that node to satisfy equilibrium.
%
% PCapFbrLeft(i,j): force at node i from the LEFT PSTB boundary (row 2 of
PSTBStrains).
% The left cap bends with tension at the bottom → compression in the pile
% bearing zone at the TOP of the embedment, tension at the BOTTOM.
% Sign: -fc_ij × Area → negative for compressive fibers (concrete pushes pile).
% Strain at pile node i on the LEFT side:
%      ec = PSTBStrains(2,j) * (h_cap - PSTBYbarCap(2,j) - FbrLocCap(i)) /
PSTBYbarCap(2,j)
% The term (h_cap - PSTBYbarCap - FbrLocCap(i)) gives the distance from the
% LEFT idealized section NA to fiber i, measured downward from the top.
%
% PCapFbrRight(i,j): force at node i from the RIGHT PSTB boundary (row 4).
% Right cap bends with tension at the bottom (sagging). The concrete
% pushes down (positive) at the bottom of the pile and pulls up at the top.
% Strain:
%      ec = PSTBStrains(4,j) * (FbrLocCap(i) - PSTBYbarCap(4,j)) / PSTBYbarCap(4,j)
% Sign: +fc_ij × Area → positive for compressive fibers.
%% =====

% Left PSTB boundary forces (node i, load case j)
PCapFbrLeft = zeros(DOFS/2, NoPileForceEvaluationPoints);
for i = 1:DOFS/2
```



```

for j = 1:NoPileForceEvaluationPoints
    % Strain at pile node i at the LEFT idealized beam section (M_Beam row 2):
    % The left cap sags; bottom is in tension. The pile face is on the RIGHT
    % side of the left cap segment, so the hogging convention applies here.
    ec_ij = PSTBStrains(2,j) * ...
        (h_cap - PSTBYbarCap(2,j) - FbrLocCap(i)) / PSTBYbarCap(2,j);
    fc_ij = concrete_stress(ec_ij, f_co, f_c1, f_ct, ...
        eps_co, eps_c1, eps_sp, eps_ct, n_c, n_t);
    % Negative sign: the cap concrete pushes (compresses) the pile bearing face.
    % Compressive fiber force = reaction force acting on pile = -fc × A.
    PCapFbrLeft(i,j) = -fc_ij * TribAreaPSTB(i);
end
end

fprintf('\n--- PCapFbrLeft (max load case) ---\n')
fprintf(' Node 1 (bottom of cap): %.4f lb\n', PCapFbrLeft(1, end))
fprintf(' Node 2 (interior): %.4f lb\n', PCapFbrLeft(2, end))
fprintf(' Node %d (top of pile): %.4f lb\n', DOFS/2, PCapFbrLeft(DOFS/2, end))

% Right PSTB boundary forces (node i, load case j)
PCapFbrRight = zeros(DOFS/2, NoPileForceEvaluationPoints);
for i = 1:DOFS/2
    for j = 1:NoPileForceEvaluationPoints
        % Strain at pile node i at the RIGHT idealized beam section (M_Beam row 4):
        % The right cap also sags; FbrLocCap(i) measured from bottom of cap.
        % Positive strain (compression) above NA, negative (tension) below NA.
        ec_ij = PSTBStrains(4,j) * ...
            (FbrLocCap(i) - PSTBYbarCap(4,j)) / PSTBYbarCap(4,j);
        fc_ij = concrete_stress(ec_ij, f_co, f_c1, f_ct, ...
            eps_co, eps_c1, eps_sp, eps_ct, n_c, n_t);
        % Positive sign: compressive fiber force contributes positively to pile reaction.
    end
end

```

```

    PCapFbrRight(i,j) = fc_ij * TribAreaPSTB(i);
end
end

fprintf('\n--- PCapFbrRight (max load case) ---\n')
fprintf(' Node 1 (bottom of cap): %.4f lb\n', PCapFbrRight(1, end))
fprintf(' Node 2 (interior):    %.4f lb\n', PCapFbrRight(2, end))
fprintf(' Node %d (top of pile): %.4f lb\n', DOFS/2, PCapFbrRight(DOFS/2, end))

%% =====
% SECTION 14 - METHOD 1: ITERATIVE NLA WITH LINEAR TRACTION
%     FORCE, PEAK AT PILE FACE (NLAMethod1)
%
% PHYSICAL MODEL (Hammett 2017, Section 3.8.3):
% The link element has a LINEAR traction force distribution
% that PEAKS AT THE PILE FACE and is zero at the idealized
% beam section. This means most load sharing between the PSTB
% and surrounding cap concrete occurs very close to the pile
% face → minimizes deflection at the pile face → highest
% stiffness of the three link element methods.
%
% TRACTION DISTRIBUTION:
% The total traction on the element = Q_LinkTotal (lb).
% With a triangular distribution peaking at the pile face:
% Force at pile face node = (2/3) × Q_LinkTotal
% Force at idealized node = (1/3) × Q_LinkTotal
% The midpoint of the element (x = L_pst/2) carries:
% P_at_midpoint = P_pile_face + (3/4) × Q_LinkTotal
% (The 3/4 factor is the integral-average position for a
% triangle peaking at x=0: centroid at x = L_pst/3 from face,
% so midpoint correction = 3/4 of Q.)

```

```
%
% ITERATION ALGORITHM:
% 1. Solve  $K \times \Delta = F + Q\_LinkPileFace$  for displacements  $\Delta$ .
% 2. Compute spring (bearing) forces  $P\_PileFace = K\_spring \times \Delta - Q\_LinkPileFace$ .
% 3. For each node i:
%     a. Compute ReactionError = required link reaction – current link reaction.
%         Required reaction = PCapFbrLeft or PCapFbrRight (from cap fiber forces).
%         Current reaction =  $-(P\_PileFace + Q\_LinkTotal)$  at that node.
%     b. Update Q_LinkTotal and Q_LinkPileFace with the error fraction.
%     c. Compute midpoint stress and update spring tangent stiffness.
% 4. Reassemble K_GlobalComplete_local with new spring stiffnesses.
% 5. Repeat until  $\max(|ReactionError|) < 0.1$  lb.
%
% OUTPUTS stored in CombinedResults_M1:
% Row 1: iteration count
% Row 2: max pile-face bearing stress (psi)
% Row 3: moment equilibrium check (should  $\approx 0$ )
% Row 4: shear equilibrium check (should  $\approx 0$ )
% Rows 5..end: full displacement vector  $\Delta$  (DOFS entries)
%% =====

fprintf('\n--- Running Method 1 ---\n')

K_GSO_init_M1 = K_GSO; % save original elastic spring matrix for Method 1

% Results matrix: 4 summary rows + DOFS displacement rows, one column per load
case
CombinedResults_M1 = zeros(4 + DOFS, NoPileForceEvaluationPoints);

for j = 1:NoPileForceEvaluationPoints
```

```
Count_j      = 0;          % iteration counter for this load case
Q_LinkTotal  = zeros(DOFS, 1); % total traction force on each link element (lb)
                % Q_LinkTotal(dof) = total load-sharing force at node dof
Q_LinkPileFace = zeros(DOFS, 1); % portion of Q_LinkTotal applied at the pile face
node

                % = (2/3) × Q_LinkTotal for Method 1
ReactionError = 1;          % initialize > convergence tolerance (0.1 lb)

% Local spring and global stiffness matrices (reset each load case)
K_SpringsLocal      = K_GSO_init_M1;
K_GlobalComplete_local = K_GlobalElementsOnly + K_SpringsLocal;

% Working arrays updated each iteration
Q_LinkTotalNew      = zeros(DOFS/2, 1); % updated Q_LinkTotal (per node)
Q_LinkPileFaceNew    = zeros(DOFS/2, 1); % updated Q_LinkPileFace (per node)
k_SpringMidpoint     = zeros(DOFS/2, 1); % updated spring stiffness at midpoint strain
sigma_PileFace       = zeros(DOFS/2, 1); % bearing stress at pile face (psi)

% Solution vectors (pre-allocated)
Delta_j              = zeros(DOFS, 1); % displacement vector (in, rad)
P_PileFace_j         = zeros(DOFS, 1); % bearing force vector (lb)
SumMoments_j         = 0;              % moment equilibrium residual (lb-in)
SumShears_j          = 0;              % shear equilibrium residual (lb)
sigma_PileFaceMax_j  = 0;              % max bearing stress this load case (psi)

while ReactionError >= 0.1 % convergence criterion: max error < 0.1 lb

    % --- Step 1: Solve for displacements ---
    % F_Pile(:,j) = applied external load (shear + moment at base node).
    % Q_LinkPileFace = equivalent nodal forces from traction distribution
    % applied AT the pile face nodes (corrects spring forces at pile face).
```

```

Delta_j = K_GlobalComplete_local \ (F_Pile(:,j) + Q_LinkPileFace);

% --- Step 2: Compute bearing forces at pile face ---
% P = K_spring × Δ - Q_LinkPileFace
% Subtracting Q_LinkPileFace removes the artificially added face force
% to recover the true bearing force at the pile face.
P_PileFace_j = K_SpringsLocal * Delta_j - Q_LinkPileFace;

% --- Step 3: Compute reaction error and update traction forces ---
ReactionError_vec = zeros(DOFS/2, 1);
for i = 1:DOFS/2
    dof = 2*i - 1; % translational DOF index for node i

    % Required link element reaction at the idealized section =
    % cap fiber force from frame analysis (PCapFbrLeft or PCapFbrRight),
    % depending on which side of the pile node i is bearing against.
    % If P_PileFace > 0 (pile pushes left → bearing on left cap concrete):
    % ReqReaction = PCapFbrLeft (negative = cap pushes back)
    % Current reaction at idealized section = -(P_PileFace + Q_LinkTotal)
    % Error = current - required = -(P_PileFace + Q_LinkTotal) + PCapFbrLeft
    %      = -P_PileFace_j(dof) - Q_LinkTotal(dof) + PCapFbrLeft(i,j)
    ReactionAtIdealizedBeam = -P_PileFace_j(dof) - Q_LinkTotal(dof);
    if P_PileFace_j(dof) >= 0
        ReactionError_vec(i) = ReactionAtIdealizedBeam + PCapFbrLeft(i,j);
    else
        ReactionError_vec(i) = ReactionAtIdealizedBeam - PCapFbrRight(i,j);
    end

    % Update total traction force: add full error
    Q_LinkTotalNew(i) = Q_LinkTotal(dof) + ReactionError_vec(i);

```

```

% Update pile-face traction: Method 1 assigns (2/3) of Q_LinkTotal
% to the pile face node (peak of the triangular distribution at x=0).
Q_LinkPileFaceNew(i) = Q_LinkPileFace(dof) + (2/3) * ReactionError_vec(i);

% --- Update spring stiffness at midpoint strain ---
% Bearing stress at pile face (psi)
sigma_PileFace(i) = P_PileFace_j(dof) / TribAreaPSTB(i);

% Midpoint force = pile face force + (3/4) × total traction
% The (3/4) factor is the centroid position for a triangle peaking at x=0
% over length L_pst: the midpoint (x=L_pst/2) carries 3/4 of the total
% traction integral because the triangle centroid is at L_pst/3 from the face.
P_Midpoint_i = abs(P_PileFace_j(dof) + (3/4) * Q_LinkTotalNew(i));
sigma_Midpoint_i = P_Midpoint_i / TribAreaPSTB(i); % midpoint stress (psi)

% Clamp stress to just below f_co to avoid singularity in strain inversion
sigma_clamped = min(sigma_Midpoint_i, f_co * 0.9999);

% Invert K&M pre-peak formula to get midpoint strain:
%  $\sigma = f_{co} \times [1 - (1 - \epsilon/\epsilon_{co})^{n_c}]$ 
%  $\rightarrow \epsilon = \epsilon_{co} \times [1 - (1 - \sigma/f_{co})^{(1/n_c)}]$ 
eps_Midpoint_i = eps_co * (1 - (1 - sigma_clamped/f_co)^(1/n_c));

% Tangent modulus at midpoint strain (first derivative of K&M curve):
%  $E_{tan} = d\sigma/d\epsilon = (f_{co} \times n_c / \epsilon_{co}) \times (1 - \epsilon/\epsilon_{co})^{(n_c-1)}$ 
E_MidpointTangent_i = (f_co * n_c / eps_co) * ...
    (1 - eps_Midpoint_i/eps_co)^(n_c - 1);

% Updated spring stiffness = (Area × E_tangent) / L_pst
% (axial spring: k = EA/L, where E is now the secant/tangent modulus)
k_SpringMidpoint(i) = (TribAreaPSTB(i) * E_MidpointTangent_i) / L_pst;

```

```
end

% Convergence check: max absolute error across all nodes
REMin    = abs(min(ReactionError_vec));
REMax    = max(ReactionError_vec);
ReactionError = max(REMin, REMax);

% --- Rebuild global stiffness and traction vectors for next iteration ---
Q_LinkTotal  = zeros(DOFS, 1);
Q_LinkPileFace = zeros(DOFS, 1);
K_SpringsLocal = zeros(DOFS, DOFS);
for i = 1:DOFS/2
    dof = 2*i - 1;
    Q_LinkTotal(dof)    = Q_LinkTotalNew(i);
    Q_LinkPileFace(dof) = Q_LinkPileFaceNew(i);
    K_SpringsLocal(dof, dof) = k_SpringMidpoint(i); % updated tangent stiffness
end
K_GlobalComplete_local = K_GlobalElementsOnly + K_SpringsLocal;

% --- Equilibrium check (moment and shear at bottom of cap) ---
% Sum all bearing forces and their moments about the bottom of the cap.
% At convergence, these sums should equal the applied shear and moment
% (within numerical tolerance).
SumMoments_j = -F_Pile(2,j); % initialize with applied moment (opposite sign)
SumShears_j = -F_Pile(1,j); % initialize with applied shear
for m = 1:DOFS/2
    dof_m = 2*m - 1;
    % Moment contribution: bearing force × distance from bottom node
    % distance = (m-1) × L_PE (m=1 is at the bottom face, m=26 at top of pile)
    SumMoments_j = SumMoments_j + P_PileFace_j(dof_m) * (m-1) * L_PE;
    SumShears_j = SumShears_j + P_PileFace_j(dof_m);
end
```

```
end

sigma_PileFaceMax_j = max(sigma_PileFace); % track max bearing stress

Count_j = Count_j + 1;
if Count_j > 1000
    warning('Method 1: iteration limit (1000) exceeded at load case j=%d.', j)
    break
end
end % while loop

% Store results for load case j
CombinedResults_M1(1, j) = Count_j; % iterations to converge
CombinedResults_M1(2, j) = sigma_PileFaceMax_j; % max concrete bearing stress
(psi)
CombinedResults_M1(3, j) = SumMoments_j; % moment equilibrium residual
(lb-in)
CombinedResults_M1(4, j) = SumShears_j; % shear equilibrium residual (lb)
CombinedResults_M1(5:end, j) = Delta_j; % full displacement vector

fprintf(' j=%2d: %3d iters,  $\sigma_{PFmax}$ =%.1f psi, SumM=%.2f lb-in, SumV=%.2f lb\n', ...
        j, Count_j, sigma_PileFaceMax_j, SumMoments_j, SumShears_j)
end

% Unpack Method 1 results
Count_M1 = CombinedResults_M1(1, :);
SigPFMax_M1 = CombinedResults_M1(2, :);
SumMoments_M1 = CombinedResults_M1(3, :);
SumShears_M1 = CombinedResults_M1(4, :);
Delta_Global_M1 = CombinedResults_M1(5:end, :);
```



```
% Rotation at the base of the cap (DOF 2 = rotational DOF of the bottom node)
Rotation_M1 = Delta_Global_M1(2, :);

fprintf('\n--- Method 1 Summary ---\n')
fprintf(' %4s %14s %14s %10s\n', 'j', 'Moment(kip-ft)', 'Rot(mrad)', 'Iterations')
for j = 1:NoPileForceEvaluationPoints
    % Only print rows where concrete has not been overstressed
    if CombinedResults_M1(2, j) <= f_co
        % Connection rotation = arctangent of (relative displacement / embedment length)
        %  $\Delta_{bot}$  = displacement at the bottom face (row 5 = DOF 1)
        %  $\Delta_{top}$  = displacement at the top of pile (row 5+DOFS-2 = DOF DOFS-1)
        %  $\theta = \text{atan}[(\Delta_{bot} - \Delta_{top}) / p_e]$  (rigid body approximation)
        delta_bot = Delta_Global_M1(1, j);
        delta_top = Delta_Global_M1(51, j); % DOF 51 = translational DOF at pile top
        theta_j = atan((delta_bot - delta_top) / p_e);
        fprintf(' %4d %14.2f %14.4f %10d\n', ...
            j, abs(F_Pile(2,j))/12000, abs(theta_j)*1000, Count_M1(j))
    end
end

%% =====
% SECTION 15 - METHOD 2: ITERATIVE NLA WITH UNIFORM TRACTION
%     FORCE (NLAMethod2)
%
% PHYSICAL MODEL (Hammett 2017, Section 3.8.4):
% Uniform traction force along the link element: load sharing
% between the PSTB and surrounding cap concrete is equal at
% every point. Deflection at the pile face is between Methods
% 1 and 3 → stiffness is intermediate.
%
% TRACTION DISTRIBUTION:
```

```
% Force at pile face node = (1/2) × Q_LinkTotal (uniform distribution:
% Force at idealized node = (1/2) × Q_LinkTotal half of total to each end)
% The midpoint carries the average of the two end forces:
% P_at_midpoint = P_pile_face + (1/2) × Q_LinkTotal
% (Midpoint of a uniform distribution = midpoint force level.)
%
% Only the traction fractions differ from Method 1:
% Q_LinkPileFaceNew = Q_LinkPileFace + (1/2) × ReactionError [vs. 2/3 in M1]
% P_Midpoint = |P_PileFace + (1/2) × Q_LinkTotalNew| [vs. 3/4 in M1]
%
% All other logic (convergence criterion, spring update, equilibrium
% check) is identical to Method 1.
%% =====

fprintf('\n--- Running Method 2 ---\n')
K_GSO_init_M2 = K_GSO;
CombinedResults_M2 = zeros(4 + DOFS, NoPileForceEvaluationPoints);

for j = 1:NoPileForceEvaluationPoints
    Count_j = 0;
    Q_LinkTotal = zeros(DOFS, 1);
    Q_LinkPileFace = zeros(DOFS, 1);
    ReactionError = 1;
    K_SpringsLocal = K_GSO_init_M2;
    K_GlobalComplete_local = K_GlobalElementsOnly + K_SpringsLocal;
    Q_LinkTotalNew = zeros(DOFS/2, 1);
    Q_LinkPileFaceNew = zeros(DOFS/2, 1);
    k_SpringMidpoint = zeros(DOFS/2, 1);
    sigma_PileFace = zeros(DOFS/2, 1);
    Delta_j = zeros(DOFS, 1);
    P_PileFace_j = zeros(DOFS, 1);
```

```

SumMoments_j      = 0;
SumShears_j       = 0;
sigma_PileFaceMax_j = 0;

while ReactionError >= 0.1
    Delta_j      = K_GlobalComplete_local \ (F_Pile(:,j) + Q_LinkPileFace);
    P_PileFace_j = K_SpringsLocal * Delta_j - Q_LinkPileFace;

    ReactionError_vec = zeros(DOFS/2, 1);
    for i = 1:DOFS/2
        dof = 2*i - 1;
        ReactionAtIdealizedBeam = -P_PileFace_j(dof) - Q_LinkTotal(dof);
        if P_PileFace_j(dof) >= 0
            ReactionError_vec(i) = ReactionAtIdealizedBeam + PCapFbrLeft(i,j);
        else
            ReactionError_vec(i) = ReactionAtIdealizedBeam - PCapFbrRight(i,j);
        end
        Q_LinkTotalNew(i) = Q_LinkTotal(dof) + ReactionError_vec(i);
        % METHOD 2 TRACTION: uniform → (1/2) of total to pile face node
        Q_LinkPileFaceNew(i) = Q_LinkPileFace(dof) + (1/2) * ReactionError_vec(i);

        sigma_PileFace(i) = P_PileFace_j(dof) / TribAreaPSTB(i);
        % METHOD 2 MIDPOINT: uniform → midpoint = pile face + (1/2) × Q_Total
        P_Midpoint_i = abs(P_PileFace_j(dof) + (1/2) * Q_LinkTotalNew(i));
        sigma_Midpoint_i = P_Midpoint_i / TribAreaPSTB(i);
        sigma_clamped = min(sigma_Midpoint_i, f_co * 0.9999);
        eps_Midpoint_i = eps_co * (1 - (1 - sigma_clamped/f_co)^(1/n_c));
        E_MidpointTangent_i = (f_co * n_c / eps_co) * ...
            (1 - eps_Midpoint_i/eps_co)^(n_c - 1);
        k_SpringMidpoint(i) = (TribAreaPSTB(i) * E_MidpointTangent_i) / L_pst;
    end
end

```

```
REMin      = abs(min(ReactionError_vec));
REMax      = max(ReactionError_vec);
ReactionError = max(REMin, REMax);

Q_LinkTotal = zeros(DOFS, 1);
Q_LinkPileFace = zeros(DOFS, 1);
K_SpringsLocal = zeros(DOFS, DOFS);
for i = 1:DOFS/2
    dof = 2*i - 1;
    Q_LinkTotal(dof) = Q_LinkTotalNew(i);
    Q_LinkPileFace(dof) = Q_LinkPileFaceNew(i);
    K_SpringsLocal(dof, dof) = k_SpringMidpoint(i);
end
K_GlobalComplete_local = K_GlobalElementsOnly + K_SpringsLocal;

SumMoments_j = -F_Pile(2,j);
SumShears_j = -F_Pile(1,j);
for m = 1:DOFS/2
    dof_m = 2*m - 1;
    SumMoments_j = SumMoments_j + P_PileFace_j(dof_m) * (m-1) * L_PE;
    SumShears_j = SumShears_j + P_PileFace_j(dof_m);
end

sigma_PileFaceMax_j = max(sigma_PileFace);
Count_j = Count_j + 1;
if Count_j > 1000
    warning('Method 2: iteration limit (1000) exceeded at load case j=%d.', j)
    break
end
% Early exit if concrete crushes: bearing stress  $\geq f_{co}$ 
```

```

        if sigma_PileFaceMax_j > f_co
            break
        end
    end
end

CombinedResults_M2(1, j) = Count_j;
CombinedResults_M2(2, j) = sigma_PileFaceMax_j;
CombinedResults_M2(3, j) = SumMoments_j;
CombinedResults_M2(4, j) = SumShears_j;
CombinedResults_M2(5:end, j) = Delta_j;
fprintf(' j=%2d: %3d iters,  $\sigma_{PFmax}$ =%.1f psi, SumM=%.2f lb-in, SumV=%.2f lb\n', ...
        j, Count_j, sigma_PileFaceMax_j, SumMoments_j, SumShears_j)
end

Count_M2      = CombinedResults_M2(1, :);
SigPFMax_M2   = CombinedResults_M2(2, :);
SumMoments_M2 = CombinedResults_M2(3, :);
SumShears_M2  = CombinedResults_M2(4, :);
Delta_Global_M2 = CombinedResults_M2(5:end, :);
Rotation_M2 = Delta_Global_M2(2, :);

fprintf('\n--- Method 2 Summary ---\n')
fprintf(' %-4s %-14s %-14s %-10s\n', 'j', 'Moment(kip-ft)', 'Rot(mrad)', 'Iterations')
for j = 1:NoPileForceEvaluationPoints
    if CombinedResults_M2(2, j) <= f_co
        delta_bot = Delta_Global_M2(1, j);
        delta_top = Delta_Global_M2(51, j);
        theta_j = atan((delta_bot - delta_top) / p_e);
        fprintf(' %-4d %-14.2f %-14.4f %-10d\n', ...
                j, abs(F_Pile(2,j))/12000, abs(theta_j)*1000, Count_M2(j))
    end
end

```

```

end

%% =====
% SECTION 16 - METHOD 3: ITERATIVE NLA WITH LINEAR TRACTION
%     FORCE, PEAK AT IDEALIZED BEAM SECTION (NLAMethod3)
%
% PHYSICAL MODEL (Hammett 2017, Section 3.8.5):
%   Linear traction force PEAKING AT THE IDEALIZED BEAM SECTION
%   and zero at the pile face. Most load sharing occurs far from
%   the pile face → pile face deflection is maximized → connection
%   is most flexible (lowest stiffness of the three link methods).
%   This physically corresponds to a scenario where the surrounding
%   cap concrete is relatively weak and the PSTB spreads the
%   stresses gradually with little concentration at the pile face.
%
% TRACTION DISTRIBUTION:
%   Force at pile face node =  $(1/3) \times Q\_LinkTotal$  (peak far from face)
%   Force at idealized node =  $(2/3) \times Q\_LinkTotal$ 
%   The midpoint carries:
%    $P\_at\_midpoint = P\_pile\_face + (1/4) \times Q\_LinkTotal$ 
%   (For a triangle peaking at  $x=L\_pst$ : centroid at  $2L\_pst/3$  from face,
%   so the midpoint at  $L\_pst/2$  has  $1/4$  of total traction added to the
%   pile face value.)
%
% Only the traction fractions differ from Methods 1 and 2:
%    $Q\_LinkPileFaceNew = Q\_LinkPileFace + (1/3) \times ReactionError$  [vs.  $2/3$  in M1,  $1/2$ 
in M2]
%    $P\_Midpoint = |P\_PileFace + (1/4) \times Q\_LinkTotalNew|$  [vs.  $3/4$  in M1,  $1/2$  in M2]
%% =====

fprintf('\n--- Running Method 3 ---\n')

```

```
K_GSO_init_M3    = K_GSO;
CombinedResults_M3 = zeros(4 + DOFS, NoPileForceEvaluationPoints);

for j = 1:NoPileForceEvaluationPoints
    Count_j      = 0;
    Q_LinkTotal   = zeros(DOFS, 1);
    Q_LinkPileFace = zeros(DOFS, 1);
    ReactionError = 1;
    K_SpringsLocal    = K_GSO_init_M3;
    K_GlobalComplete_local = K_GlobalElementsOnly + K_SpringsLocal;
    Q_LinkTotalNew     = zeros(DOFS/2, 1);
    Q_LinkPileFaceNew  = zeros(DOFS/2, 1);
    k_SpringMidpoint   = zeros(DOFS/2, 1);
    sigma_PileFace     = zeros(DOFS/2, 1);
    Delta_j           = zeros(DOFS, 1);
    P_PileFace_j      = zeros(DOFS, 1);
    SumMoments_j       = 0;
    SumShears_j        = 0;
    sigma_PileFaceMax_j = 0;

    while ReactionError >= 0.1
        Delta_j      = K_GlobalComplete_local \ (F_Pile(:,j) + Q_LinkPileFace);
        P_PileFace_j = K_SpringsLocal * Delta_j - Q_LinkPileFace;

        ReactionError_vec = zeros(DOFS/2, 1);
        for i = 1:DOFS/2
            dof = 2*i - 1;
            ReactionAtIdealizedBeam = -P_PileFace_j(dof) - Q_LinkTotal(dof);
            if P_PileFace_j(dof) >= 0
                ReactionError_vec(i) = ReactionAtIdealizedBeam + PCapFbrLeft(i,j);
            else
```

```

    ReactionError_vec(i) = ReactionAtIdealizedBeam - PCapFbrRight(i,j);
end
Q_LinkTotalNew(i) = Q_LinkTotal(dof) + ReactionError_vec(i);
% METHOD 3 TRACTION: peak at idealized section → (1/3) to pile face
Q_LinkPileFaceNew(i) = Q_LinkPileFace(dof) + (1/3) * ReactionError_vec(i);

sigma_PileFace(i) = P_PileFace_j(dof) / TribAreaPSTB(i);
% METHOD 3 MIDPOINT: triangle peak at far end → midpoint = (1/4) of Q_Total
P_Midpoint_i = abs(P_PileFace_j(dof) + (1/4) * Q_LinkTotalNew(i));
sigma_Midpoint_i = P_Midpoint_i / TribAreaPSTB(i);
sigma_clamped = min(sigma_Midpoint_i, f_co * 0.9999);
eps_Midpoint_i = eps_co * (1 - (1 - sigma_clamped/f_co)^(1/n_c));
E_MidpointTangent_i = (f_co * n_c / eps_co) * ...
    (1 - eps_Midpoint_i/eps_co)^(n_c - 1);
k_SpringMidpoint(i) = (TribAreaPSTB(i) * E_MidpointTangent_i) / L_pst;
end

REMin = abs(min(ReactionError_vec));
REMax = max(ReactionError_vec);
ReactionError = max(REMin, REMax);

Q_LinkTotal = zeros(DOFS, 1);
Q_LinkPileFace = zeros(DOFS, 1);
K_SpringsLocal = zeros(DOFS, DOFS);
for i = 1:DOFS/2
    dof = 2*i - 1;
    Q_LinkTotal(dof) = Q_LinkTotalNew(i);
    Q_LinkPileFace(dof) = Q_LinkPileFaceNew(i);
    K_SpringsLocal(dof, dof) = k_SpringMidpoint(i);
end
K_GlobalComplete_local = K_GlobalElementsOnly + K_SpringsLocal;

```



```
SumMoments_j = -F_Pile(2,j);
SumShears_j = -F_Pile(1,j);
for m = 1:DOFS/2
    dof_m = 2*m - 1;
    SumMoments_j = SumMoments_j + P_PileFace_j(dof_m) * (m-1) * L_PE;
    SumShears_j = SumShears_j + P_PileFace_j(dof_m);
end

sigma_PileFaceMax_j = max(sigma_PileFace);
Count_j = Count_j + 1;
if Count_j > 1000
    warning('Method 3: iteration limit (1000) exceeded at load case j=%d.', j)
    break
end
end

CombinedResults_M3(1, j) = Count_j;
CombinedResults_M3(2, j) = sigma_PileFaceMax_j;
CombinedResults_M3(3, j) = SumMoments_j;
CombinedResults_M3(4, j) = SumShears_j;
CombinedResults_M3(5:end, j) = Delta_j;
fprintf(' j=%2d: %3d iters,  $\sigma_{PFmax}$ =%.1f psi, SumM=%.2f lb-in, SumV=%.2f lb\n', ...
        j, Count_j, sigma_PileFaceMax_j, SumMoments_j, SumShears_j)
end

Count_M3 = CombinedResults_M3(1, :);
SigPFMax_M3 = CombinedResults_M3(2, :);
SumMoments_M3 = CombinedResults_M3(3, :);
SumShears_M3 = CombinedResults_M3(4, :);
Delta_Global_M3 = CombinedResults_M3(5:end, :);
```

```

Rotation_M3 = Delta_Global_M3(2, :);

fprintf('\n--- Method 3 Summary ---\n')
fprintf(' %-4s %-14s %-14s %-10s\n', 'j', 'Moment(kip-ft)', 'Rot(mrad)', 'Iterations')
for j = 1:NoPileForceEvaluationPoints
    if CombinedResults_M3(2, j) <= f_co
        delta_bot = Delta_Global_M3(1, j);
        delta_top = Delta_Global_M3(51, j);
        theta_j = atan((delta_bot - delta_top) / p_e);
        fprintf(' %-4d %-14.2f %-14.4f %-10d\n', ...
            j, abs(F_Pile(2,j))/12000, abs(theta_j)*1000, Count_M3(j))
    end
end

%% =====
% SECTION 17 - ROTATION VECTORS, MOMENT VECTORS, NEUTRAL AXIS
%           & RIGID BODY HEIGHT (Methods 1, 2, 3)
%
% Post-processes results from Methods 1–3:
% ThetaM1/2/3: connection rotation (rad) at each valid load case.
% M1/2/3: applied moment (lb-in) at each valid load case.
% YBar_M1/2/3: neutral axis location (in from bottom of cap) at
%           the maximum valid load case.
% h_RB: rigid body height = p_e (embedment depth). Used
%           as the rotation lever arm in Method 4.
%
% "Valid" load cases are those where the maximum bearing stress
% does not exceed f_co (CombinedResults_Mx(2,j) ≤ f_co).
% RC_Mx = index of the last valid load case.
%
% Rotation computation (rigid body approximation):

```

```
%  $\theta = \text{atan}[(\Delta_{\text{bottom}} - \Delta_{\text{top}}) / p_e]$ 
%  $\Delta_{\text{bottom}}$  = lateral displacement at the bottom face of the cap (node 1)
%  $\Delta_{\text{top}}$  = lateral displacement at the top of the pile (node 26)
%  $p_e$  = embedment depth (distance between these two nodes)
%
% Neutral axis:
% At the maximum load, the NA is found from the linear displacement profile:
% If the NA is at height YBar_M1 from the bottom, then displacements vary
% linearly from  $\Delta_{\text{bot}}$  (pile pushes left  $\rightarrow$  compression left) to  $\Delta_{\text{top}}$ 
% (pile pushes right  $\rightarrow$  compression right). The NA is where  $\Delta = 0$ :
%  $Y_{\text{Bar}} = p_e \times \Delta_{\text{bot}} / (\Delta_{\text{bot}} - \Delta_{\text{top}})$ 
% (by similar triangles on the displacement diagram)
%% =====

row_bottom = 5; % row in CombinedResults corresponding to DOF 1 ( $\Delta_{\text{bottom}}$ )
row_top = DOFS + 3; % row corresponding to DOF DOFS-1 ( $\Delta_{\text{top}}$  of pile)
% = 5 + DOFS - 2 = DOFS + 3 (0-based: rows 5..4+DOFS for  $\Delta$ )

% METHOD 1
RC_M1 = find(CombinedResults_M1(2, :) <= f_co, 1, 'last'); % last valid load case index
if isempty(RC_M1); RC_M1 = 0; end
ThetaM1 = zeros(RC_M1, 1);
for j = 1:RC_M1
    delta_bot = CombinedResults_M1(row_bottom, j);
    delta_top = CombinedResults_M1(row_top, j);
    ThetaM1(j) = atan((delta_bot - delta_top) / p_e); % (rad), positive = clockwise
end
M1 = -F_Pile(2, 1:RC_M1)';
% M1 = applied moment magnitude (lb-in), positive, for valid load cases
M1 = -F_Pile(2, 1:RC_M1)';
```

% Neutral axis at maximum valid load (similar triangles on displacement diagram):

% $Y_{\text{Bar}} = p_e \times \Delta_{\text{bot}} / (\Delta_{\text{bot}} - \Delta_{\text{top}})$

db_max = CombinedResults_M1(row_bottom, RC_M1);

dt_max = CombinedResults_M1(row_top, RC_M1);

YBar_M1 = p_e * db_max / (db_max - dt_max);

fprintf('\n--- Method 1: Rotation & Neutral Axis ---\n')

fprintf(' ResultsColumn = %d (of %d load cases)\n', RC_M1, NoPileForceEvaluationPoints)

fprintf(' Max rotation = %.6f rad (%.3f mrad)\n', ThetaM1(end), ThetaM1(end)*1000)

fprintf(' Max moment = %.0f lb-in (%.1f kip-ft)\n', M1(end), M1(end)/12000)

fprintf(' YBar_M1 = %.4f in (neutral axis from bottom of cap)\n', YBar_M1)

% METHOD 2

RC_M2 = find(CombinedResults_M2(2, :) <= f_co, 1, 'last');

if isempty(RC_M2); RC_M2 = 0; end

ThetaM2 = zeros(RC_M2, 1);

for j = 1:RC_M2

 delta_bot = CombinedResults_M2(row_bottom, j);

 delta_top = CombinedResults_M2(row_top, j);

 ThetaM2(j) = atan((delta_bot - delta_top) / p_e);

end

M2 = -F_Pile(2, 1:RC_M2)';

db_max = CombinedResults_M2(row_bottom, RC_M2);

dt_max = CombinedResults_M2(row_top, RC_M2);

YBar_M2 = p_e * db_max / (db_max - dt_max);

fprintf('\n--- Method 2: Rotation & Neutral Axis ---\n')

fprintf(' ResultsColumn = %d (of %d load cases)\n', RC_M2, NoPileForceEvaluationPoints)

fprintf(' Max rotation = %.6f rad (%.3f mrad)\n', ThetaM2(end), ThetaM2(end)*1000)

```

fprintf(' Max moment   = %.0f lb-in  (%.1f kip-ft)\n', M2(end), M2(end)/12000)
fprintf(' YBar_M2      = %.4f in\n', YBar_M2)

% METHOD 3
RC_M3 = find(CombinedResults_M3(2, :) <= f_co, 1, 'last');
if isempty(RC_M3); RC_M3 = 0; end
ThetaM3 = zeros(RC_M3, 1);
for j = 1:RC_M3
    delta_bot = CombinedResults_M3(row_bottom, j);
    delta_top = CombinedResults_M3(row_top, j);
    ThetaM3(j) = atan((delta_bot - delta_top) / p_e);
end
M3 = -F_Pile(2, 1:RC_M3);
db_max = CombinedResults_M3(row_bottom, RC_M3);
dt_max = CombinedResults_M3(row_top, RC_M3);
YBar_M3 = p_e * db_max / (db_max - dt_max);

fprintf('\n--- Method 3: Rotation & Neutral Axis ---\n')
fprintf('      ResultsColumn   = %d      (of %d load cases)\n', RC_M3,
NoPileForceEvaluationPoints)
fprintf(' Max rotation = %.6f rad (%.3f mrad)\n', ThetaM3(end), ThetaM3(end)*1000)
fprintf(' Max moment   = %.0f lb-in (%.1f kip-ft)\n', M3(end), M3(end)/12000)
fprintf(' YBar_M3      = %.4f in\n', YBar_M3)

% h_RB: the rigid body height used in Method 4.
% Equals p_e because the pile segment rotating as a rigid body spans
% exactly the embedment depth, from y=0 (bottom of cap) to y=p_e (top of pile).
h_RB = p_e;
fprintf('\nh_RB (rigid body height) = %.4f in (= p_e)\n', h_RB)

%% =====

```

```
% SECTION 18 - METHOD 4: RIGID BODY BEHAVIOR (AnalysisRB)
%
% PHYSICAL MODEL (Hammett 2017, Section 3.8.6):
%   The embedded pile segment rotates as a rigid body about a
%   neutral axis at height Y_RB from the bottom face of the cap.
%   The rotation induces compressive bearing stresses in the
%   concrete on BOTH sides of the pile (resisting couple), since
%   the pile face presses into the concrete above the NA on one
%   side and below the NA on the other.
%
%   All strains within the embedded zone are compressive (no
%   tension zone at the pile face). The strain profile is linear:
%    $\epsilon(y) = \epsilon_{bf} \times (Y_{RB} - y) / Y_{RB}$     for  $0 \leq y \leq Y_{RB}$  (lower portion)
%    $\epsilon(y) = \epsilon_{bf} \times (y - Y_{RB}) / Y_{RB}$     for  $Y_{RB} \leq y \leq h_{RB}$  (upper portion)
%   where  $\epsilon_{bf}$  = strain at the bottom face ( $y=0$ ), the peak strain.
%
% EQUILIBRIUM SEARCH:
%   Given the applied V and M at the pile base, find the pair
%   ( $\epsilon_{bf}$ ,  $Y_{RB}$ ) that simultaneously satisfies:
%    $\Sigma V = 0$ :  $V_{load} + V_{upper}(\epsilon_{bf}, Y_{RB}) - V_{lower}(\epsilon_{bf}, Y_{RB}) = 0$ 
%    $\Sigma M = 0$ :  $M_{load} + M_{upper}(\epsilon_{bf}, Y_{RB}) + M_{lower}(\epsilon_{bf}, Y_{RB}) = 0$ 
%   (see Hammett 2017 Appendix C for the closed-form integrals of
%   the K&M nonlinear stress profile).
%
%   The search is brute-force on a 2-D grid:
%   NAPrecision    = 501 trial neutral axis locations
%   StrainPrecision = 1500 trial bottom-face strain values
%   Total grid: 501  $\times$  1499 = ~751,000 combinations evaluated.
%   The entry minimizing max(|VEQ|, |MEQ|) is the solution.
%
% VECTORIZED IMPLEMENTATION:
```

```
% All 501 × 1499 combinations are computed simultaneously using
% matrix operations (repmat + elementwise arithmetic), avoiding
% nested loops for speed.
%
% AnalysisRB matrix: (NoPileForceEvaluationPoints+1) × 6
% Column 1: row index of best ( $\epsilon_{bf}$ ,  $Y_{RB}$ ) in the search grid
% Column 2: column index of best ( $\epsilon_{bf}$ ,  $Y_{RB}$ )
% Column 3:  $Y_{RB}$  (in) - neutral axis height from bottom of cap
% Column 4:  $\epsilon_{bf}$  (in/in) - bottom face compressive strain
% Column 5:  $\sigma_{bf}$  (psi) - bottom face compressive stress
% Column 6:  $EQ_{min}$  (lb or lb-in) - minimum residual (goodness of fit)
%% =====

fprintf('\n--- Running Method 4 (Rigid Body Analysis) ---\n')

NARange      = 0.15 * h_RB; % search range for neutral axis location (in).
                % The NA is expected near the midpoint of the
                % embedded segment ( $h_{RB}/2 = p_e/2 = 6$  in).
                %  $\pm 15\%$  of  $h_{RB} = \pm 1.8$  in on each side.
                % If YLocationCheck_RB fails, increase this.
% NAPrecision parameter defined in section 0.18
                % number of trial NA locations in the search grid.
                % Hammett uses 501; finer grids improve accuracy
                % at the cost of memory and compute time.
% StrainPrecision parameter defined in section 0.18
                % number of trial  $\epsilon_{bf}$  values from 0 to  $\epsilon_{co}$ .
                % Hammett uses 1500. The strain step is  $\epsilon_{co}/1499$ .

n_c_RB = E_c * eps_co / f_co; % shape exponent for the rigid body analysis.
                % Same as n_c computed in Section 4; recomputed
                % here explicitly for clarity in the RB context.
```

```

fprintf(' NARange=%g in, NAPrecision=%d, StrainPrecision=%d\n', ...
        NARange, NAPrecision, StrainPrecision)

% --- Build the 2-D search grid (vectorized) ---
% YRB_vec: column vector of NAPrecision trial NA locations (in).
%   Centered on h_RB/2, spanning  $\pm$ NARange/2 (only upward: h_RB/2 to h_RB/2 +
NARange).
%   The NA is expected to be above mid-height when shear shifts it upward.
YRB_vec = h_RB/2 + (NARange/(NAPrecision-1)) .* (0:NAPrecision-1)';

% epsbf_vec2: row vector of StrainPrecision-1 trial bottom-face strains (in/in).
%   Starts at  $\epsilon_{co}/1499$  (excludes zero to avoid division-by-zero in integrals).
%   Upper limit =  $\epsilon_{co}$  (peak concrete strain; beyond this the formulas diverge).
epsbf_vec2 = (eps_co/(StrainPrecision-1)) .* (1:StrainPrecision-1)';

% Expand both vectors to full (NAPrecision  $\times$  (StrainPrecision-1)) matrices
% for vectorized computation of all grid entries simultaneously.
YRB_mat = repmat(YRB_vec, 1, StrainPrecision-1); % each row = same Y_RB value
EBF_mat = repmat(epsbf_vec2', NAPrecision, 1);    % each col = same  $\epsilon_{bf}$  value

% --- Pre-compute integration coefficients ---
% arg_up: normalized strain argument at the TOP of the upper portion (y = h_RB):
%    $\epsilon(h\_RB) / \epsilon_{co} = \epsilon_{bf} \times (h\_RB - Y\_RB) / (\epsilon_{co} \times Y\_RB)$ 
%   Used in the K&M pre-peak formula for the upper compression zone.
arg_up = EBF_mat .* (h_RB - YRB_mat) ./ (eps_co .* YRB_mat);

% arg_lo: normalized strain argument at the BOTTOM face (y = 0):
%    $\epsilon(0) / \epsilon_{co} = \epsilon_{bf} / \epsilon_{co}$ 
%   Constant for all rows (independent of Y_RB) since  $\epsilon(0) = \epsilon_{bf}$  always.
arg_lo = EBF_mat ./ eps_co;

```



```
% coeff_V: coefficient appearing in both VUpper and VLower after K&M integration.
%   = ( $\epsilon_{co} \times Y_{RB}$ ) / ( $\epsilon_{bf} \times (n_c + 1)$ )
%   Comes from integrating  $(1 - \epsilon/\epsilon_{co})^{n_c}$  over a linear strain profile.
coeff_V = (eps_co .* YRB_mat) ./ (EBF_mat .* (n_c_RB + 1));

% coeff_M2: coefficient for the second moment integration term.
%   =  $\epsilon_{co}^2 \times Y_{RB}^2 / (\epsilon_{bf}^2 \times (n_c + 1) \times (n_c + 2))$ 
coeff_M2 = eps_co^2 .* YRB_mat.^2 ./ ...
    (EBF_mat.^2 .* (n_c_RB + 1) .* (n_c_RB + 2));

% Initialize results matrix; row 1 is a zero-state placeholder
AnalysisRB = nan(NoPileForceEvaluationPoints + 1, 6);
AnalysisRB(1, :) = [NaN, NaN, h_RB/2, 0, 0, 0];

for h_load = 1:NoPileForceEvaluationPoints

    if F_Pile(1, h_load) == 0 && F_Pile(2, h_load) == 0
        % Zero load → no rotation, NA stays at centroid, zero strains
        AnalysisRB(h_load+1, :) = [NaN, NaN, h_RB/2, 0, 0, 0];
        continue
    end

    V_load = F_Pile(1, h_load); % applied shear at base of cap (lb)
    M_load = F_Pile(2, h_load); % applied moment at base of cap (lb-in, negative)

    % --- Internal shear forces from rigid body concrete stresses ---
    % VUpper: shear from the concrete in the UPPER zone (y = Y_RB to h_RB).
    %   Integral of  $f_{co} \times [1 - (\epsilon_{bf} \times (y - Y_{RB}) / (\epsilon_{co} \times Y_{RB}))^{n_c}] \times d \times dy$ 
    %   from Y_RB to h_RB, evaluated analytically via K&M integration formula.
    %   d = pile depth (bearing width).
```

```

VUpper = f_co * d * ((h_RB - YRB_mat) + ...
    coeff_V .* abs(1 - arg_up).^(n_c_RB+1) - coeff_V);

% VLower: shear from the concrete in the LOWER zone (y = 0 to Y_RB).
% Integral of f_co × [1 - (ε_bf × (Y_RB-y)/(ε_co×Y_RB))n_c] × d × dy
% from 0 to Y_RB.
VLower = f_co * d * (YRB_mat + ...
    coeff_V .* abs(1 - arg_lo).^(n_c_RB+1) - coeff_V);

% --- Internal moment contributions ---
% MUpper: moment about the bottom face from the upper compression zone.
% MUP1 = geometric term (moment arm × area without stress)
% MUP2 = correction for the nonlinear stress reduction (K&M shape)
MUP1 = (h_RB - YRB_mat).^2/2 + ...
    (h_RB - YRB_mat) .* eps_co .* YRB_mat .* ...
    abs(1 - arg_up).^(n_c_RB+1) ./ (EBF_mat .* (n_c_RB+1));
MUP2 = coeff_M2 .* abs(1 - arg_up).^(n_c_RB+2) - coeff_M2;
MUpper = f_co * d * (MUP1 + MUP2);

% MLower: moment about the bottom face from the lower compression zone.
MLP1 = YRB_mat.^2/2 + ...
    eps_co .* YRB_mat.^2 .* ...
    abs(1 - arg_lo).^(n_c_RB+1) ./ (EBF_mat .* (n_c_RB+1));
MLP2 = coeff_M2 .* abs(1 - arg_lo).^(n_c_RB+2) - coeff_M2;
MLower = f_co * d * (MLP1 + MLP2);

% --- Equilibrium residuals ---
% VEQ: shear residual [V_load + V_upper - V_lower].
% At equilibrium, external shear = net concrete shear force.
% Upper pushes one way, lower pushes the other (resisting couple).
VEQ = abs(V_load + VUpper - VLower);

```

```
% MEQ: moment residual |M_load + M_upper + M_lower|.
% IMPORTANT (v25 fix): MEQ is purely the couple equilibrium.
% M_load (negative) + M_upper + M_lower = 0 at the true equilibrium.
% Both M_upper and M_lower are positive (both zones create moments
% in the same rotational sense about the bottom face). No -V×h/2 term:
% the shear is already handled by VEQ; including it here double-counts
% and shifts the solution by ~28% in rotation.
MEQ = abs(M_load + MUpper + MLower);

% Combined residual: take the larger of shear and moment imbalance at each grid
point.
% The solution is the grid point minimizing this combined residual.
EQReduced = max(VEQ, MEQ);

% Find the grid point with the smallest residual
[EQ_min_h, lin_idx] = min(EQReduced(:));
[row_idx, col_idx] = ind2sub(size(EQReduced), lin_idx);

% Recover the physical values at the best-fit grid point
Y_RB_eq = h_RB/2 + (NARange/(NAPrecision-1)) * (row_idx - 1); % NA height (in)
eps_bf_eq = (eps_co/(StrainPrecision-1)) * col_idx; % bottom strain (in/in)
sigma_bf_eq = f_co * (1 - (1 - eps_bf_eq/eps_co)^n_c_RB); % bottom stress (psi)

AnalysisRB(h_load+1, :) = [row_idx, col_idx, Y_RB_eq, eps_bf_eq, sigma_bf_eq,
EQ_min_h];

fprintf(' h=%2d: Y_RB=%.4f in, eps_bf=%.6f, sigma_bf=%.1f psi, residual=%.2f\n', ...
h_load, Y_RB_eq, eps_bf_eq, sigma_bf_eq, EQ_min_h)

% Stop if the bottom face concrete has crushed ( $\sigma_{bf} \geq f_{co}$ )
```

```
    if sigma_bf_eq >= f_co
        break
    end
end

fprintf('\nAnalysis_RB complete. %d load cases processed.\n', ...
        sum(~isnan(AnalysisRB(2:end,3))))

% --- Identify the last valid load case for Method 4 ---
% A load case is "valid" if:
% (a)  $\epsilon_{bf} > 0$  (non-trivial result), AND
% (b)  $\sigma_{bf} < f_{co}$  (concrete has not crushed), AND
% (c)  $Y_{RB} \neq h_{RB}/2$  (NA has actually moved from the zero-load centroid position)
% Condition (c) excludes the trivially zero-load entry at row 1.
last_data = find(~isnan(AnalysisRB(:,4)), 1, 'last');
if AnalysisRB(last_data, 5) < f_co && abs(AnalysisRB(last_data, 3) - h_RB/2) > 1e-8
    RR = last_data;
else
    RR = last_data - 1;
end
MaxFC_RB = AnalysisRB(RR, 5); % max concrete stress at the last valid load case (psi)

if AnalysisRB(last_data, 5) < f_co && abs(AnalysisRB(last_data, 3) - h_RB/2) > 1e-8
    ResultsRow_RB = last_data;
else
    ResultsRow_RB = last_data - 1;
end
YBar_RB = AnalysisRB(ResultsRow_RB, 3); % NA height at max valid load (in)

fprintf('\n--- Method 4 Key Results ---\n')
fprintf(' MaxFC_RB = %.2f psi (max valid bottom fiber stress)\n', MaxFC_RB)
```

```

fprintf(' YBar_RB = %.4f in (neutral axis from bottom of cap at max load)\n', YBar_RB)

% Sanity check: confirm the NA search range was wide enough.
% If YMax_RB > NALimit_RB, the true NA is outside the search window → increase
NARange.
valid_YRB = AnalysisRB(2:end, 3);
valid_YRB = valid_YRB(~isnan(valid_YRB));
YMax_RB = max(valid_YRB);
NALimit_RB = 0.15 * h_RB + h_RB/2; % upper edge of the search range (in)
if YMax_RB <= NALimit_RB
    YLocationCheck_RB = 'OK';
    fprintf(' YLocationCheck_RB = OK (YMax=%.4f ≤ NALimit=%.4f)\n', YMax_RB,
NALimit_RB)
else
    YLocationCheck_RB = 'NG';
    warning('YLocationCheck_RB = NG: YMax_RB (%.4f) > NALimit_RB (%.4f). Increase
NARange.', ...
        YMax_RB, NALimit_RB)
end

%% =====
% SECTION 19 - METHOD 4: ROTATION, MOMENT VECTORS,
%     FINAL PLOT & MOMENT CAPACITY (CapacityRB)
%
% ROTATION CALCULATION - STRAIN DENSITY (Hammett 2017, Fig. 3-10):
% The connection rotation  $\theta_{RB}$  is computed via the "strain density"
% method: the translational displacement at a point equals the area
% of the strain diagram between the rigid body strain profile and
% the idealized beam strain profile over length  $L_{pst}$ .
%
% Two displacement components are computed:

```

```
%
% A_BottomFace: displacement at y=0 (bottom face of cap), LEFT side.
% The rigid body strain at y=0 is  $\epsilon_{bf}$  (compressive).
% The idealized beam strain at y=0 (LEFT PILE FACE, M_Beam row 1) is:
%  $\epsilon_{beam\_bot} = PSTBStrains(1,i) \times (h_{cap}/YBC(1,i) - 1)$ 
% (This is the hogging beam tensile strain at the bottom face,
% referenced to the left pile face section NA.)
% The area of the triangular strain difference diagram over L_pst:
%  $A\_BottomFace = \frac{1}{2} \times L\_pst \times (\epsilon_{bf} - \epsilon_{beam\_bot})$ 
% Physical meaning: as the pile pushes left at the bottom,
% the concrete deforms. The net deformation beyond what the
% idealized beam already has is the pile face displacement.
%
% A_TopOfPile: displacement at y=p_e (top of pile), RIGHT side.
% The rigid body strain at y=p_e is:
%  $\epsilon_{tp} = \epsilon_{bf} \times (h_{RB}/Y_{RB} - 1)$ 
% (compressive, from linear profile above NA)
% The idealized beam strain at y=p_e (RIGHT PILE FACE, M_Beam row 3) is:
%  $PSTBStrains(3,i) \times (1 - p_e/YBC(3,i))$ 
% (tensile on the sagging right cap; note: at y=p_e above the NA
% on the right side the beam is in tension, adding to the RB strain.)
%  $A\_TopOfPile = \frac{1}{2} \times L\_pst \times (\epsilon_{tp} + \epsilon_{beam\_top})$ 
%
%  $\theta_{RB} = \text{atan}[(A\_BottomFace + A\_TopOfPile) / h_{RB}]$ 
%
% v24 FIX: Both A_BottomFace and A_TopOfPile use the idealized beam
% strains AT THE PILE FACE (M_Beam rows 1 and 3), not at the idealized
% section (rows 2 and 4). The pile face is where the strain density
% triangle baseline is defined (Hammett Appendix C,  $\theta_{RB}$  routine).
%
% CAPACITY (CapacityRB):
```

```
% The maximum moment capacity is found by setting  $\epsilon_{bf} = \epsilon_{co}$ 
% (peak strain) and searching for the NA location that satisfies
% pure moment equilibrium (no applied shear). The resulting moment
% =  $M_{upper} + M_{lower}$  is the theoretical flexural strength.
%% =====

% Strip the placeholder row 1 and work with rows 2..end
AnalysisRB_clean = AnalysisRB(2:end, :);
last_valid = find(AnalysisRB_clean(:,4) > 0, 1, 'last');
if AnalysisRB_clean(last_valid, 5) < f_co && ...
    abs(AnalysisRB_clean(last_valid, 3) - h_RB/2) > 1e-8
    RC_RB = last_valid; % number of valid load cases for Method 4
else
    RC_RB = last_valid - 1;
end

if isempty(last_valid) || RC_RB == 0
    warning('Method 4: no valid load cases found. Check NARange and StrainPrecision.')
    ThetaRB = zeros(1,1);
    MRB = zeros(1,1);
else
    % --- Compute rotation for each valid load case ---
    ThetaRB = zeros(RC_RB, 1);
    for i = 1:RC_RB
        eps_bf_i = AnalysisRB_clean(i, 4); % bottom-face compressive strain (in/in)
        Y_RB_i = AnalysisRB_clean(i, 3); % neutral axis height (in)

        if eps_bf_i == 0
            ThetaRB(i) = 0;
            continue
        end
    end
end
```

```

% Rigid body strain at top of pile (y = p_e = h_RB):
% From linear profile:  $\epsilon(y) = \epsilon_{bf} \times (y - Y_{RB}) / Y_{RB}$  for  $y > Y_{RB}$ 
% At  $y = h_{RB}$ :  $\epsilon_{tp} = \epsilon_{bf} \times (h_{RB}/Y_{RB} - 1)$ 
eps_tp_i = eps_bf_i * (h_RB / Y_RB_i - 1);

% Idealized beam strain at the bottom face (y=0) from the LEFT PILE FACE section.
% PSTBStrains(1,i) = bottom-fiber tensile strain in the cap at left pile face.
% PSTBYbarCap(1,i) = NA height for the left pile face section.
% Strain at y=0 in the hogging section:
%  $\epsilon_c(y=0) = \epsilon_b \times (0 - y_{bar}) / y_{bar} = -\epsilon_b$  (tension at bottom, negative in RB
convention)
% But for the strain density diagram we need the compressive side analog:
%  $\epsilon_{beam\_bot} = PSTBStrains(1,i) \times (h_{cap}/PSTBYbarCap(1,i) - 1)$ 
% This gives the COMPRESSIVE strain at the TOP of the cap at the left pile face
% section, which is the idealized beam strain contribution at y=0 from the right
% (compressive) side. The sign convention aligns with the rigid body compression.
eps_beam_bot = PSTBStrains(1, i) * (h_cap / PSTBYbarCap(1, i) - 1);

% A_BottomFace: triangular area of strain difference at the bottom face.
% The rigid body pushes the bottom left by  $\epsilon_{bf}$ .
% The idealized beam already has  $\epsilon_{beam\_bot}$  at the same point.
% Net strain difference =  $\epsilon_{bf} - \epsilon_{beam\_bot}$ .
% Area of triangle over  $L_{pst} = \frac{1}{2} \times L_{pst} \times (\epsilon_{bf} - \epsilon_{beam\_bot})$ .
A_BottomFace = 0.5 * L_pst * (eps_bf_i - eps_beam_bot);

% Idealized beam strain at the top of the pile (y = p_e) from the RIGHT PILE FACE.
% PSTBStrains(3,i) = bottom-fiber tensile strain at right pile face.
% At  $y = p_e$  (above NA on sagging right cap), the strain is tensile:
%  $\epsilon_{beam\_top} = PSTBStrains(3,i) \times (1 - p_e/PSTBYbarCap(3,i))$ 
% When  $p_e < PSTBYbarCap(3,i)$ , this is positive (tension), which ADDS to

```



```

% the RB compressive strain at the top of the pile (the two effects reinforce).

% A_TopOfPile: triangular area at the top of the pile.
% Rigid body compressive strain at  $y=p_e = \epsilon_{tp_i}$ .
% Idealized beam tensile strain adds to the effective deformation.
A_TopOfPile = 0.5 * L_pst * ...
    (eps_tp_i + PSTBStrains(3, i) * (1 - p_e / PSTBYbarCap(3, i)));

% Connection rotation = atan of total relative displacement / embedment depth
ThetaRB(i) = atan((A_BottomFace + A_TopOfPile) / h_RB);
end
end

MRB = -F_Pile(2, 1:RC_RB); % applied moment magnitude for valid load cases (lb-in)

fprintf('\n--- Method 4: Rotation & Moment ---\n')
fprintf(' RC_RB      = %d (of %d load cases)\n', RC_RB, NoPileForceEvaluationPoints)
fprintf(' Max rotation = %.6f rad (%.3f mrad)\n', ThetaRB(end), ThetaRB(end)*1000)
fprintf(' Max moment   = %.0f lb-in (%.1f kip-ft)\n', MRB(end), MRB(end)/12000)
fprintf(' %-4s %-16s %-16s %-14s\n', 'i', 'Moment(in-lbs)', 'Moment(kip-ft)', 'Rot(rad)')
for i = 1:RC_RB
    fprintf(' %-4d %-16.1f %-16.2f %-14.6f\n', i, MRB(i), MRB(i)/12000, ThetaRB(i))
end

% --- CapacityRB: pure moment capacity at  $\epsilon_{bf} = \epsilon_{co}$  ---
% Set the bottom-face strain to  $\epsilon_{co}$  (peak concrete strain) and search
% only for the NA location that balances moments. This gives the theoretical
% flexural strength of the connection (Hammett 2017, Section 3.9).
NARange_Cap = 0.10 * h_RB; % narrower search range for capacity ( $\pm 10\%$  of  $h_{RB}$ )
% NAPrecision_Cap parameter defined in section 0.19
    % number of trial NA locations for capacity search

```

```

eps_bf_cap    = eps_co;      % fix bottom-face strain at peak value

% Trial NA locations for capacity search
YRB_cap_vec    =    h_RB/2    +    (NARange_Cap/(NAPrecision_Cap-1))    .*
(0:NAPrecision_Cap-1)';

% Strain arguments at the upper and lower limits of the embedded zone
arg_up_cap    = eps_bf_cap * (h_RB - YRB_cap_vec) ./ (eps_co .* YRB_cap_vec);
arg_lo_cap    = eps_bf_cap / eps_co; % = 1.0 by definition ( $\epsilon_{bf} = \epsilon_{co} \rightarrow \arg_{lo} = 1$ )

% Integration coefficients for capacity
coeff_V_cap    = (eps_co .* YRB_cap_vec) ./ (eps_bf_cap .* (n_c_RB + 1));
coeff_M2_cap    = eps_co^2 .* YRB_cap_vec.^2 ./ ...
    (eps_bf_cap^2 .* (n_c_RB+1) .* (n_c_RB+2));

% Upper zone moment contribution at  $\epsilon_{bf} = \epsilon_{co}$ 
MUP1_cap    = (h_RB - YRB_cap_vec).^2/2 + ...
    (h_RB - YRB_cap_vec) .* eps_co .* YRB_cap_vec .* ...
    abs(1 - arg_up_cap).^(n_c_RB+1) ./ (eps_bf_cap .* (n_c_RB+1));
MUP2_cap    = coeff_M2_cap .* abs(1 - arg_up_cap).^(n_c_RB+2) - coeff_M2_cap;
MUpper_cap    = f_co * d * (MUP1_cap + MUP2_cap);

% Lower zone moment contribution at  $\epsilon_{bf} = \epsilon_{co}$ 
% Note: arg_lo_cap = 1  $\rightarrow$  (1 - arg_lo_cap) = 0  $\rightarrow$  both MLP2_cap terms cancel.
% The lower zone effectively becomes a rectangular stress block (all at f_co).
MLP1_cap    = YRB_cap_vec.^2/2 + ...
    eps_co .* YRB_cap_vec.^2 .* ...
    abs(1 - arg_lo_cap).^(n_c_RB+1) ./ (eps_bf_cap .* (n_c_RB+1));
MLP2_cap    = coeff_M2_cap .* abs(1 - arg_lo_cap).^(n_c_RB+2) - coeff_M2_cap;
MLower_cap    = f_co * d * (MLP1_cap + MLP2_cap);

```

```
% For pure moment (no shear):  $\Sigma V = 0 \rightarrow V_{Upper} = V_{Lower}$ .
% MEQ_cap = |MUpper - MLower|: moment imbalance used to find the NA that
% gives equal upper and lower shear forces (pure couple condition).
MEQ_cap      = abs(MUpper_cap - MLower_cap);

% Total resisting moment = MUpper + MLower at the equilibrium NA
MomentCapacity_vec = MUpper_cap + MLower_cap;

% Find the NA location where the couple is most nearly balanced
[MEQ_min_cap, Row_cap] = min(MEQ_cap);
Y_RB_cap = YRB_cap_vec(Row_cap);      % NA height at capacity (in)
MomCap_RB = MomentCapacity_vec(Row_cap); % moment capacity (lb-in)

Results_Cap = [Row_cap, Y_RB_cap, eps_bf_cap, MEQ_min_cap, MomCap_RB];

fprintf('\n--- CapacityRB (Pure Moment Capacity, Method 4) ---\n')
fprintf(' Y_RB    = %.4f in (neutral axis at capacity)\n', Y_RB_cap)
fprintf(' eps_bf    = %.6f (= eps_co, peak strain)\n', eps_bf_cap)
fprintf(' MEQ_min    = %.2f (force balance residual)\n', MEQ_min_cap)
fprintf(' MomCap_RB = %.0f lb-in (0.1f kip-ft)\n', MomCap_RB, MomCap_RB/12000)

% --- Final combined moment-rotation plot ---
% All four methods on one plot for direct comparison.
% Valid data only (load cases where concrete has not crushed).
% M_Yield_NL horizontal line = the design limit for this connection scenario.
figure
plot(ThetaM1, M1, ...
     'g-.', 'LineWidth', 2, 'MarkerSize', 6); hold on
plot(ThetaM2, M2, ...
     'r-', 'LineWidth', 2)
plot(ThetaM3, M3, ...
```

```

    'b--', 'LineWidth', 2)
plot(ThetaRB, MRB, ...
    'k-', 'LineWidth', 2)
yline(M_Yield_NL, 'k--', 'LineWidth', 1.5, 'Label', 'M_{Yield}', ...
    'LabelHorizontalAlignment', 'right')
xlabel('Rotation (radians)')
ylabel('Moment (in-lbs)')
title(sprintf('Bending Case %d, %s, %g" Embed., f'_c = %g psi', ...
    CapBendingScenario, Pile_Name, p_e, f_co)))
legend('Method 1 (Link Element w/LTF - Peak at Pile Face)', ...
    'Method 2 (Link Element with Uniform Traction Force)', ...
    'Method 3 (Link Element w/LTF - Peak at Idealized Beam Section)', ...
    'Method 4 (Rigid Body Behavior)', ...
    'Location', 'northwest')
grid on
xlim([0, max([ThetaM1(end), ThetaM2(end), ThetaM3(end), ThetaRB(end))] * 1.1)])
ylim([0, M_Yield_NL * 1.15])

figure
plot(ThetaM1, M1/12000, 'g-.', 'LineWidth', 2); hold on
plot(ThetaM2, M2/12000, 'r-', 'LineWidth', 2)
plot(ThetaM3, M3/12000, 'b--', 'LineWidth', 2)
plot(ThetaRB, MRB/12000, 'k-', 'LineWidth', 2)
yline(M_Yield_NL/12000, 'k--', 'LineWidth', 1.5, 'Label', 'M_{Yield}', ...
    'LabelHorizontalAlignment', 'right')
xlabel('Rotation (radians)')
ylabel('Moment (kip-ft)')
title(sprintf('Bending Case %d, %s, %g" Embed., f'_c = %g psi', ...
    CapBendingScenario, Pile_Name, p_e, f_co)))
legend('Method 1 (Link Element w/LTF - Peak at Pile Face)', ...
    'Method 2 (Link Element with Uniform Traction Force)', ...

```

```

'Method 3 (Link Element w/LTF - Peak at Idealized Beam Section)', ...
'Method 4 (Rigid Body Behavior)', ...
'Location', 'northwest')

grid on
xlim([0, max([ThetaM1(end), ThetaM2(end), ThetaM3(end), ThetaRB(end))] * 1.1)])
ylim([0, M_Yield_NL/12000 * 1.15])

%% =====
% SECTION 20 - ROTATIONAL STIFFNESS SUMMARY
%
% Secant rotational stiffness at the last valid load case:
% K_rot = M / theta (lb-in/rad)
% Reported for all four methods.
%% =====
fprintf('\n--- Section 20: Rotational Stiffness Summary ---\n')
fprintf(' %-12s %-22s %-18s\n', 'Method', 'K_rot (lb-in/rad)', 'K_rot (kip-ft/rad)')

K_rot_M1 = M1(end) / ThetaM1(end);
K_rot_M2 = M2(end) / ThetaM2(end);
K_rot_M3 = M3(end) / ThetaM3(end);
K_rot_RB = MRB(end) / ThetaRB(end);

fprintf(' %-12s %-22.0f %-18.2f\n', 'Method 1', K_rot_M1, K_rot_M1/12000)
fprintf(' %-12s %-22.0f %-18.2f\n', 'Method 2', K_rot_M2, K_rot_M2/12000)
fprintf(' %-12s %-22.0f %-18.2f\n', 'Method 3', K_rot_M3, K_rot_M3/12000)
fprintf(' %-12s %-22.0f %-18.2f\n', 'Method 4', K_rot_RB, K_rot_RB/12000)

%% =====
% LOCAL FUNCTIONS (must remain at the end of the script file;
% MATLAB requires all local functions to follow the last line
% of the script body)

```

```
%% =====

function fs = steel_stress(es, E_s, f_y, f_su, eps_sh, eps_su, P)
% STEEL_STRESS Karthik & Mander (2011) nonlinear steel stress-strain model.
%
% INPUTS:
% es    – strain (in/in). Scalar or array. Positive = tension.
% E_s   – elastic modulus (psi)
% f_y   – yield stress (psi)
% f_su  – ultimate (rupture) stress (psi)
% eps_sh – strain at onset of strain hardening (in/in)
% eps_su – ultimate rupture strain (in/in)
% P     – K&M shape exponent (dimensionless); controls hardening curvature.
%        Computed in the main script as  $P = E_{sh} * (eps_{su} - eps_{sh}) / (f_{su} - f_y)$ .
%
% OUTPUT:
% fs    – stress (psi), same size as es.
%
% MODEL STRUCTURE:
% Two additive terms:
% Term 1: elastic-to-yield transition (sigmoid-like, sharp at  $\epsilon_y$ ).
%  $= E_s * \epsilon / (1 + |E_s * \epsilon / f_y|^{20})^{0.05}$ 
% The ^20 exponent makes the transition very abrupt; ^0.05 = 1/20
% ensures the formula is self-consistent (slope at origin =  $E_s$ ).
% Term 2: strain-hardening branch (rises from 0 at  $\epsilon_{sh}$  to  $(f_{su} - f_y)$  at  $\epsilon_{su}$ ).
%  $= (f_{su} - f_y) \times [1 - |\epsilon_{su} - \epsilon|^P / (|\epsilon_{su} - \epsilon_{sh}|^{(20P)} + |\epsilon_{su} - \epsilon|^{(20P)})]^{0.05}$ 
% At  $\epsilon = \epsilon_{sh}$ : numerator  $\approx$  denominator  $\rightarrow$  Term 2  $\approx$  0 (hardening not yet active).
% At  $\epsilon = \epsilon_{su}$ : numerator  $\rightarrow$  0  $\rightarrow$  Term 2  $\rightarrow$   $(f_{su} - f_y)$  (full hardening reached).
%
% NOTE: The function is symmetric (handles both tension and compression by
% the absolute value in Term 1). For compression, pass negative es.
```

```

fs = E_s.*es ./ (1 + abs(E_s.*es./f_y).^20).^0.05 ...
+ (f_su - f_y) .* (1 - abs(eps_su - es).^P ...
./ (abs(eps_su - eps_sh).^(20*P) + abs(eps_su - es).^(20*P)).^0.05);
end

function fc = concrete_stress(ec, f_co, f_c1, f_ct, eps_co, eps_c1, eps_sp, eps_ct, n_c,
n_t)
% CONCRETE_STRESS Karthik & Mander (2011) four-branch concrete model.
%
% INPUTS:
% ec    – strain (in/in). Scalar only (called via arrayfun or in loops).
%       SIGN CONVENTION: positive ec = compression, negative ec = tension.
% f_co   – peak compressive stress (psi)
% f_c1   – stress at end of post-peak softening branch (psi)
% f_ct   – tensile cracking stress / modulus of rupture (psi)
% eps_co – strain at peak compressive stress (in/in)
% eps_c1 – strain at end of post-peak softening (in/in)
% eps_sp – spalling strain (in/in); stress = 0 beyond this
% eps_ct – tensile cracking strain (in/in)
% n_c    – compressive branch shape exponent (dimensionless)
% n_t    – tensile branch shape exponent (dimensionless)
%
% OUTPUT:
% fc     – stress (psi). Positive = compression, negative = tension.
%
% FOUR BRANCHES:
% Branch 1 (tension): ec < 0
% fc = -f_ct × (1 - |1 + ec/eps_ct|^n_t)
% Rises from 0 at ec=0 to -f_ct at ec=-eps_ct (modulus of rupture).
% Post-cracking: stress falls back toward zero (tension softening).
%
```

```
% Branch 2 (pre-peak compression):  $0 \leq ec < eps\_co$ 
%  $fc = f\_co \times (1 - |1 - ec/eps\_co|^{n\_c})$ 
% K&M unified formula: rises from 0 at  $ec=0$ , peaks at  $f\_co$  when  $ec=eps\_co$ .
% Shape controlled by  $n\_c$ ; higher  $n\_c$  = sharper peak.
%
% Branch 3 (post-peak softening):  $eps\_co \leq ec < eps\_c1$ 
%  $fc = f\_co - (f\_co - f\_c1) / (eps\_c1/eps\_co - 1) \times (ec/eps\_co - 1)$ 
% Linear descent from  $f\_co$  at  $ec=eps\_co$  to  $f\_c1$  at  $ec=eps\_c1$ .
%
% Branch 4 (crush to zero):  $eps\_c1 \leq ec < eps\_sp$ 
%  $fc = f\_c1 \times (ec/eps\_co - eps\_sp/eps\_co) / (eps\_c1/eps\_co - eps\_sp/eps\_co)$ 
% Linear descent from  $f\_c1$  at  $ec=eps\_c1$  to 0 at  $ec=eps\_sp$ .
%
% Beyond  $eps\_sp$ :  $fc = 0$  (spalled; no stress carried).
%
% NOTE: The cap analysis limits tensile strain to  $eps\_ct$  and compressive
% stress to  $f\_co$ ; Branches 3 and 4 are never reached in Methods 1–3.
% Method 4 similarly limits  $eps\_bf$  to  $eps\_co$ . Branch 3 and 4 behavior
% is implemented for completeness and for the Section 4 stress-strain plot.
if  $ec/eps\_ct < 0$ 
    % Branch 1: tension softening
     $fc = -f\_ct \times (1 - \text{abs}(1 + ec/eps\_ct)^{n\_t})$ ;
elseif  $ec/eps\_co < 1$ 
    % Branch 2: pre-peak compression (includes  $ec=0 \rightarrow fc=0$ )
     $fc = f\_co \times (1 - \text{abs}(1 - ec/eps\_co)^{n\_c})$ ;
elseif  $ec/eps\_co < eps\_c1/eps\_co$ 
    % Branch 3: post-peak linear softening
     $fc = f\_co - (f\_co - f\_c1) / (eps\_c1/eps\_co - 1) \times (ec/eps\_co - 1)$ ;
elseif  $ec/eps\_co < eps\_sp/eps\_co$ 
    % Branch 4: linear crush to zero
     $fc = f\_c1 \times (ec/eps\_co - eps\_sp/eps\_co) / (eps\_c1/eps\_co - eps\_sp/eps\_co)$ ;
```



```
else
    % Spalled: no stress
    fc = 0;
end
end
```